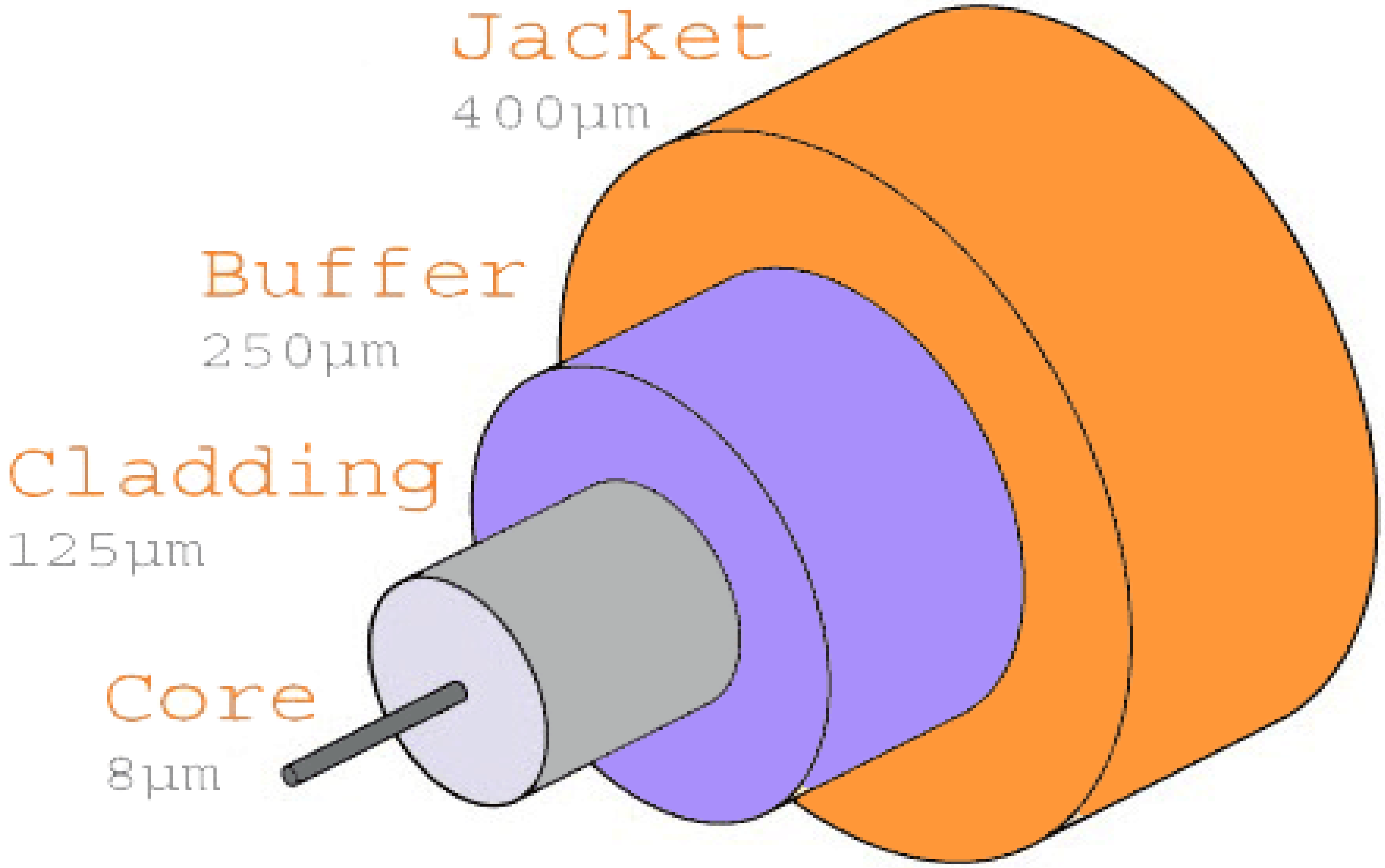
PRINCIPLES OF LIGHT PROPAGATION THROUGH OPTICAL FIBER

introduction

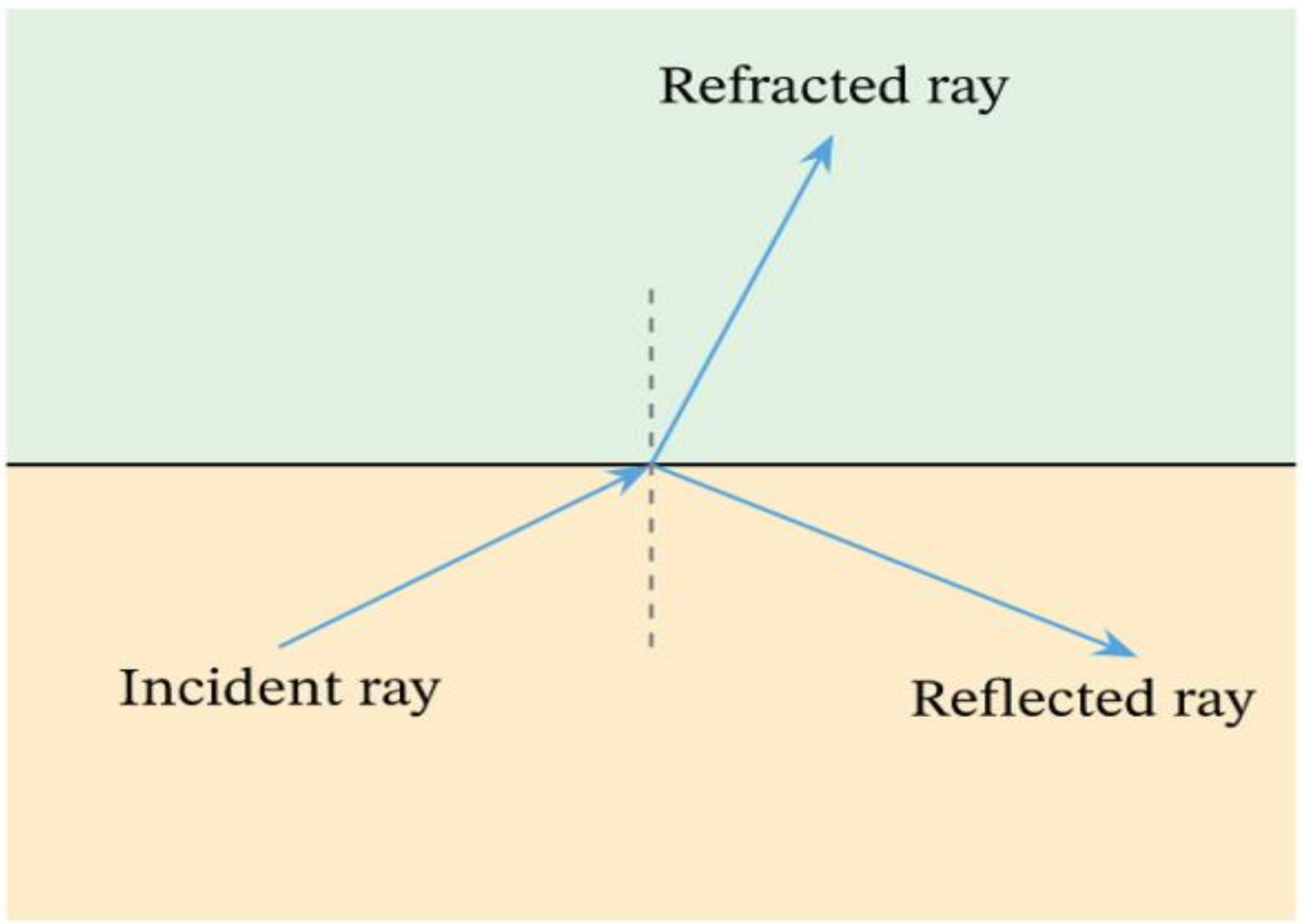
Fiber optic Cable construction



Optical fiber construction

- **Optical fiber construction shows that, the cable consists of two main parts, core and cladding.**
- **Theses two materials have different refractive properties(Refractive Index).**
- **Refractive index (n) tells how *fast or slow* light travels through the material.**
- **Optical boundary is a surface that separates two materials with different refractive indexes.**
- **Most light rays both reflect and(or) refract when they encounter a boundary between two materials.**
- **These depends on refractive index of the material and angle of ray incident.**

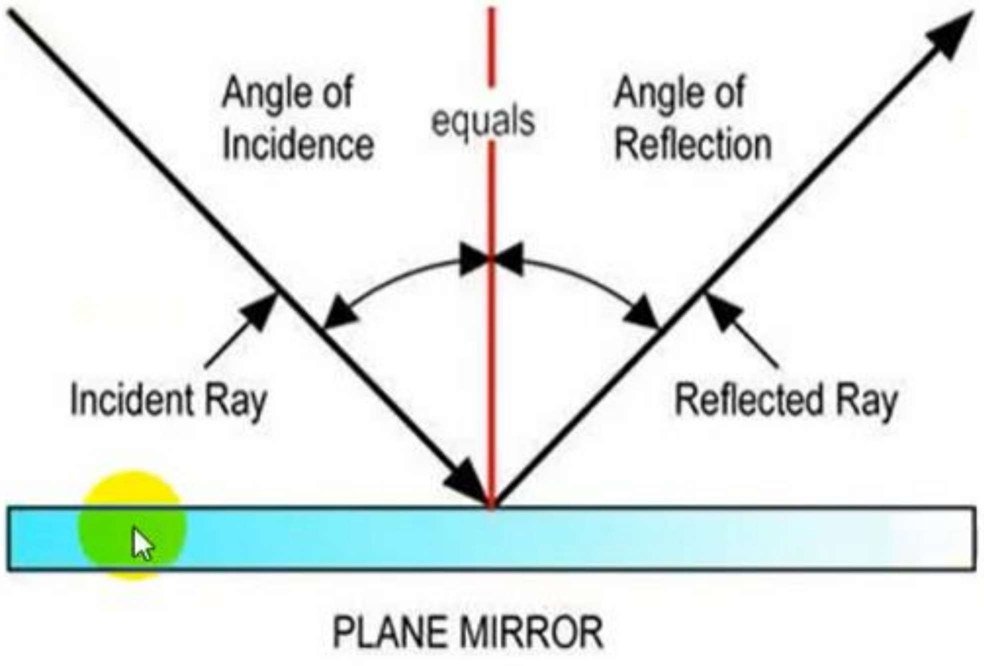
Light propagation through different media with Different Refractive Index



Reflection, Refraction, and Snell's Law

Law of Reflection:

- The angle of incidence equals to the angle of reflection



Reflection, Refraction, and Snell's Law

Refraction:

- Light will bend when it changes mediums because its velocity is changed once it leaves the vacuum.

Refractive Index (n):

- Is the ratio of the speed of light in vacuum to the speed of light in a medium.

$$n = \frac{speed\ of\ light\ in\ the\ vacuum}{speed\ of\ light\ in\ the\ medium}$$

Reflection, Refraction, and Snell's Law

Snell's Law

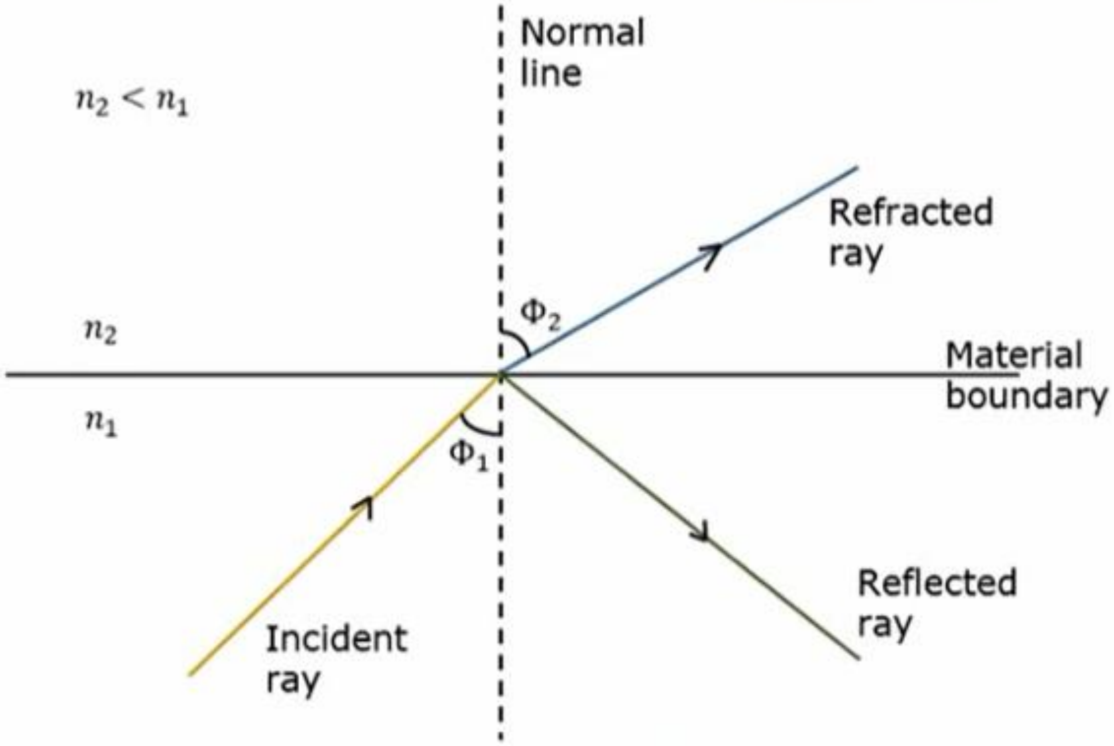
$$n_1 \sin \emptyset_1 = n_2 \sin \emptyset_2$$

n_1: refractive index for incident medium

n_2: refractive index for refracted medium

$\emptyset_1$: angle of incidence

$\emptyset_2$: angle of refraction

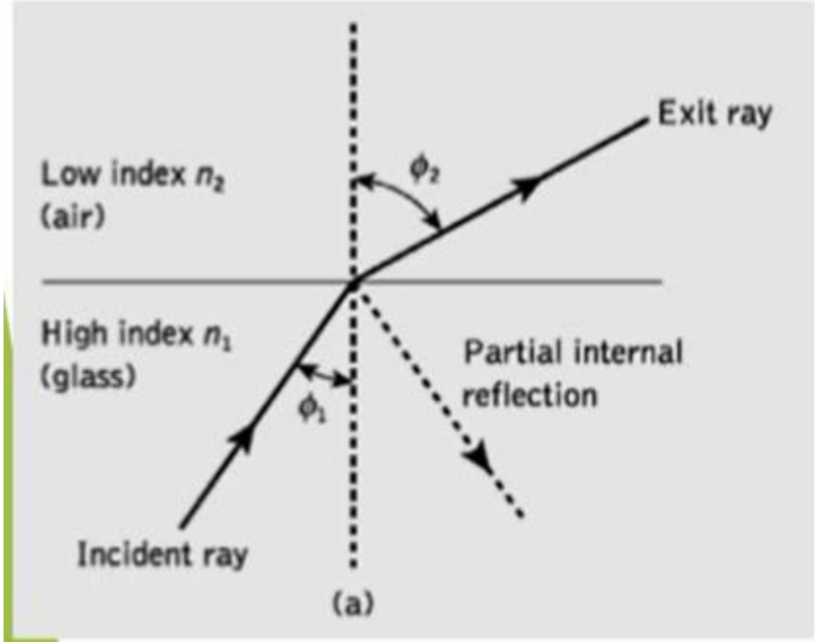


Ray Transmission Theorem/Refraction

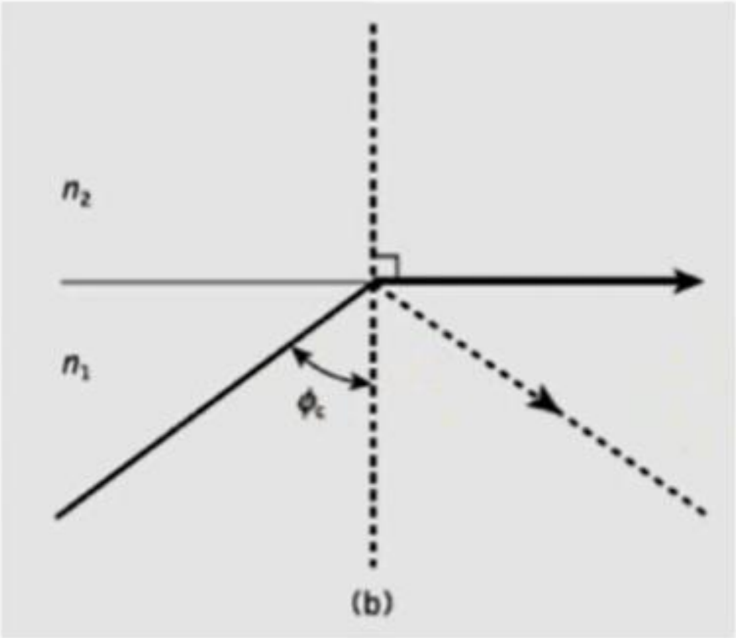
- **When light travels from one medium to another it changes speed and is refracted.**
- **If the light rays are travelling from a less dense material to a dense medium they are refracted towards the normal and if they are travelling from a dense to less dense medium they are refracted away from the normal.**
- **As the angle of incidence increases so does the angle of refraction.**
- **When the angle of incidence reaches a value known as the critical angle the refracted rays travel along the surface of the medium or in other words are refracted to an angle of 90°.**

Ray Transmission Theory

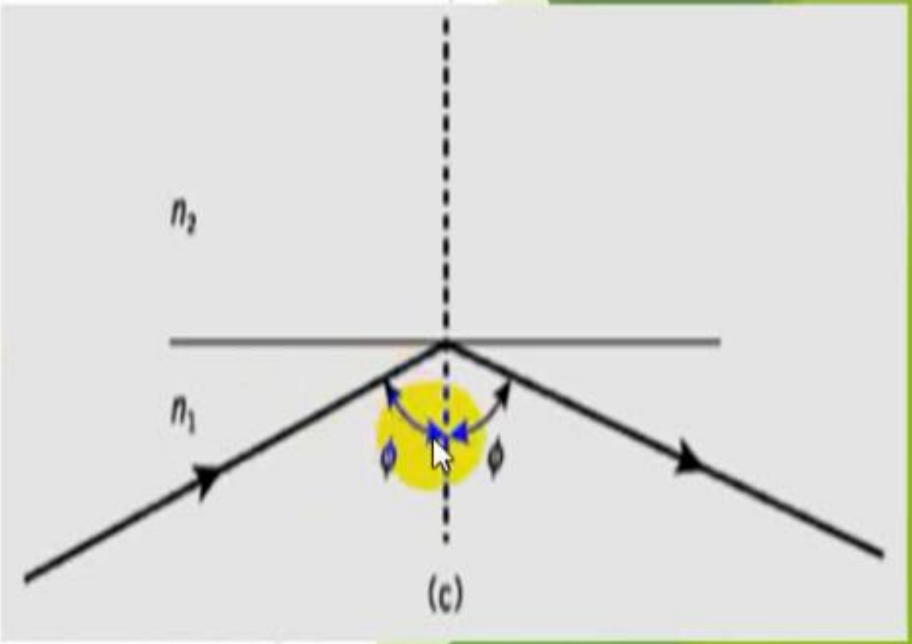
A: Partial Internal Reflection



B: Critical Angle



C: Total Internal Reflection



Critical Angle

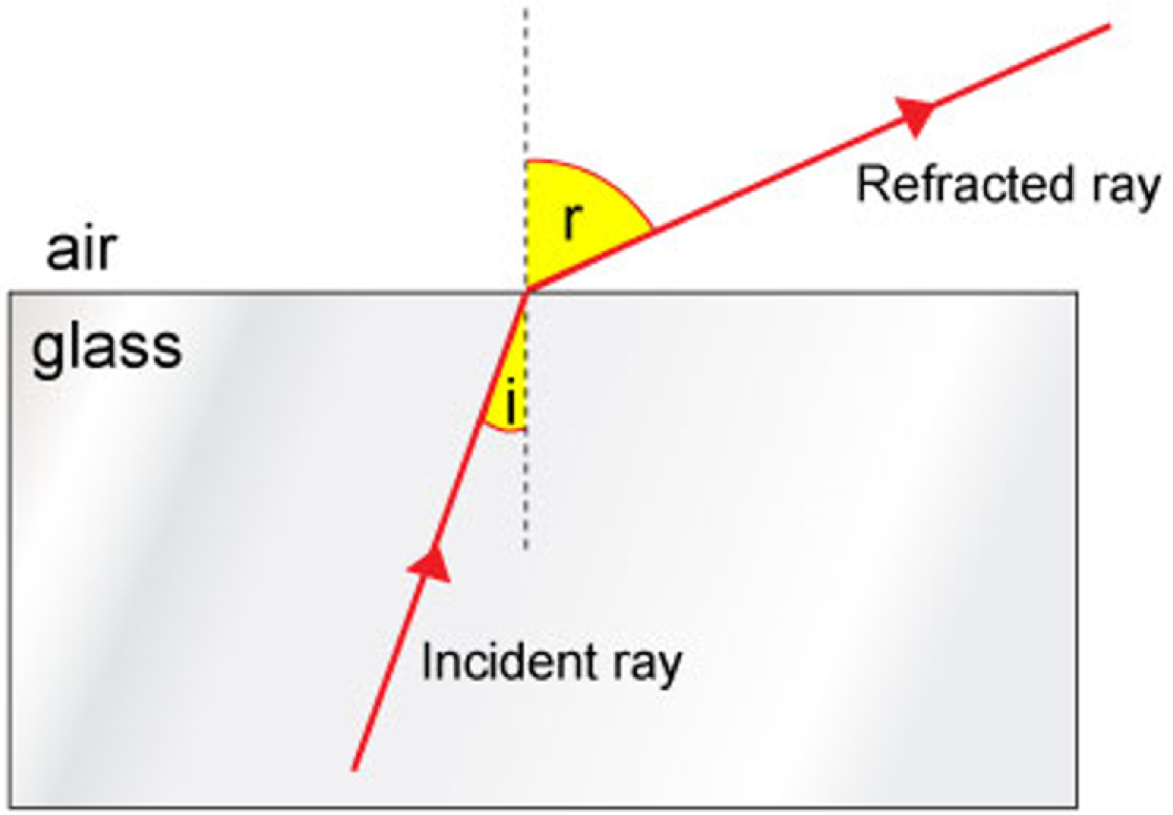
- **When the angle of incidence(Φ1) is progressively increased, there will be progressive increase in refractive angle (Φ2)**
- **At some condition (Φ1) ,** the refractive angle (Φ2) becomes 90 degree to the normal.
- When this happens the refracted ray travels along the interface.
- The angle Φ1 at which Φ2 become 90 is called critical angle and denoted by **Φc**.

Angle of Incident – Critical angle relation

- **There are three cases:**

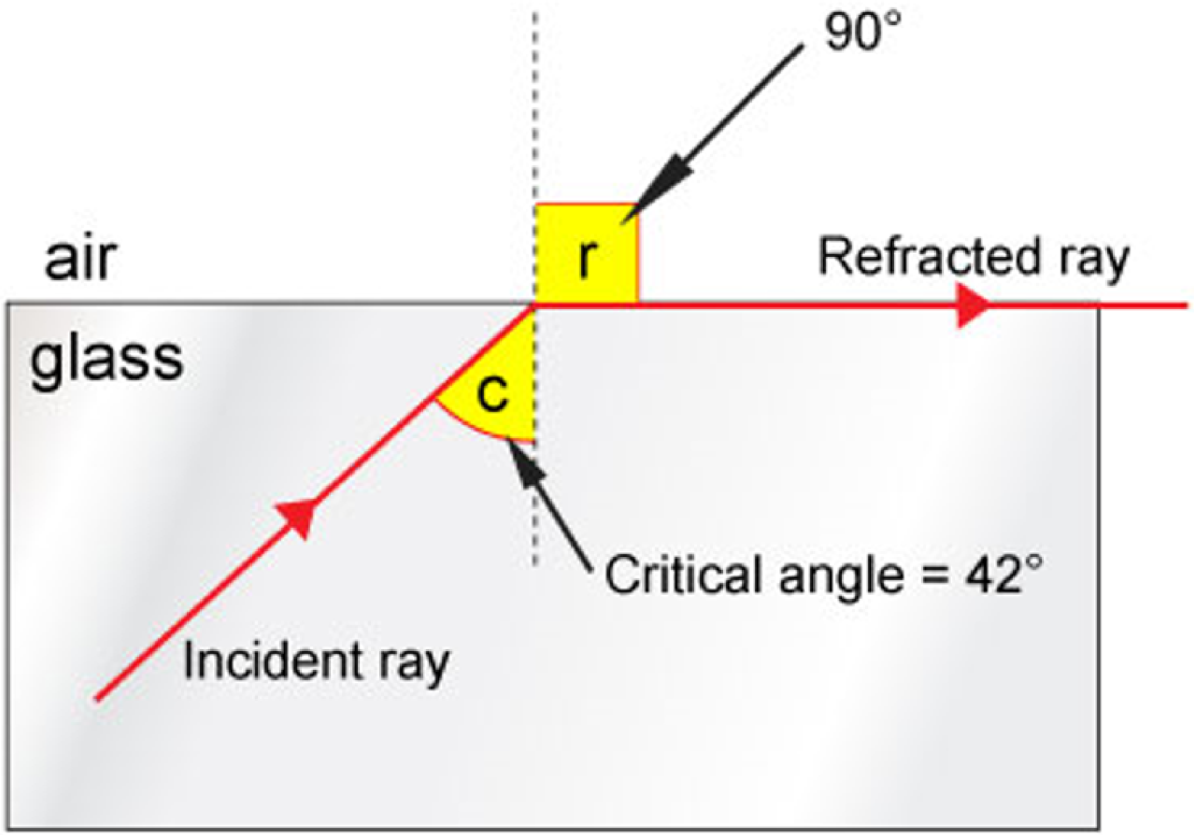
1. **Angle of incidence Less than the critical angle**
2. **Angle of incidence Equal to the critical angle**
3. **Angle of incidence Greater than the critical angle**

1. Angle of incidence less than the critical angle



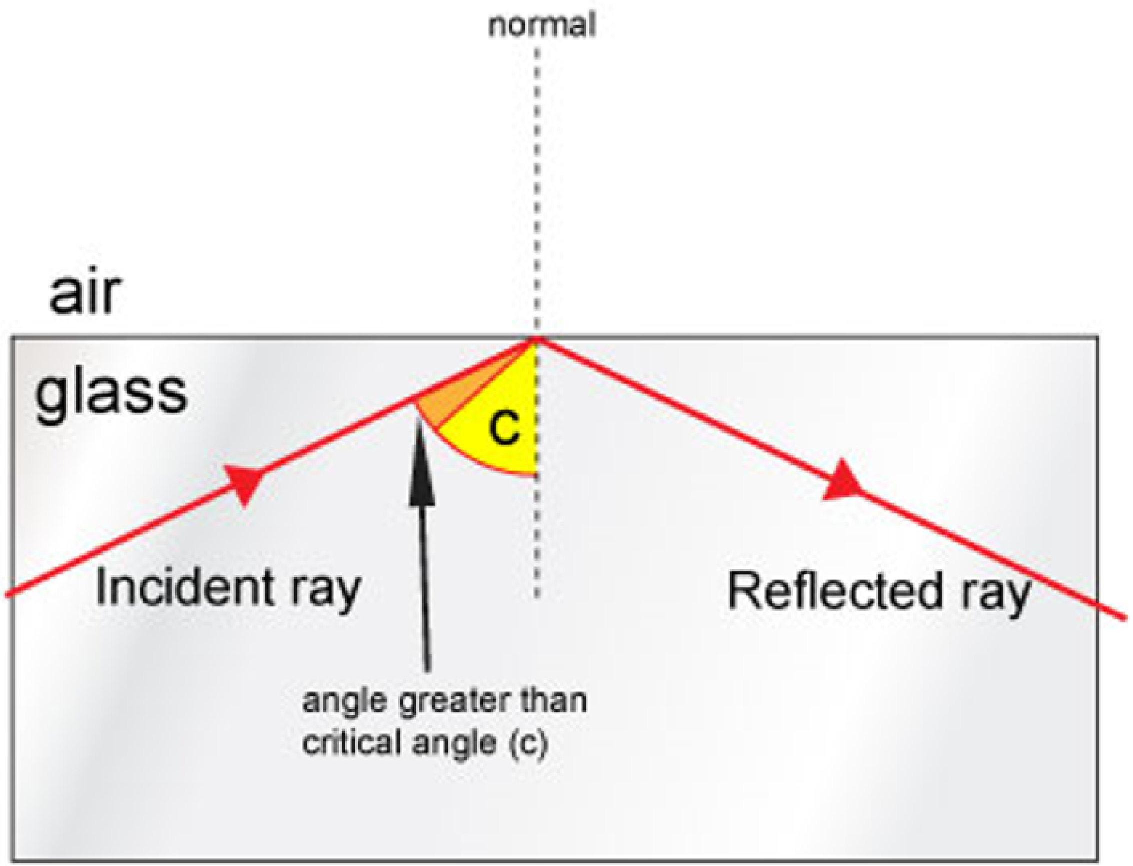
When the angle of incidence of the light ray leaving the glass is less than the critical angle, the light ray speeds up on leaving the glass and is refracted away from the normal.

2. Angle of incidence equal to the critical angle



When the angle of incidence of the light ray reaches the critical angle (42°) the angle of refraction is 90°. The refracted ray travels along the surface of the denser medium in this case the glass.

3. Angle of incidence greater than the critical angle



When the angle of incidence of the light ray is greater than the critical angle then no refraction takes place. Instead, all the light is reflected back into the denser material in this case the glass. This is called **total internal reflection**.

- The **critical angle** is defined as the minimum angle of incidence (ϕ_1) at which the ray strikes the interface of two media and causes an angle of refraction (ϕ_2) equal to 90°. Fig. 2.2.4 shows critical angle refraction.

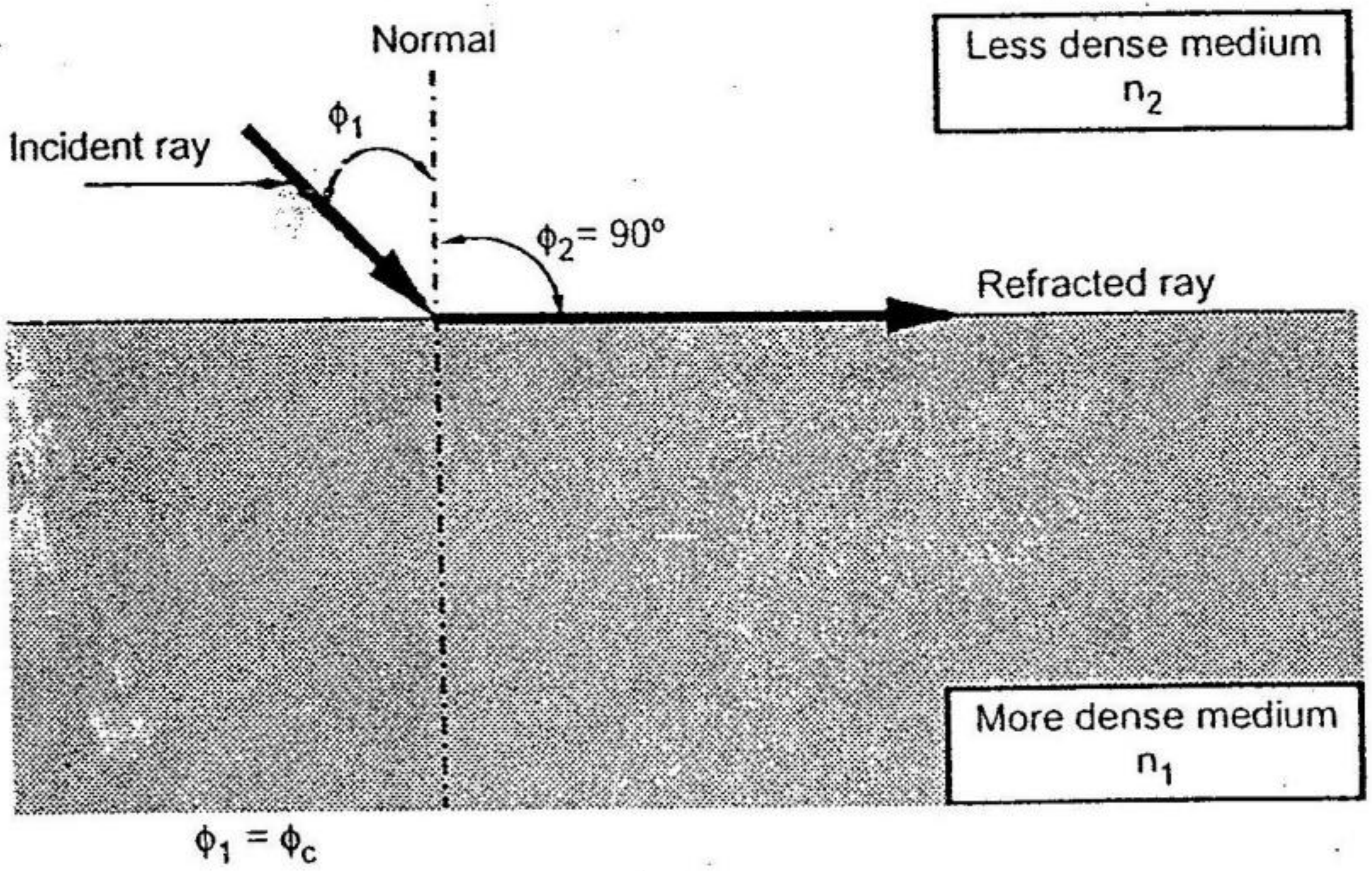


Fig. 2.2.4 Critical angle

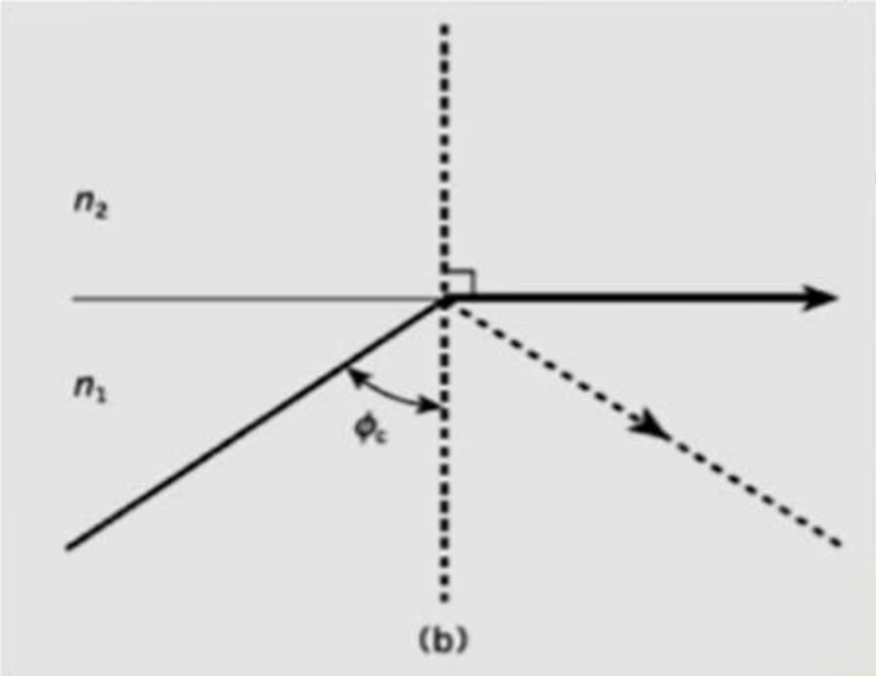
Critical Angle

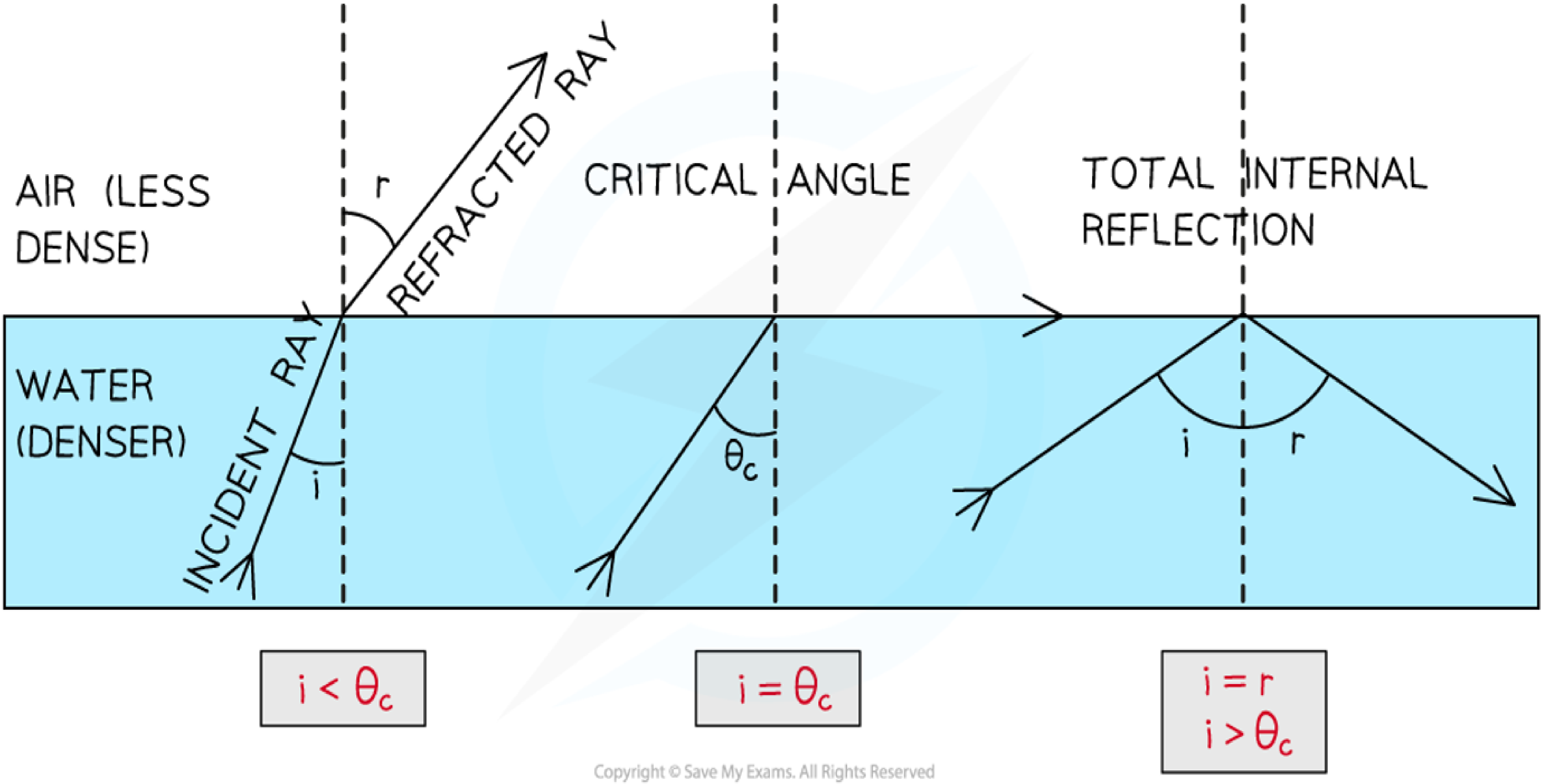
$$n_1 \sin \emptyset_1 = n_2 \sin \emptyset_2$$

$$n_1 \sin \emptyset_c = n_2 \sin 90$$

$$\sin \emptyset_c = \frac{n_2}{n_1}$$

$$\emptyset_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$




AIR (LESS DENSE)
WATER (DENSER)
REFRACTED RAY
INCIDENT RAY
r
i
CRITICAL ANGLE
θc
TOTAL INTERNAL REFLECTION
i
r
i < θc
i = θc
i = r
i > θc
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Total Internal Reflection (TIR)

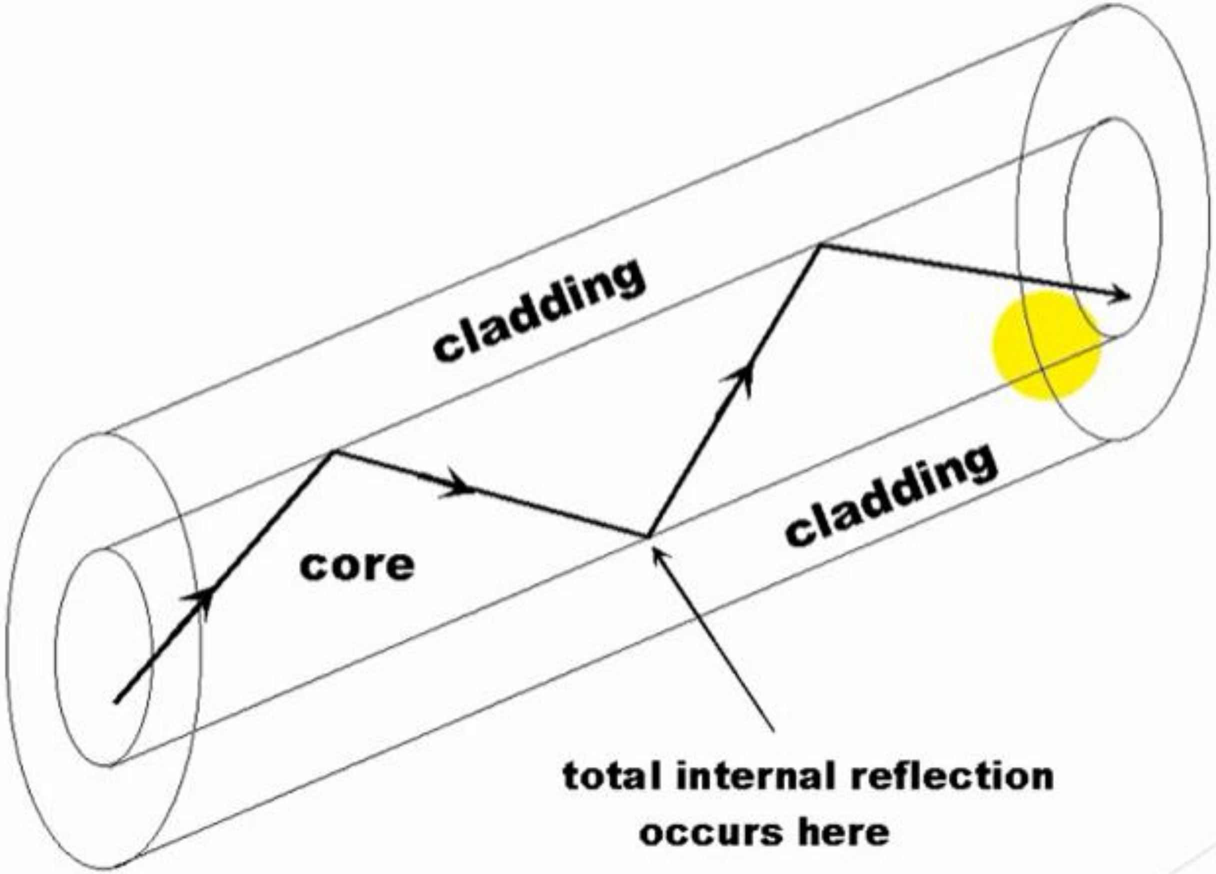
- **When the incident angle is increased beyond the critical angle, the light ray does not pass through the interface into the other medium.**
- **This gives the effect of mirror exist at the interface with no possibility of light escaping outside the medium.**
- **In this condition angle of reflection ($\varnothing_2$) is equal to angle of incidence ($\varnothing_1$) .**
- **This action is called as Total Internal Reflection (TIR) of the beam.**
- **It is TIR that leads to the propagation of waves within fiber-cable medium.**
- **TIR can be observed only in materials in which the velocity of light is less than in air.**

Continue

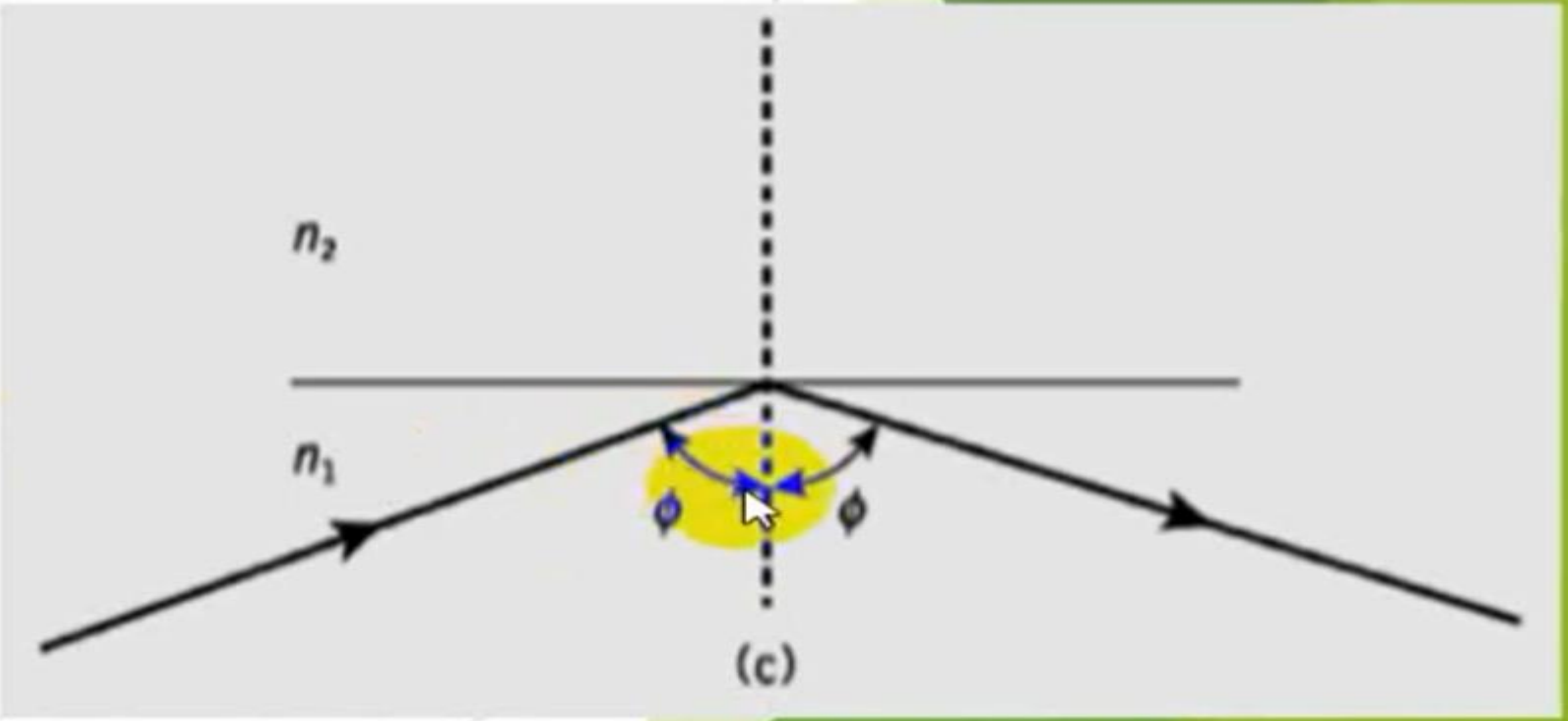
- **Sometimes, when light is moving from a denser medium towards a less dense one, instead of being refracted, all of the light is reflected.**
- **This phenomenon is called total internal reflection**
- Total internal reflection (TIR) occurs when:
  - The angle of incidence is greater than the critical angle and
  - The incident material is denser than the second material
- Therefore, the **two conditions** for total internal reflection are:
  - **The angle of incidence > the critical angle**
  - **The incident material is denser than the second material**

Principle of Transmitting Light in Fiber Optic

- Reflection (Total Internal Reflection)



C: Total Internal Reflection



Example 2.2.1 : *A light ray is incident from medium-1 to medium-2. If the refractive indices of medium-1 and medium-2 are 1.5 and 1.36 respectively then determine the angle of refraction for an angle of incidence of 30°.*

Solution : Medium-1 $n_1 = 1.5$

Medium-2 $n_2 = 1.36$

Angle of incidence $\phi_1 = 30°$

Angle of refraction $\phi_2 = ?$

$$\boxed{\text{Snell's law : } n_1 \sin\phi_1 = n_2 \sin\phi_2}$$

$$1.5 \sin 30° = 1.36 \sin\phi_2$$

$$\sin\phi_2 = \frac{1.5}{1.36} \sin 30°$$

$$\sin\phi_2 = 0.55147$$

$$\therefore \quad \phi_2 = 33.46°$$

Angle of refraction = 33.46° from normal. .. Ans.

Example 2.2.2 : *A light ray is incident from glass to air. Calculate the critical angle* (ϕ_c).

Solution : Refractive index of glass $n_1 = 1.50$

Refractive index of air $n_2 = 1.00$

$$\boxed{\text{Snell's law : } n_1 \sin\phi_1 = n_2 \sin\phi_2}$$

$$\sin\phi_1 = \frac{n_2}{n_1} \sin\phi_2$$

From definition of critical angle, $\phi_2 = 90°$ and $\phi_1 = \phi_c$.

$$\therefore \quad \sin\phi_c = \frac{n_2}{n_1} \sin 90°$$

$$\sin\phi_c = \left(\frac{1.0}{1.5}\right) \times 1 = 0.67$$

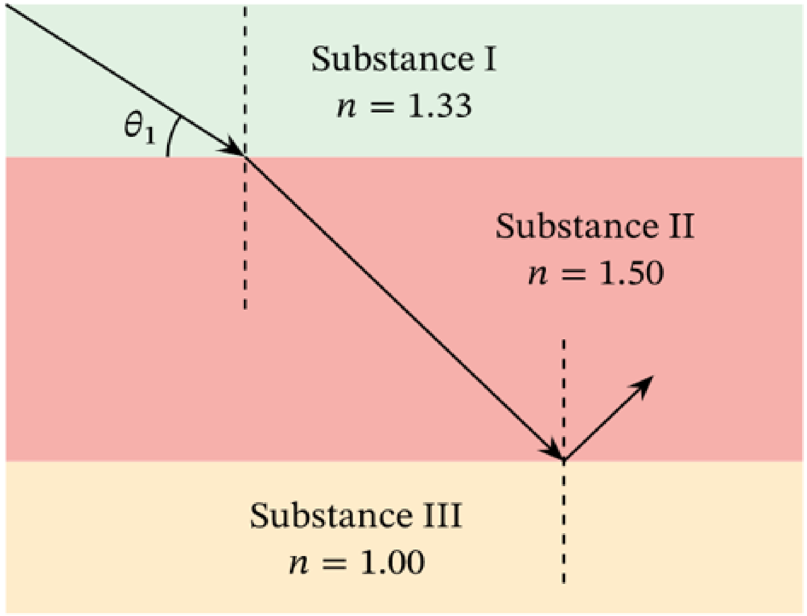
$$\therefore \quad \phi_c = \sin^{-1} 0.67$$

$$\phi_c = 41.81°$$

Critical angle ϕ_c = **41.81°** ... **Ans.**

Example 4: Relating Angles in Several Optically Different Materials

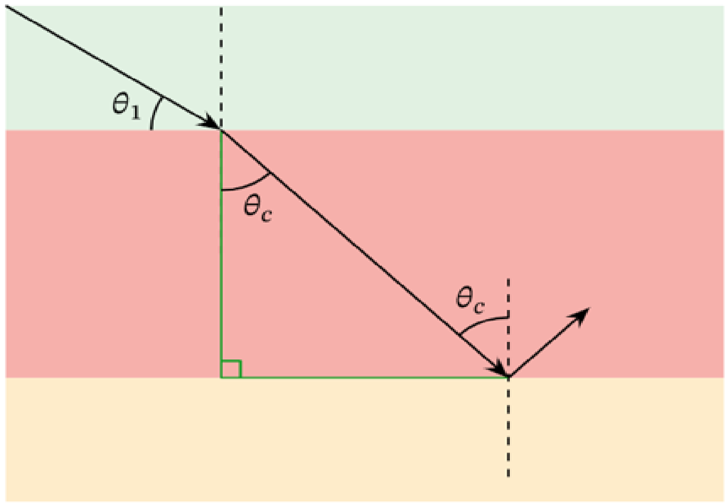
The diagram shows a light ray that is transmitted from substance I to substance II at angle θ_1 to the boundary between the substances. The ray is totally internally reflected back into substance II at the boundary to substance III. For any angle of θ greater than θ_1, the light ray is transmitted to substance II. Find the angle θ_1 to the nearest degree.



The ray experiences total internal reflection at the boundary between substances II and III, so it must be incident on the boundary at an angle no smaller than the critical angle. We also know that the ray transmits through this boundary when it is incident on the first boundary between substances I and II at an angle larger than θ_1, so let us think about what this implies. If the angle at θ_1 is made larger, the angle of incidence at the boundary of substances II and III is made smaller. We know that if this angle is made any smaller, the light will not experience total internal reflection; thus, θ_1 is set at a special value so that the angle of incidence between substances II and III is approximate to the critical angle. Since we know the refractive indices of the materials on either side of the boundary, we can calculate the critical angle to learn more about this setup. To calculate the critical angle here, we can rearrange the critical angle formula (from Snell's law) to solve for θ_c and plug in the values $n_r = 1.00$ and $n_i = 1.50$:

$$\theta_c = \arcsin \frac{n_r}{n_i} = \arcsin \frac{1.00}{1.50} = 41.8^\circ.$$

Knowing this, we can use the properties of a right triangle to determine the angle of refraction as the ray passes from substance I to II. Below is a diagram that helps relate these angles to each other using a right triangle, shown in blue. Notice that the vertical leg of the blue triangle is parallel to the surface normal between substances II and III. Because of the alternate interior angles theorem, we know that the top internal angle of the right triangle, which is the angle of refraction for the ray passing from substance I to II, is congruent, or equal, to θ_c. This is shown in the diagram below.



Thus, at this boundary, $\theta_r = 41.8°$, and we also know the indices of refraction for the materials on either side of the boundary, so we can use Snell's law to solve for the angle of incidence there:

$$n_i \sin\theta_i = n_r \sin\theta_r$$

$$\theta_i = \arcsin\left(\frac{n_r}{n_i}\sin\theta_r\right)$$

$$= \arcsin\left(\frac{1.50}{1.33}\sin 41.8°\right)$$

$$= \arcsin\left(\frac{1.50}{1.33}\sin 41.8^\circ\right)$$

$$= 48.7^\circ.$$

Because θ_1 is complementary to θ_i, we can find the value of θ_1 using $90^\circ - 48.7^\circ = 41.3^\circ$.

Rounding to the nearest degree, we have found that $\theta_1 = 41^\circ$.

Beyond using n values to determine the critical angle, it is important to be able to use the critical angle equation to solve for different values, such as the refractive index of a given material. We will practice this in the following example.

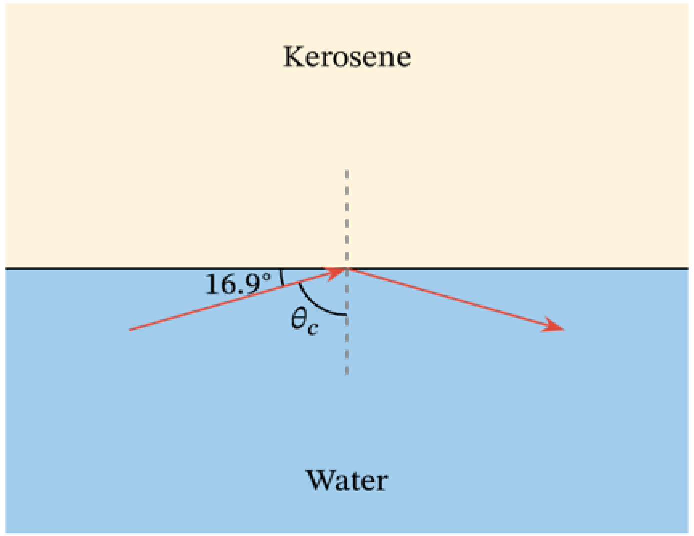
Example 5: Calculating a Refractive Index Using the Critical Angle

Light rays travel through a layer of kerosene floating on the surface of water that has a refractive index of 1.33. Light rays that are incident on the interface of kerosene and water at angles of 16.9° from the surface or less are totally internally reflected. What is the refractive index of the kerosene? Give your answer to two decimal places.

Answer

We have been given the value of an angle, but it is not equal to the critical angle because it is measured relative to the surface rather than the surface normal. However, the critical angle can be easily computed from the angle given because they are complementary:

$$90^\circ - 16.9^\circ = 73.1^\circ.$$



Thus, we know that $\theta_c = 73.1^\circ$. Now that we have a value for the critical angle, and since we know that $n_{\text{water}} = 1.33$, we have two out of the three variables that appear in our equation for the critical angle:

$$\sin\theta_c = \frac{n_r}{n_i}.$$

We can rearrange this equation to solve for one of the n values. Because the light ray is traveling from the kerosene to water, we know that $n_{\text{water}} = n_r$, and we need to solve for n_i, the refractive index of kerosene. To do this, we can multiply both sides of the equation by $\frac{n_i}{\sin\theta_c}$:

$$n_i = \frac{n_r}{\sin\theta_c}.$$

We are now ready to plug in $\theta_c = 73.1^\circ$ and $n_r = 1.33$:

$$n_i = \frac{n_r}{\sin\theta_c} = \frac{1.33}{\sin 73.1^\circ} = 1.39.$$

Therefore, kerosene has a refractive index of 1.39.

Example 2.2.3 : *A lightwave is travelling in a semiconductor medium (GaAs) of refractive index 3.6. It is incident on a different semiconductor medium (AlGaAs) of refractive index 3.4 and the angle of incidence is 80°. Will this result in total internal reflection.*

Solution : Refractive index of GaAs (n_1) = 3.6

Refractive index of AlGaAs (n_2) = 3.4

Angle of incidence $\phi_1 = 80°$

$$\boxed{\text{Snell's law : } n_1 \sin\phi_1 = n_2 \sin\phi_2}$$

At critical angle $\phi_1 = \phi_c$ and $\phi_2 = 90°$

$$\sin\phi_c = \frac{n_2}{n_1}\sin 90°$$

$$= \frac{3.4}{3.6}\sin 90°$$

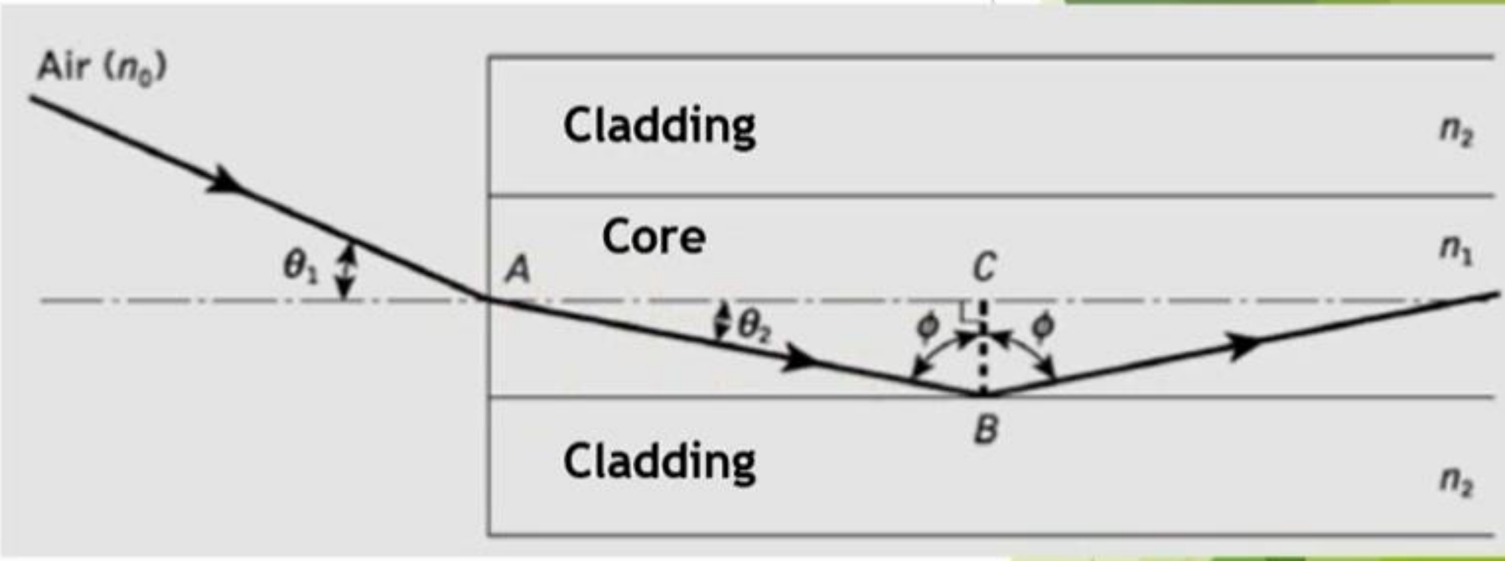
$$= 0.945$$

$$\therefore \quad \phi_c = \sin^{-1} 0.945 = 70.81°$$ ... Ans.

Since critical angle θ_c is less than angle of incidence ϕ_1 but for total internal reflection (TIR) to exist the incident angle (ϕ_1) must be greater than critical angle (θ_c). In the given problem $\phi_1 = 80°$ and $\theta_c = 71°$, hence TIR will take place.

Acceptance Angle (θ_a)

it is the maximum angle of a ray hitting the fiber core which allows the incident light to propagate into the fiber.



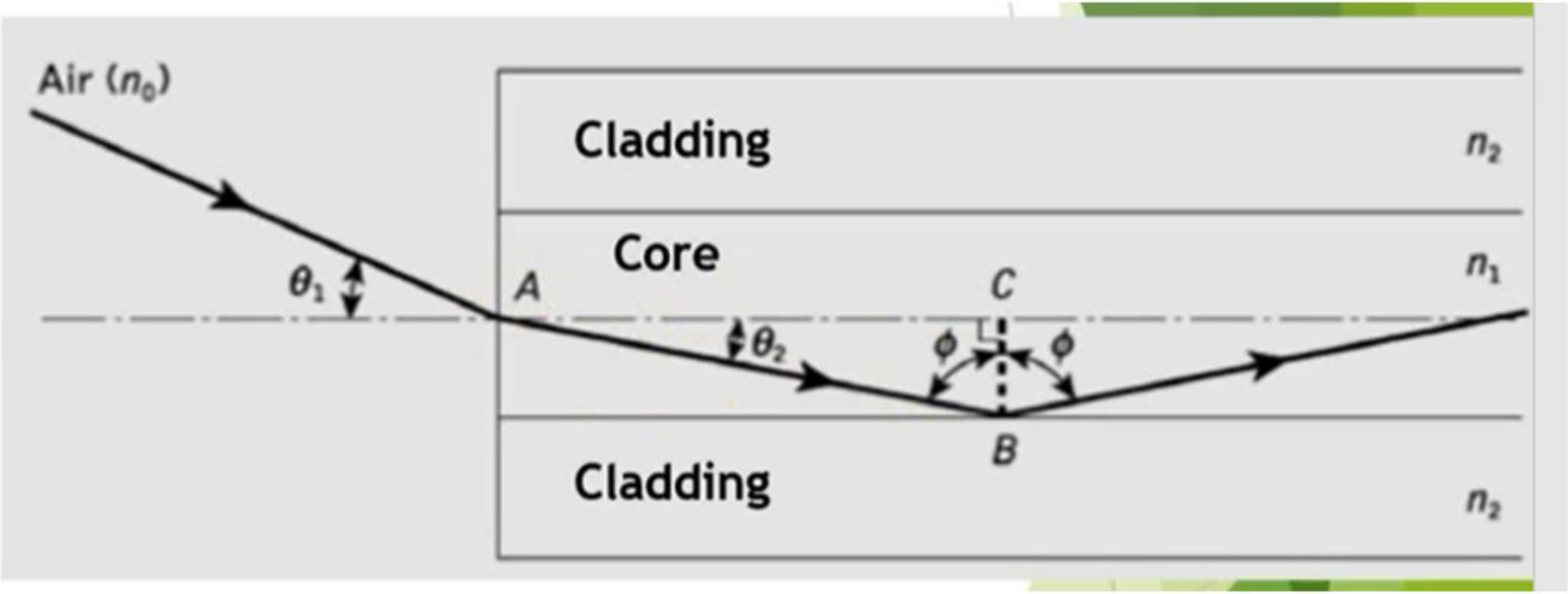
$$n_0 \sin \theta_1 = n_1 \sin \theta_2$$

$$\emptyset = \frac{\pi}{2} - \theta_2$$

$$n_0 \sin \theta_1 = n_1 \cos \emptyset$$

Acceptance Angle

When a light ray hits Fiber optic core with an incident angle ϴ1 , that means it transmits from air to the fiber core (two materials with different refraction index), it refracted with an angle ϴ2.



In this case, Snell's law can be applied

Snell's Law

$$n_1 \sin \varnothing_1 = n_2 \sin \varnothing_2$$

n_1: refractive index for incident medium

n_2: refractive index for refracted medium

$\varnothing_1$: angle of incidence

$\varnothing_2$: angle of refraction

If incidence angle is ϴ1 and refraction angle is ϴ2, Then

$$n_1 \mathrm{Sin}\ \Theta_1 = n_2\ \mathrm{Sin}\Theta_2$$

- Since the incidence ray comes from air (refraction index of air n_0), then $n_1=n_0$
- The refraction index of core is n_1, then $n_2=n_1$

Thus,

$$n_0 \text{ Sin } \Theta_1 = n_1 \text{ Sin}\Theta_2$$

- Since the ray transmission in optic fiber depends on total internal reflection, i.e., the ray should be reflected at the interface surface between core and clad (point B)

for the tringle ABC, $\Theta_2 = 90 - \phi$

$$n_0 \text{ Sin } \Theta 1 = n_1 \text{ Sin } (90 - \phi) = n_1 \text{ Cos}(\phi)$$

by using the trigonometric relationship,

$\sin^2 \phi + \cos^2 \phi = 1$ then $\cos \phi = (1 - \sin^2 \phi)^{1/2}$

$n_0 \sin \Theta_1 = n_1 (1 - \sin^2 \phi)^{1/2} = (n_1^2 - n_1^2 \sin^2 \phi)^{1/2}$

for total internal Reflection, the angle $\phi \geq \phi_c$

were ϕ_c is the critical angle and $\sin \phi_c = n_2/n_1$

then $\sin \phi = n_2/n_1$, $\sin^2 \phi = (n_2/n_1)^2 = n_2^2/n_1^2$

then $n_0 \sin \Theta_1 = (n_1^2 - n_1^2 . n_2^2/n_1^2)^{1/2} = (n_1^2 - n_2^2)^{1/2}$

$n_0 \sin \Theta_1 = (n_1^2 - n_2^2)^{1/2}$

$\sin \Theta_1 = (1/n_0) . (n_1^2 - n_2^2)^{1/2}$

$\Theta_1 = \sin^{-1}((1/n_0) . (n_1^2 - n_2^2)^{1/2})$

The above equation gives the incidence angle Θ_1 in function of refraction index of air, Cor and cladding

This angle is known as acceptance angle Θ_a, which is the maximum angle of a ray hitting the fiber core which allows the incident light to propagate into the fiber.

$$\Theta_a = \mathrm{Sin}^{-1}((1/ n_0).\ (n_1^2 - n_2^2)^{1/2})$$

With $n_0 = 1$ (air)

$$\Theta_a = \mathrm{Sin}^{-1}\ (n_1^2 - n_2^2)^{1/2})$$

Or Sin $\Theta_a = (n_1^2 - n_2^2)^{1/2}$ = Numerical aperture (NA)

For calculating acceptance angle from NA

$\Theta_a = Sin^{-1}\,(n_1^2 - n_2^2)^{1/2}) = Sin^{-1}\,NA$

Let us defined Index difference $\Delta_n = n_1 - n_2$

$\Theta_a = Sin^{-1}\,(n_1^2 - n_2^2)^{1/2}) = Sin^{-1}\,((n_1 - n_2)(n_1 + n_2))^{1/2}$

If n_1 is near n_2 then $n_1 + n_2 = 2n_1$

Hence $NA = Sin\,\Theta_a = (2n_1\Delta_n)^{1/2}$

If we define relative Refractive Index $\Delta = (n_1 - n_2)/n_1$

Then, $\Delta.\,n_1 = n_1 - n_2$

$NA = Sin\,\Theta_a = (n_1^2 - n_2^2)^{1/2} = ((n_1 - n_2)(n_1 + n_2))^{1/2}$

$NA = Sin\,\Theta_a = ((\Delta.\,n_1)\,(2\,n_1))^{1/2} = n_1(2.\,\Delta)$

Numerical aperture (Na) and its relation to acceptance angle θ_a

Again, the acceptance angle was defined as the maximum angle to the fiber core axis at which light ray may enter the fiber axis in order to get propagation in side it.

While Numerical aperture is used to describe the light gathering or light collection ability of optical fiber cable.

Acceptance Angle (θ_a)

$n_0 \sin\theta_1 = n_1 \cos\emptyset$

By using the trigonometrical relationship: $sin^2\emptyset + cos^2\emptyset = 1$

$n_0 \sin\theta_1 = n_1(1 - sin^2\emptyset)^{1/2}$

While $\sin\emptyset_c = \frac{n_2}{n_1}$, and $\emptyset$ becomes equal to the critical angle $\emptyset_c$

So, $\sin\emptyset = \frac{n_2}{n_1}$

θ_1 becomes the acceptance angle for the fiber θ_a

$n_0 \sin\theta_a = (n_1^2 - n_2^2)^{1/2}$

$NA = n_0 \sin\theta_a = (n_1^2 - n_2^2)^{1/2}$

Numerical Aperture (NA)

The numerical aperture (NA) of a fiber is defined as the sine of the largest angle an incident ray can have for total internal reflectance in the core.

$$NA = n_0 \sin\theta_a = (n_1^2 - n_2^2)^{1/2}$$

$$NA = n_1 (2\Delta)^{1/2}$$

n_1: Core refractive index

n_2: Cladding refractive index

Δ: relative refractive index difference

θ_a : acceptance angle

Summary

Critical Angle

$$\emptyset_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Acceptance Angle

$$\theta_a = \sin^{-1}\left(\frac{NA}{n_\circ}\right)$$

Numerical Aperture

$$NA = n_0 \sin\theta_a = (n_1^2 - n_2^2)^{1/2}$$

$$NA = n_1\,(2\Delta)^{1/2}$$

Example:

A silica optical fiber with a core diameter large enough to be considered by the ray theory analysis has a core refractive index of **1.50** and a cladding refractive index of **1.47**.

Determine:

1. The **critical angle** at the core-cladding interface.
2. The **NA** for the fiber.
3. The **acceptance angle** in air for the fiber.

Answer:

1. The critical angle at the core-cladding interface = $\emptyset_c$

$$\emptyset_c = \sin^{-1}\frac{n_2}{n_1}$$

$$\emptyset_c = \sin^{-1}\frac{1.47}{1.50}$$

$$\emptyset_c = \mathbf{78.5}^{\circ}$$

Answer:

2. The **NA** for the fiber.

$$NA = (n_1^2 - n_2^2)^{1/2}$$

$$NA = (1.50^{\,2} - 1.47^{\,2})^{1/2}$$

$$\boldsymbol{NA = 0.3}$$

3.The acceptance angle in air for the fiber = θ_a

$$\theta_a = \sin^{-1} NA$$

$$\theta_a = \sin^{-1} 0.3$$

$$\boldsymbol{\theta_a = 17.4°}$$

Homework:

An optical fiber has a numerical aperture of 0.20 and a cladding refractive index of 1.59. Determine:

1. The acceptance angle for the fiber in water which has a refractive index of 1.33.
2. The critical angle at the core-cladding interface.