Optics

Second year / First semester
Lecture 2

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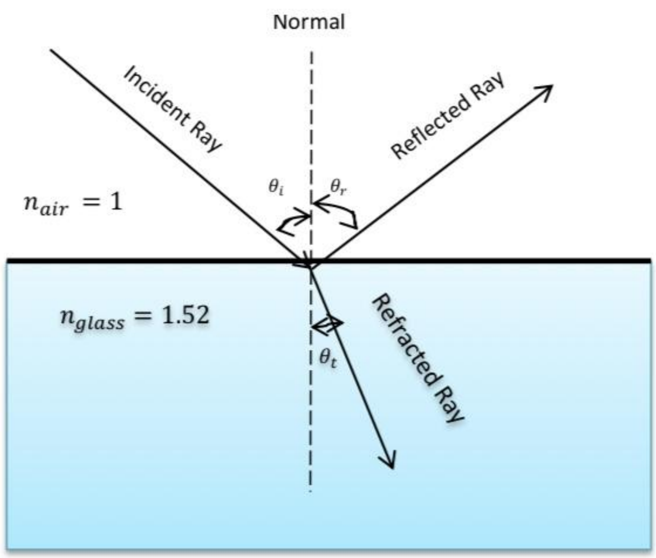
Reflection and refraction at plane surfaces

Ligth rays

A light ray is a line (straight or curved) that is perpendicular to the light's wavefronts; Light rays in homogeneous media are straight. They bend at the interface between two different media and may be curved in a medium in which the refractive index changes.



- An incident ray is a ray of light that strikes a surface. The angle between this ray and the perpendicular is the angle of incidence.
- The reflected ray Compatible with to a given incident ray, is the ray that represents the light reflected by the surface. The angle between the surface normal and the reflected ray is known as the angle of reflection.
- The refracted ray the ray which has entered the second medium and curved away from the normal or towards the normal depending on the density of the new medium in comparison to the old medium.

Refractive Index

The index of refraction, or refractive index, of any optical medium is defined as the ratio between the speed of light in a vacuum and the speed of light in the medium called absolute refractive index. .

- θ_i is the angle of incidence of a ray in vacuum (The angle between the incident ray and the normal to the surface of the medium) and θ_t is the angle of refraction (angle between the ray in the medium and the normal).
- The optical density of any transparent medium is a measure of its **refractive index**. A medium with a relatively high refractive index is said to have a high optical density, while one with a low index is said to have a low optical density.

Refractive index = $\frac{speed\ in\ vacuum}{speed\ in\ medium}$

$$\boxed{n = \frac{c}{V}}$$

For glass: n = 1.520
For water: n = 1.333
For air: n = 1.000

It is also called on the refractive index of medium two with respect to medium one is the relative refractive index can be expressed in the following relationship,

Relative refractive index = $\frac{velocity\ of\ ligth\ in\ the\ medium\ one}{velocity\ of\ ligth\ in\ the\ medium\ two}$

- The refractive index of a medium depends on:

(1) Nature of the medium.

(2) Physical conditions.
(3) Color or wavelength of light used, where the refractive index of a medium is more for violet light and less for red light.

Laws of Refraction:

Snell's law is $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \text{constant}$$

$$n = \frac{c}{V} = \frac{f\,\lambda_{air}}{f\,\lambda_{medium}} = \frac{\lambda_{air}}{\lambda_{medium}}$$

$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{\frac{c}{V_1}}{\frac{c}{V_2}} = \frac{V_2}{V_1} = \frac{\lambda_2}{\lambda_1}$$

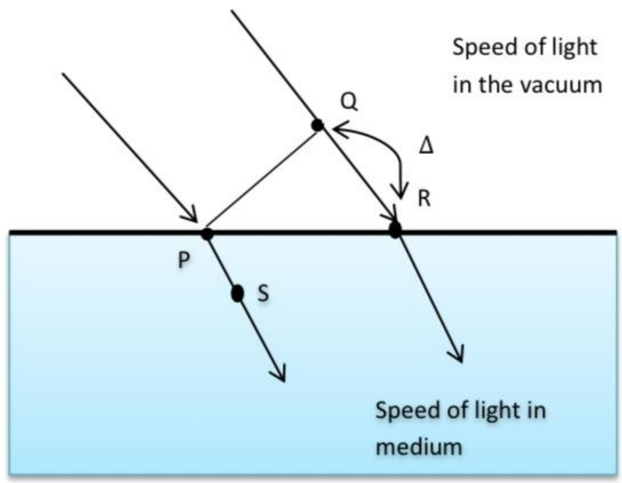
$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{V_2}{V_1}$$

λ_{air} and λ_{medium}, Being wavelength of light in air and medium respectively.

Optical Path Length

Optical path length or optical distance is the product of the geometric length of light's path through a system and the medium's index of refraction.

Optical path length is significant because it helps to determine the phase of light and control the interference and diffraction of light as it propagates. Suppose that the light has a distance QR in vacuum at the t time, in the same time (t) the light has a distance PS in the material.



$QR = ct$

PS = vt

n= $\frac{c}{V}$ **,** **v=**$\frac{c}{n}$

PS = $\frac{\text{ct}}{\text{n}} = \frac{QR}{n} \rightarrow$ QR= nPS

Let QR=Δ , PS=ι

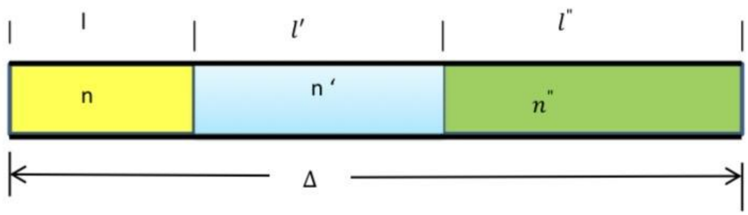
Optical path length (OPL) Δ= nι

In a homogeneous medium, Δ is Optical path length.

If a light ray travels through a series optical materials with thickness ι, ι', ι''

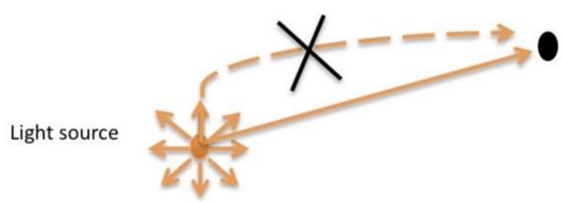
and refractive indices, n, n′, n′′, the total optical path length is just of the sum of separate values

$$\Delta = n\,\iota + n'\iota' + n''\iota''$$



Fermat's Principle

In 1650, Fermat discovered a way to explain reflection and refraction as the consequence of one single principle. It is called the principle of least time or Fermat's principle. Fermat's principle states that light will take path with the shortest travel time to go from one point to another, as shown in figure.

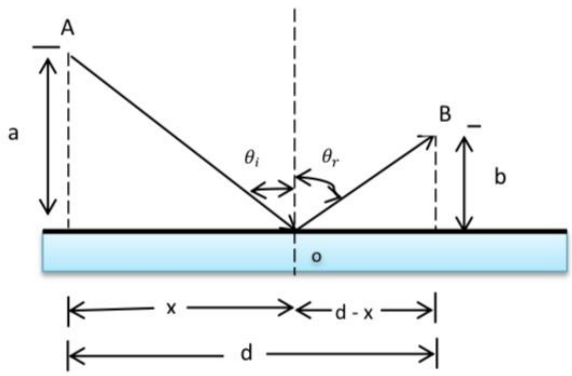


- Illustration of Fermat's principle, light will travel in a straight line from one point to another since a straight line will be the quickest path. A ray of light is incident on the boundary separating between two different medium, part of the ray is reflected back into the first medium and the remainder is refracted and enters the second medium. A light ray incident upon a reflective surface will be reflected at an angle equal to the incident angle. Both angles are typically measured with respect to the normal to the surface. This law of reflection can be derived from Fermat's principle.

Angle of incidence =Angle of reflection

$$\boldsymbol{\theta_i = \theta_r}$$

Suppose the path length from A to B is equal to (d) as shown in figure.



t= $\frac{distance}{velocity} = \frac{d}{V} = \frac{AO+OB}{V}$

d= $\sqrt{a^2 + X^2} + \sqrt{b^2 + (d-x)^2}$

Since the speed of light in vacuum is constant, the derivative of t with respect to x equal to zero.

t= $\frac{\sqrt{a^2+X^2} + \sqrt{b^2+(d-x)^2}}{V}$

t= $\frac{1}{V}\left[\sqrt{a^2 + X^2} + \sqrt{b^2 + (d-x)^2}\right]$

$\frac{\mathrm{dt}}{\mathrm{dx}} = \frac{1}{V}\left[\frac{1}{2}\frac{2x}{\sqrt{a^2+X^2}} + \frac{1}{2}\frac{2(d-x)(-1)}{\sqrt{b^2+(d-x)^2}}\right]$

But ($\frac{1}{V}$) is not equal to zero

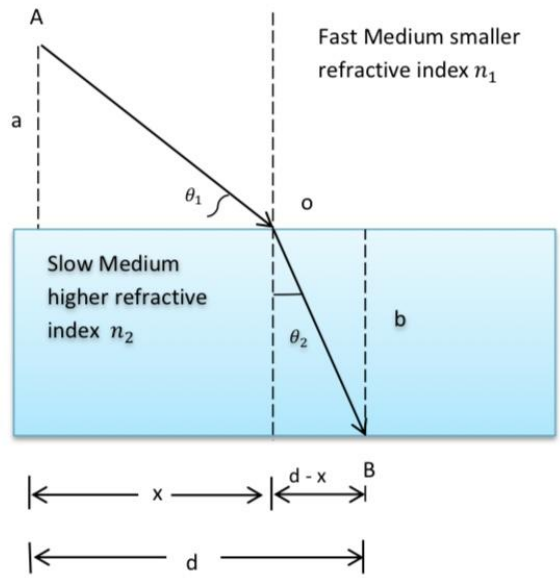
$\frac{1}{2}\frac{2x}{\sqrt{a^2+X^2}}+\frac{1}{2}\frac{2(d-x)(-1)}{\sqrt{b^2+(d-x)^2}}=0$

This reduce to $\frac{x}{\sqrt{a^2+X^2}}=\frac{(d-x)}{\sqrt{b^2+(d-x)^2}}$ which is equal to $\sin\theta_i=\sin\theta_r$

For low reflection angle

$$\theta_i = \theta_r$$

Fermat's principle: light follows the path of least time. Snell's law can be derived by setting the derivative of the time equal to zero.



$\sin\theta_1=\frac{x}{\sqrt{a^2+x^2}}$, $\sin\theta_2=\frac{d-x}{\sqrt{b^2+(d-x)^2}}$

$t=\frac{d}{V}$

$t=\frac{AO}{V}$, $t'=\frac{OB}{V'}$

$t=\frac{AO}{V}+\frac{OB}{V'}$

$$t = \frac{\sqrt{a^2+X^2}}{V} + \frac{\sqrt{b^2+(d-x)^2}}{V'}$$

$$\frac{dt}{dx} = \frac{x}{V\sqrt{a^2+X^2}} - \frac{(d-x)}{V'\sqrt{b^2+(d-x)^2}}$$

$$n_1 = \frac{c}{V} \quad , \quad n_2 = \frac{c}{V'}$$

$$V = \frac{c}{n_1} \ , \quad V' = \frac{c}{n_2}$$

$$\frac{\sin\theta_1}{\frac{c}{n_1}} - \frac{\sin\theta_2}{\frac{c}{n_2}} = 0$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\frac{n_1}{n_2} = \frac{sin\theta_2}{sin\theta_1}$$

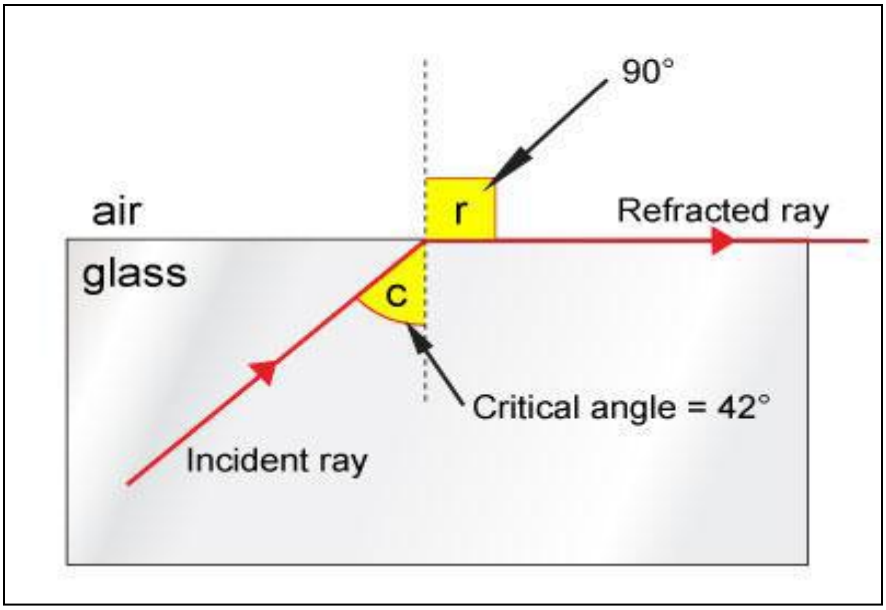
THE CRITICAL ANGLE AND TOTAL REFLECTION

The critical angle

Is the angle of incidence of a light ray which travels from high optical dense medium to the lower one $n_1 > n_2$ which results in it being refracted at 90 degree to the normal. As an example of the critical angle .

Derivation of the Critical Angle formula

Let's consider two different media and the critical angle is that gives a value of (90-degrees). If this information is substituted into Snell's Law equation, a general equation for predicting the critical angle can be derived.



Using Snell's law $n_1 > n_2$
$n_1 \sin\theta_1 = n_2 \sin\theta_2$

When incidence angle = critical angle $\rightarrow$ $\theta_1 = \theta_c$

$n_1 \sin\theta_c = n_2 \sin(90^\circ) \rightarrow \sin\theta_c = \frac{n_2}{n_1}\sin(90^\circ)$

$n_1 \sin\theta_c = n_2 \rightarrow \sin\theta_c = \frac{n_2}{n_1}$
If $n_2 = 1$ for air medium

$\sin\theta_c = \frac{1}{n_1}$
$\theta_c = \sin^{-1}\left(\frac{1}{n_1}\right)$

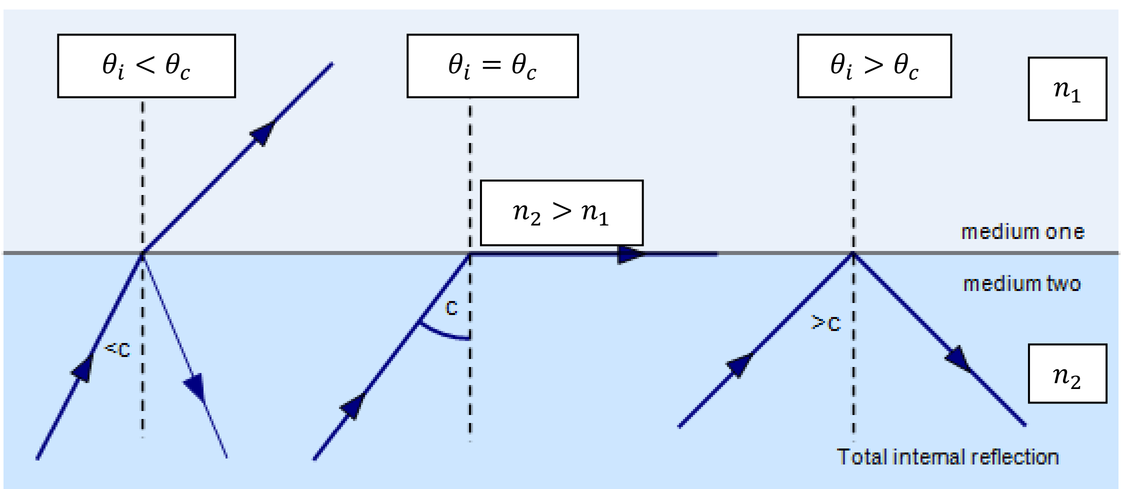
$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$ is the critical angle formula

Total internal reflection

It is the phenomenon that occurs when a wave hits at an angle greater than the critical angle relative to the normal to the surface $\theta_i > \theta c$.

- When the refractive index is lower on the other side of the boundary and the incident angle is greater than the critical angle, the wave cannot pass through and is entirely reflected.

1. If $\theta_i \leq \theta c$, the ray will split; some of the ray will reflect off the boundary, and some will refract as it passes through. This is not total internal reflection.
2. If $\theta_i > \theta c$, the entire ray reflects from the boundary. None passes through. This is called total internal reflection as shown in figure.



Laws Lecture 2

1) $n = \frac{c}{V}$

2) $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\frac{n_1}{n_2} = \frac{sin\theta_2}{sin\ \theta_1}$$

3) $\Delta = n\iota$

4) $t = \frac{L}{V}$

5) $\theta_c = sin^{-1}\left(\frac{n_2}{n_1}\right)$

n : معامل الانكسار
c : سرعة الضوء في الفراغ
v : سرعة الضوء في الوسط
Δ : طول المسار البصري
ι : المسافة
t : الزمن
θ_c: الزاوية الحرجة

Example1: A light pulse passes through a glass rod 3m long and refractive index 1.5 with another light pulse that passes through the same distance in the air which of the two pulses arrive first and what is difference time between them?

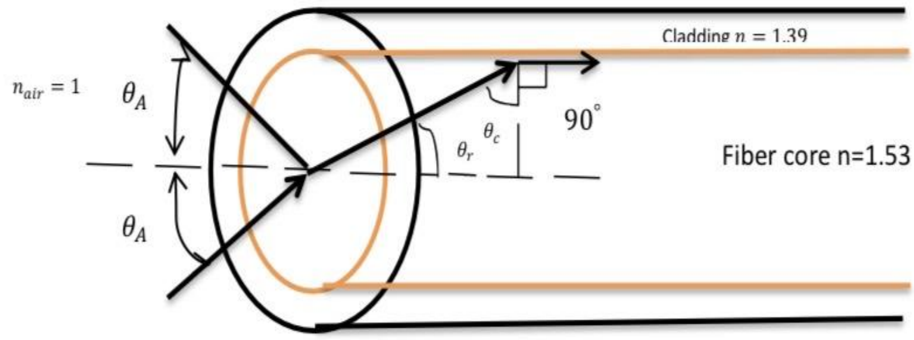
Sol:

Time of light pulse $t_{air} = \frac{distance}{c} = \frac{3\ m}{3 \times 10^8 m/sec} = 10^{-8}$sec

n= $\frac{c}{V}$ → v= $\frac{c}{n} = \frac{3 \times 10^8}{1.5} = 2\times\ 10^8\ \frac{m}{sec}$

Time of light pulse in the medium $t_{glass} = \frac{optical\ path}{V} = \frac{nl}{V} = \frac{1.5 \times 3}{2 \times 10^8} = 2.25 \times 10^{-8}$sec

Example 2: A step-index fiber 0.0025 inch in diameter has a core index of 1.53 and a cladding index of 1.39. See drawing. Such clad fibers are used frequently in applications involving communication, sensing, and medical imaging. What is the maximum acceptance angle θ_A for a cone of light rays incident on the fiber face such that the refracted ray in the core of the fiber is incident on the cladding at the critical angle?



Sol:

First find the critical angle θ_C in the core, at the core-cladding interface. Then, from geometry, identify θ_r and use Snell's law to find θ_A at the core-cladding interface

$$\theta_C = sin^{-1}\left(\frac{1.39}{1.53}\right) = 65.3^{\circ}$$

From right-triangle geometry, $\theta_r = 90 - 65.3 = 24.7°$

- A right triangle consists of two legs and a hypotenuse. The two legs meet at a 90° angle and the hypotenuse is the longest side of the right triangle and is the side opposite the right angle.

From Snell's law, at the fiber face,

$$n_{air}\sin\theta_A = n_{core}\sin\theta_r$$

$$\sin\theta_A = \left(\frac{n_{core}}{n_{air}}\right)\sin\theta_r = \left(\frac{1.53}{1}\right)\sin 24.7^{\circ}$$

$$\sin\theta_A = 0.639$$

$$\theta_A = sin^{-1} 0.639 = 39.7^{\circ}$$

Thus, the maximum acceptance angle is 39.7° and the acceptance cone is twice that, or 2 $\theta_A = 79.4^{\circ}$. The acceptance cone indicates that any light ray incident on the fiber face within the acceptance angle will undergo total internal reflection at the core-cladding face and remain trapped in the fiber as it propagates along the fiber.