Optical Fiber Communication Systems

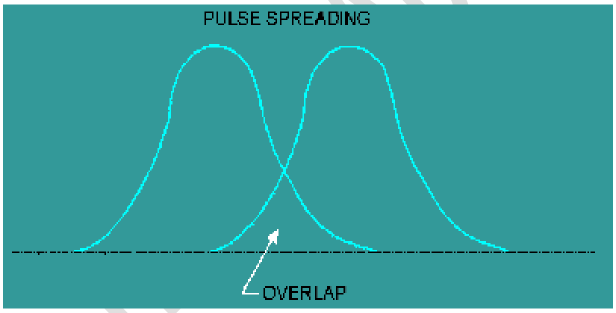
Lecture Four

DISPERSION

- **Dispersion**: Any phenomenon in which the velocity of propagation of any electromagnetic wave is wavelength dependent.
- Dispersion causes the duration and shape of an optical pulse to change in the course of propagation, causing bit errors in reception.
- In communication, dispersion is used to describe any process by which any electromagnetic signal propagating in a physical medium is degraded because the various wave characteristics (i.e., frequencies) of the signal have different propagation velocities within the physical medium.

- **Dispersion is typically measured as a time spread per distance traveled (s/km)**

Intersymbol interference



For no overlapping of light pulses down on an optical fiber link, the digital bit rate B_T must be less than the reciprocal of the broadened (through dispersion) pulse duration (2τ) . Hence:

$$B_\tau \leq \frac{1}{2\tau}$$

$$B_{\tau(\max)} = \frac{1}{2\tau}$$

This assumes that the pulse broadening due to dispersion on the channel is τ which dictates the input pulse duration is also τ.

Another more accurate estimate of the maximum bit rate for an optical channel with dispersion may be obtained by considering the light pulses at the output to have a Gaussian shape with an RMS width of σ.

The maximum bit rate is given approximately by

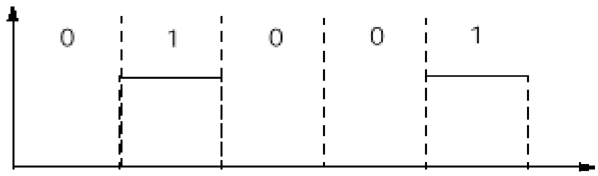
$$B_{\tau(\max)} \approx \frac{0.2}{\sigma} \qquad bit.s^{-1}$$

The conversion of bit rate to bandwidth in hertz depends on the digital coding format used.

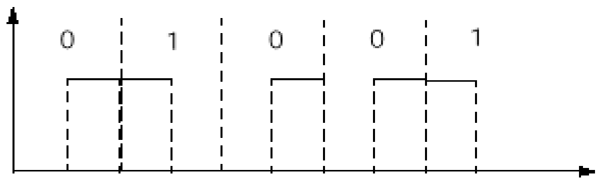
$$B_{\tau(\max)} = 2B \qquad \text{For nonreturn to zero code}$$

$$B_{\tau(\max)} = B \qquad \text{For return to zero code}$$

Nonreturn-to-zero code



Return-to-zero (Manchester) code



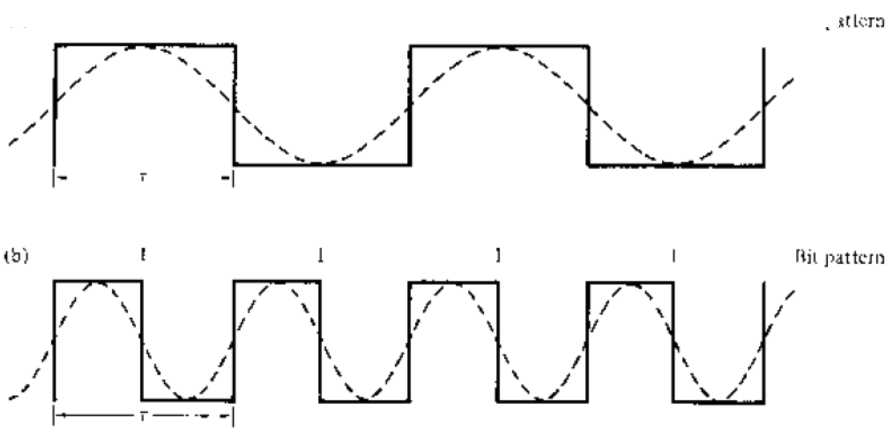


Fig. 3.6 Schematic illustration of the relationships of the bit rate to wavelength for digital codes: (a) non return to zero (NRZ); (b) return to zero (RZ).

The amount of pulse broadening is dependent upon the distance the pulse travel within the fiber. In absence of mode coupling, the pulse broadening increases linearly with fiber length and thus the bandwidth is inversely proportional to distance.

A measure of the information capacity of an optical waveguide is usually specified by **bandwidth-distance product** in MHz.km.

Fiber type	**Bandwidth-distance product**
Step index multimode	20 MHz.km
Graded index	20 GHz.km
Single mode	100 GHz.km

Example 3.5

A multimode graded index fiber exhibits total pulse broadening of 0.1 μs over a distance of 15 km. Estimate:

(a) the maximum possible bandwidth on the link assuming no intersymbol interference;
(b) the pulse dispersion per unit length;
(c) the bandwidth–length product for the fiber.

Solution: (a) The maximum possible optical bandwidth which is equivalent to the maximum possible bit rate (for return to zero pulses) assuming no ISI may be obtained from Eq. (3.9), where:

$$B_{opt} = B_T = \frac{1}{2\tau} = \frac{1}{0.2 \times 10^{-6}} = 5 \text{ MHz}$$

(b) The dispersion per unit length may be acquired simply by dividing the total dispersion by the total length of the fiber.

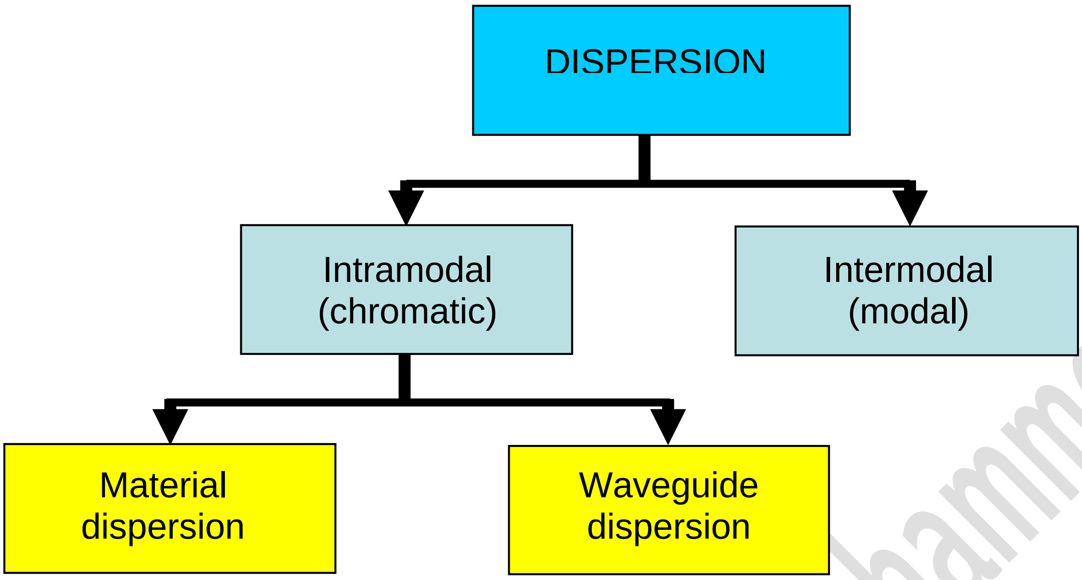
$$\text{dispersion} = \frac{0.1 \times 10^{-6}}{15} = 6.67 \text{ ns km}^{-1}$$

(c) The bandwidth–length product may be obtained in two ways. Firstly by simply multiplying the maximum bandwidth for the fiber link by its length. Hence:

$$B_{opt}L = 5 \text{ MHz} \times 15 \text{ km} = 75 \text{ MHz km}$$

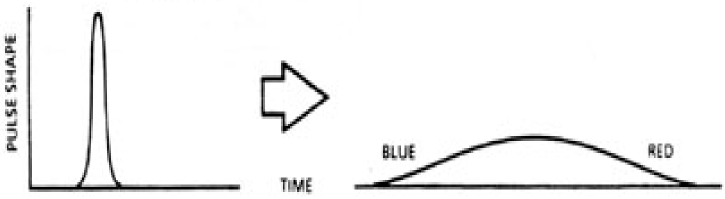
Alternatively it may be obtained from the dispersion per unit length using Eq. (3.9) where:

$$B_{opt}L = \frac{1}{2 \times 6.67 \times 10^{-9}} = 75 \text{ MHz km}$$



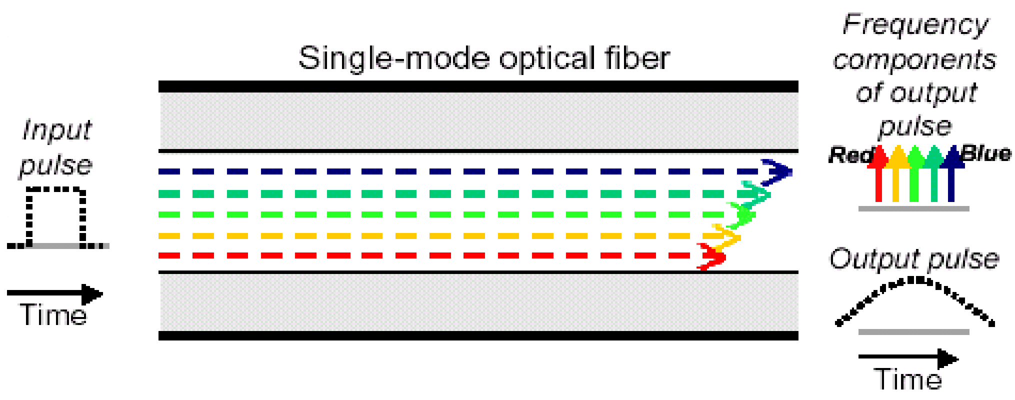
1- Intramodal (Chromatic) dispersion

There are two types of intramodal dispersion. The first type is material dispersion. The second type is waveguide dispersion.



1-1 Material dispersion occurs because the spreading of a light pulse is dependent on the wavelengths' interaction with the refractive index of the fiber core. Different wavelengths travel at different speeds in the fiber material. Different wavelengths of a light pulse that enter a fiber at one time exit the fiber at different times. Material dispersion is a function of the source spectral width. The spectral width specifies the range of wavelengths that can propagate in the fiber. Material dispersion is less at longer wavelengths.

- It is said to have material dispersion when the second derivative of refractive index with respect to wavelength is not zero **$[d^2n_1/ d\lambda^2 \neq 0]$.**



CD causes the shorter λ to travel faster than the longer λ.

What is Group Velocity ?

- Group Velocity (v_g) is Considered as the velocity of energy propagating in the direction of the axis of the guide fiber.
- In order to convey intelligence; Modulation is done. When is done, there are group velocities those must be propagating along the fiber.
- The waves of different frequencies in the group will be transmitted with slightly different velocities. $v_g = d\omega/d\beta$.

- The group delay τ_g is given by:

*

$$\tau_g = \frac{d\beta}{d\omega} = \frac{1}{C}\left(n_1 - \lambda \frac{dn_1}{d\lambda}\right) \qquad 1$$

- The pulse delay τ_m

*

$$\tau_m = \frac{L}{C}\left(n_1 - \lambda \frac{dn_1}{d\lambda}\right) \qquad 2$$

For a source with RMS spectral width σ_λ and a mean wavelength, the RMS pulse broadening σ_m due to material dispersion [which may be obtained from the expansion of Taylor series about λ as]

$$\sigma_m = \sigma_\lambda \frac{d\tau_m}{d\lambda} + \sigma_\lambda \frac{d^2\tau_m}{d\lambda^2} + \ldots\ldots\ldots\ldots$$

Hence the pulse broadening may be evaluated by considering the dependence of τm on λ where from eq (2):

$$\frac{d\tau_m}{d\lambda} = \frac{-L\lambda}{c}\frac{d^2 n_1}{d\lambda^2}$$

And approximately

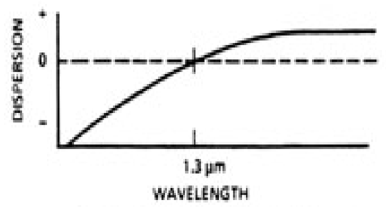
$$\sigma_m \cong \sigma_\lambda \frac{d\tau_m}{d\lambda} = \frac{\sigma_\lambda L}{c}\left|\lambda \frac{d^2 n_1}{d\lambda^2}\right| = \sigma_\lambda L M$$

where:

$$M = \frac{1}{L}\cdot\frac{d\tau m}{d\lambda} = \frac{\lambda}{c}\left|\frac{d^2 n_1}{d\lambda^2}\right|$$

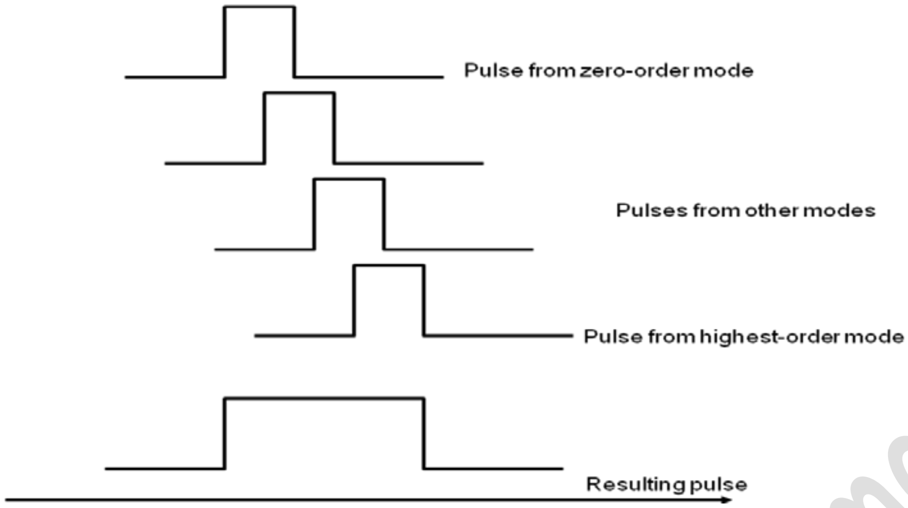
- Where M is the material dispersion parameter in ($ps.nm^{-1}.km^{-1}$). It may be observed that the material dispersion tends to zero in the longer wavelength region around 1.3 μm (for pure silica).
- It can be reduced either by choosing sources with narrower spectral output width (reducing **σ_λ**) like using injection laser diode. It is of particular importance for single–mode waveguides and LED system (since an LED has broader output spectrum than a laser diode.

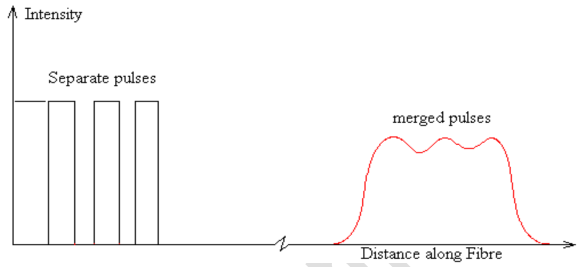
t



in a fiber of length L is

DISPERSION EXPRESSED IN PICOSECONDS OF PULSE DELAY PER KILOMETER OF FIBER LENGTH PER NANOMETER BANDWIDTH.





Example: Estimate the rms pulse broadening per kilometer for the fiber with a material dispersion parameter is given by $\left|\lambda^2 \frac{d^2 n_1}{d\lambda^2}\right|$ of 0.025, and the optical source used is an injection laser diode with a relative spectral width σ_λ / λ of 0.0012 at wavelength of 0.85μm.

Solution:

Material dispersion parameter is given by .

$$M = \frac{\lambda}{L} \cdot \left[\frac{d\tau m}{d\lambda} \right] = \frac{\lambda}{c} \left| \frac{d^2 n_1}{d\lambda^2} \right| = \frac{1}{c\lambda} \left| \lambda^2 \frac{d^2 n_1}{d\lambda^2} \right|$$

$$= \frac{1 * 0.025}{3 * 10^8 * 850} = 98.1 ps./nm.km$$

The rms spectral width may be obtained from the relative spectral width by:

$$\sigma_\lambda = 0.0012\,\lambda = 0.0012 \times 0.85 \times 10^{-6}$$
$$= 1.02\ \text{nm}.$$

The rms pulse broadening is

$$\sigma_m \simeq \sigma_\lambda L M$$

Therefore, the rms pulse broadening per kilometere due to material dispersion is:

$$\sigma_m \simeq 1.02 \times 1 \times 98.1 \times 10^{-12}$$
$$= 0.1\ \text{ns.km}^{-1}$$

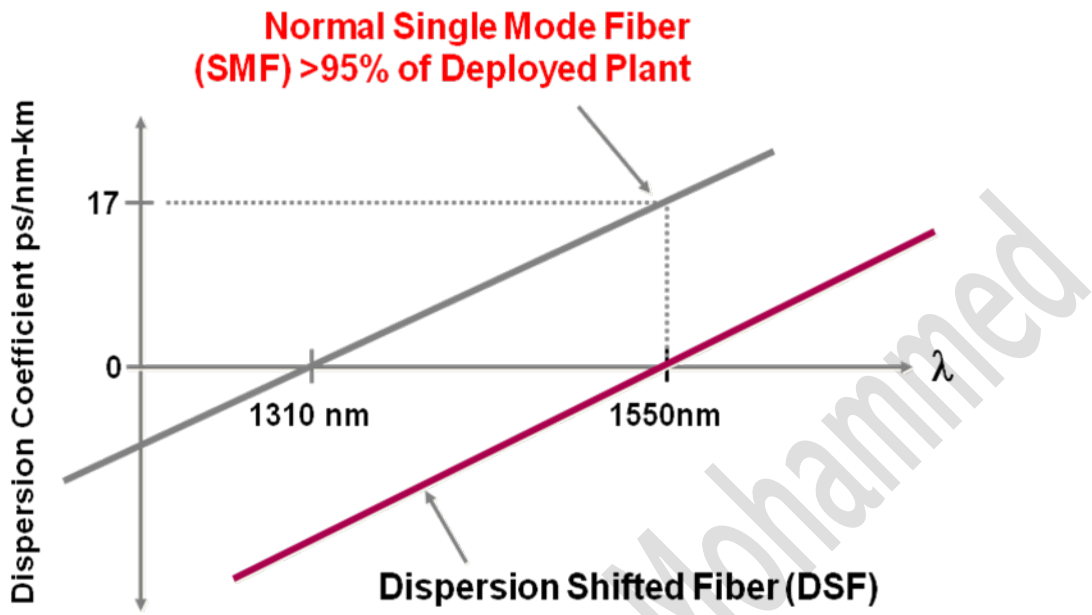
1-2 Waveguide dispersion Waveguide dispersion also occurs because light propagates differently in the core than in the cladding for a particular mode.

Waveguide dispersion depends upon the fiber design. The propagation constant which is the function of the ratio of fiber dimension (i.e. core radius) to the wavelength

In multimode fibers, waveguide dispersion and material dispersion are basically separate properties. Multimode waveguide dispersion is generally small compared to material dispersion. **Waveguide dispersion is usually neglected.**

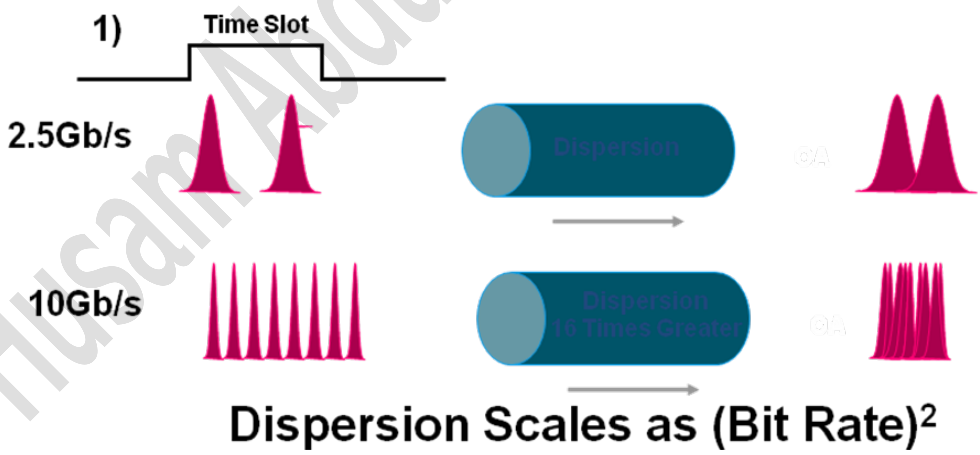
However, in single mode fibers, material and waveguide dispersion are interrelated.

Dispersion Management: Problem Fiber Dispersion Characteristic

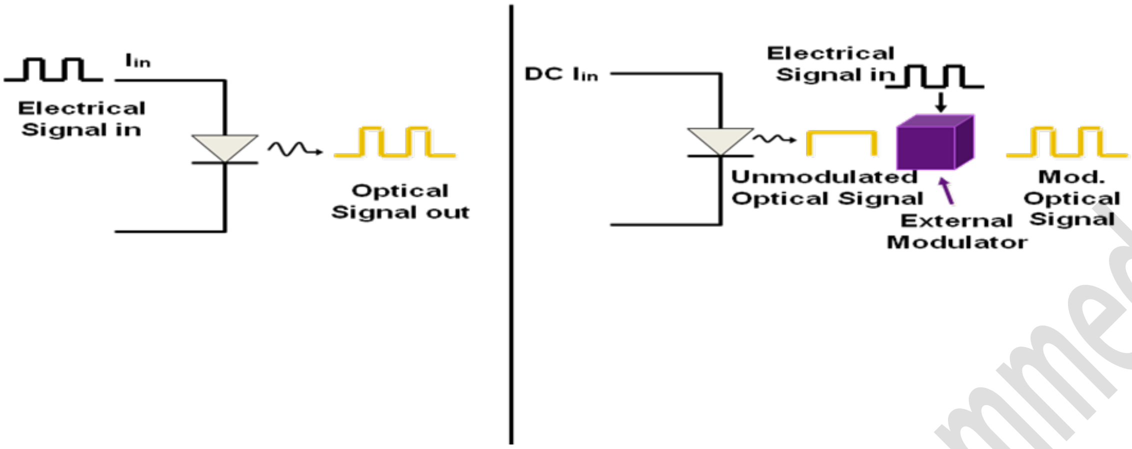


Dispersion Management: Problem Increasing the Bit Rate

Higher Bit Rates experience higher signal degradation due to Chromatic Dispersion:



Dispersion Management: Solution Direct vs. External Modulation



- Laser diode's bias current is modulated with signal input to produce modulated optical output
- Approach is straightforward and low cost, but is susceptible to chirp (spectral broadening) thus exposing the signal to higher dispersion

- The laser diode's bias current is stable
- Approach yields low chirp and better dispersion performance, but it is a more expensive approach

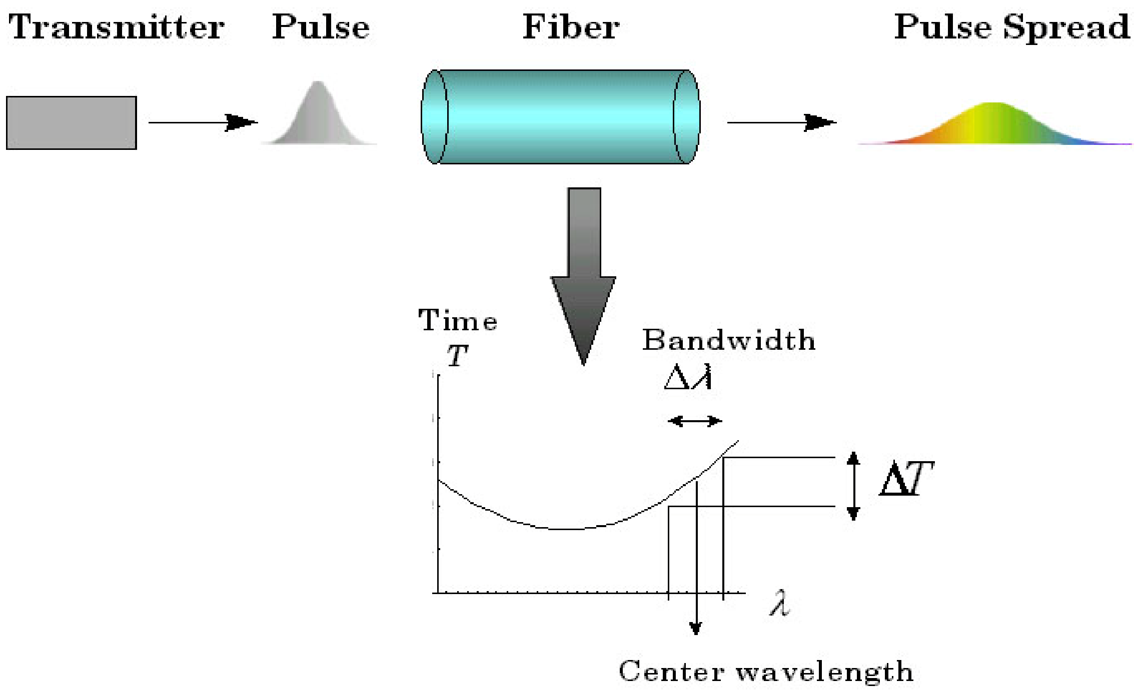
Dispersive properties

- Anomalous dispersion: $\beta 2 < 0$ or $M > 0$

— short wavelength components (blue) travel faster than long wavelength components (red)

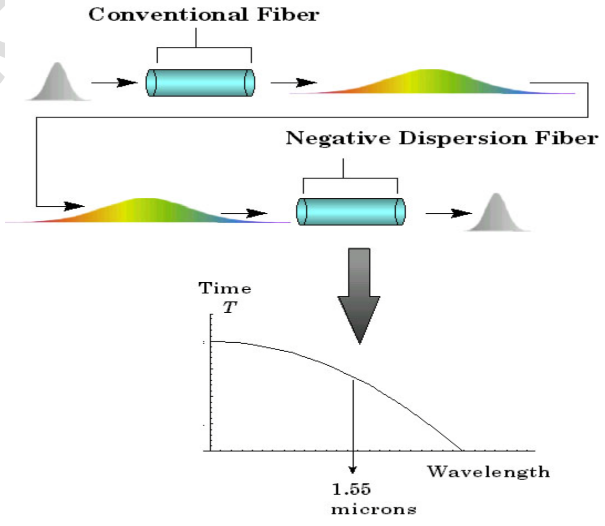
- Normal dispersion: $\beta 2 > 0$ or $M < 0$

— long wavelength components (red) travel faster than short wavelength components (blue)

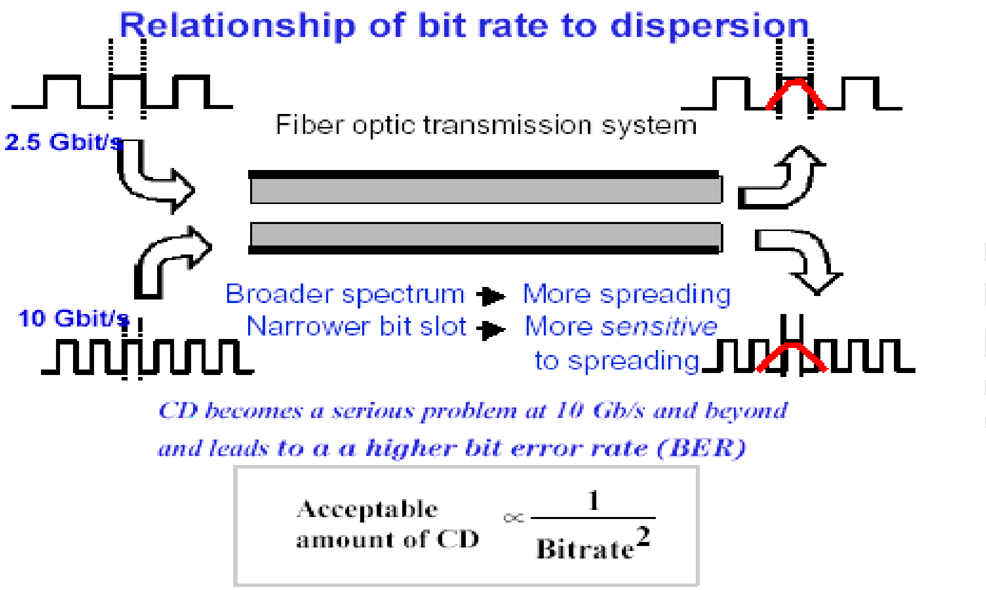


Dispersion Compensating Fiber:

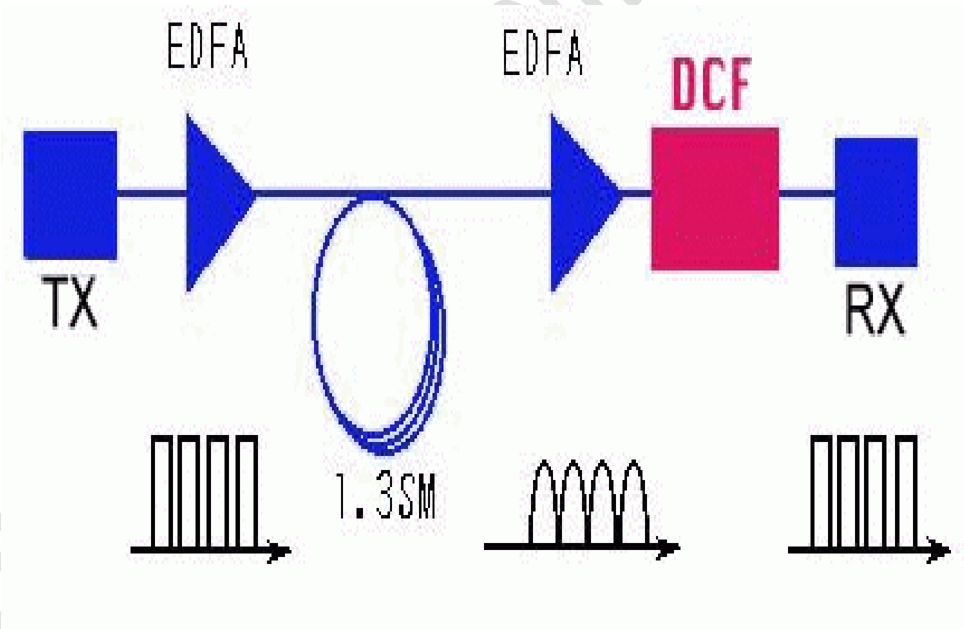
By joining fibers with CD of opposite signs and suitable lengths an average dispersion close to zero can be obtained; the compensating fiber can be several kilometers and the reel can be inserted at any point in the link, at the receiver or at the transmitter



Why Require Dispersion Compensation ?



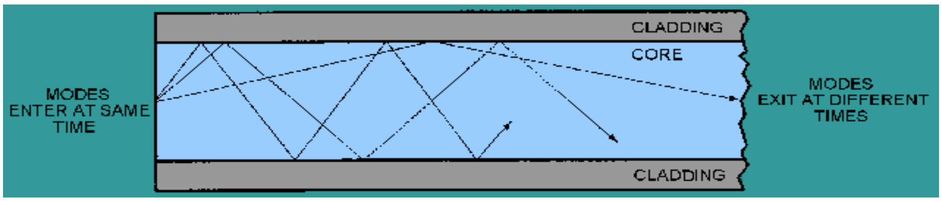
Dispersion Compensating Fiber (DCF) Application



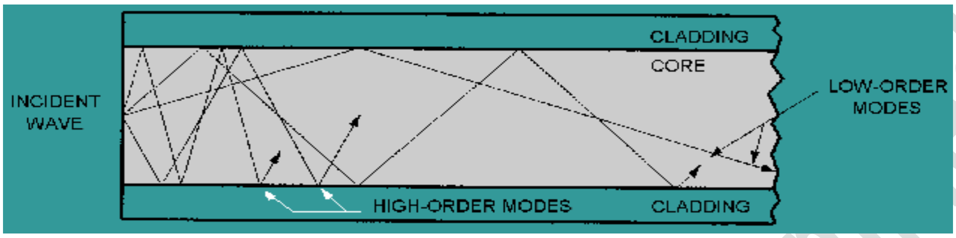
2- Intermodal (Mode or Modal) Dispersion

- Intermodal or modal dispersion results from the propagation delay difference between modes within a multimode fiber. Since modes travel in different directions, some modes travel longer distances. **Modal dispersion** occurs because each mode travels a different distance over the same time span, as shown in the figure. This condition causes the light pulse to spread.

- As the length of the fiber increases, modal dispersion increases.



Distance traveled by each mode over the same time.



- Modal dispersion is the dominant source of dispersion in multimode fibers. Modal dispersion does not exist in **single mode** fibers.
- **In multimode fiber, inter-modal dispersion is the dominant cause of dispersion, but chromatic dispersion can be important at 850 nm**
- Intermodal dispersion in a multimode fibers may be reduced by adoption of an optimum refraction index profile which is provided by the near parabolic profile of most graded index fibers.

Multimode step index fiber

The delay difference δT_s between the meridional ray and the axial ray at the critical angle is

$$\delta T_s = T_{max} - T_{min} \cong \frac{L n_1^2}{C n_2} \Delta \qquad 1$$

When $\Delta << 1$ Eq.1 may be written as:

$$\delta T_s \cong \frac{L n_1 \Delta}{C} = \approx \frac{L (NA)^2}{2 n_1 C} \qquad 2$$

The rms pulse broadening σ_s resulting from intermodal dispersion mechanism along the multimode step index fiber is

$$\sigma_s \approx \frac{L n_1 \Delta}{2\sqrt{3} C} \approx \frac{L (NA)^2}{4\sqrt{3} C n_1} \qquad 6$$

- Requirement for minimal intersymbol interference: $B \, \Delta t < 1$

Where B = bit rate

- *Numerical values for weakly guiding fiber, for which $n_1 \approx n_2 \approx 1.5$:

1. Step-index multimode ($\Delta \approx 3\times10^{-3}$): BL < 67 Mb-km/s (MHz-km).
2. Unclad multimode ($\Delta \approx .33$): BL < .4 Mb-km/s (MHz-km).

p119
Ex. (K)

A 6 km optical link consists of multimode step index fiber with a core refractive index of 1.5 and a relative refractive index difference of 1%. Estimate:

a. The delay difference between the slowest and fastest modes at the fiber output;

b. The rms pulse broadening due to intermodal dispersion on the link;

c. the maximum bit rate that may be obtained without substantial errors on the link assuming only intermodal dispersion;

d. the bandwidth-length product corresponding to (c).

Sol:

a. The dalay difference is:

$$\delta T_s \simeq \frac{L n_1 \Delta}{c} = \frac{6\times10^{3}\times1.5\times0.01}{2.998\times10^{8}}$$

$$= 300\text{ns}.$$

b. The rms pulse broadening due to intermodal dispersion is

$$\sigma_s = \frac{L n_1 \Delta}{2\sqrt{3}\,c} = \frac{1}{2\sqrt{3}} \cdot \frac{6\times10^{3}\times1.5\times0.01}{2.998\times10^{8}}$$

$$= 86.7\text{ns}.$$

c

1 The maximum bit rate may be estimated in two ways: to get an idea of the maximum bit rate when assuming no pulse overlap.

$$B_T(max) = \frac{1}{2\tau} = \frac{1}{2\delta T_s} = \frac{1}{600\times10^{-9}}$$
$$= 1.7\ \text{Mbit s}^{-1}$$

2 Alternatively an improved estimate may be obtained using the calculated rms pulse broadening.

$$B_T(max) = \frac{0.2}{\sigma_s} = \frac{0.2}{86.7\times10^{-9}}$$
$$= 2.3\ \text{Mbit s}^{-1}$$

d Using the most accurate estimate of the maximum bit rate from (c), and assuming return to zero pulses, the bandwidth-length product is

$$B_{opt} \times L = 2.3\ \text{MHz} \times 6\ \text{km} = 13.8\ \text{MHz.km}.$$

Multimode graded index fiber

- The delay difference δT_g is

$$\delta T_g \cong \frac{L n_1 \Delta^2}{2C} \approx \frac{L(NA)^4}{8 n_1^3 C} \qquad 1$$

- The rms pulse broadening of a near parabolic index profile graded index σ_g is reduced compared to the similar broadening for corresponding step index fiber σ_s (i.e. the same relative refractive index difference) following

$$\sigma_g = \frac{\Delta}{D}\sigma_s \qquad 2$$

- Where D is a constant between 4 & 10 depending on the precise evaluation and exact optimum profile chosen.
- *The best minimum theoretical rms pulse broadening for GRI fiber with an optimum characteristics refractive index profile for the core α_{op} of

$$\alpha_{op} = 2 - \frac{12\Delta}{5}$$

- The value $\alpha = 2$ produces zero intermodal dispersion in the paraxial approximation of geometrical optics

P118 Ex

Compare the rms pulse broadening per kilometere due to intermodal dispersion for the multimode step index fiber of example (K) with the corresponding rms pulse broadening for an optimum near parabolic profile graded index fiber with the same core axis refractive index and relative refractive index difference (n_1 1.5, Δ 0.01).

Sol:

In example K, σ_s over 6 km of fiber is 86.7 ns.

Hence the rms pulse broadening per kilometere for the multimode step index fiber is:

$$\frac{\sigma_s(1km)}{L} = \frac{86.7}{6} = 14.4 \ ns.km^{-1}.$$

The rms pulse broadening per kilometere for the corresponding graded index fiber is:

$$\sigma_g(1km) = \frac{L n_1 \Delta^2}{20\sqrt{3}\, c} = \frac{10^3 \times 1.5 \times (0.01)^2}{20\sqrt{3} \times 2.998 \times 10^8}$$

$$= 14.4 \ ps\, km^{-1}$$

From above the theoretical improvement factor of the graded fiber in relation to intermodal rms pulse broadening is 1000. However, this level of improvement is not usually achieved in practice due to difficulties in controlling the refractive index profile radially over long lengths of fiber.

Toatal fiber dispersion

The total dispersion is given by the following square sum expression:

$$\sigma^2_{total} = \sigma^2_M + \sigma^2_{inter}$$

where σ_{inter} is the intermodal dispersion of the fiber.

The total propagation delay difference is proportional to (σ_{total}.L), the fiber bandwidth B is defined as:

$$B = \frac{0.2}{\sigma_{total}.L}$$

This means that the larger the total dispersion and the longer the distance, the lower the transmitted bit rate.

p121

Ex. A multimode step index fiber has anumerical aperture of 0.3 and a core refractive index of 1.45. The material dispersion parameter for the fiber is 250 ps.nm^{-1}.km^{-1} which makes material dispersion the totally dominating intramodal dispersion mechanism. Estimate (a) the total rms pulse broadening per kilometere when the fiber is used with an LED source of rms spectral width 50nm and (b) the corresponding bandwidth-length product for the fiber

sol:

a The rms pulse broadening per kilometere due to material dispersion is ;

[redacted]

$$\sigma_m(1km) = \frac{\sigma_\lambda . L \lambda}{c}\left|\frac{d^2 n_1}{d\lambda^2}\right| = \sigma_\lambda L M = 50 \times 1 \times 250 \text{ ps km}^{-1}$$
$$= 12.5 \text{ ns. km}^{-1}$$

The rms pulse broadening per kilometere due to intermodal dispersion for the step index fiber is

$$\sigma_s(1km) = \frac{L(NA)^2}{4\sqrt{3}\, n_1 c} = \frac{10^3 \times 0.09}{4\sqrt{3} \times 1.45 \times 2.998 \times 10^8}$$

$$= 29.9 \text{ ns.km}^{-1}$$

The total rms pulse broadening per kilometere (as the waveguide dispersion is neglected for the multimode step index is

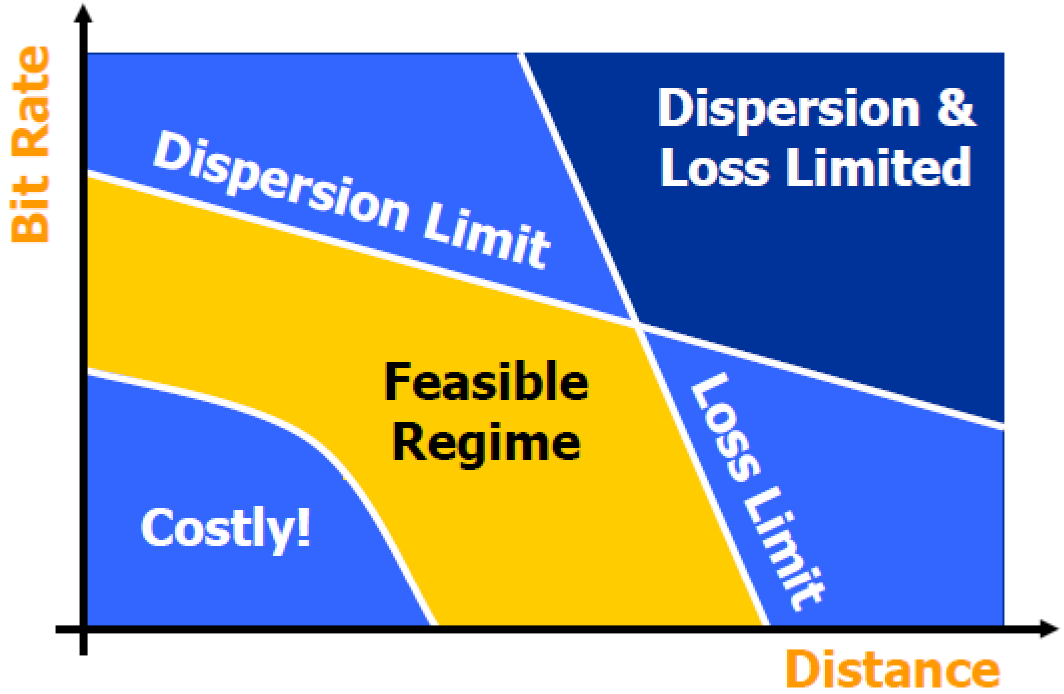
$$\sigma_T = (\sigma_m^2 + \sigma_s^2)^{1/2} = (12.5^2 + 29.9^2)^{1/2}$$

b The bandwidth-length produceisas;

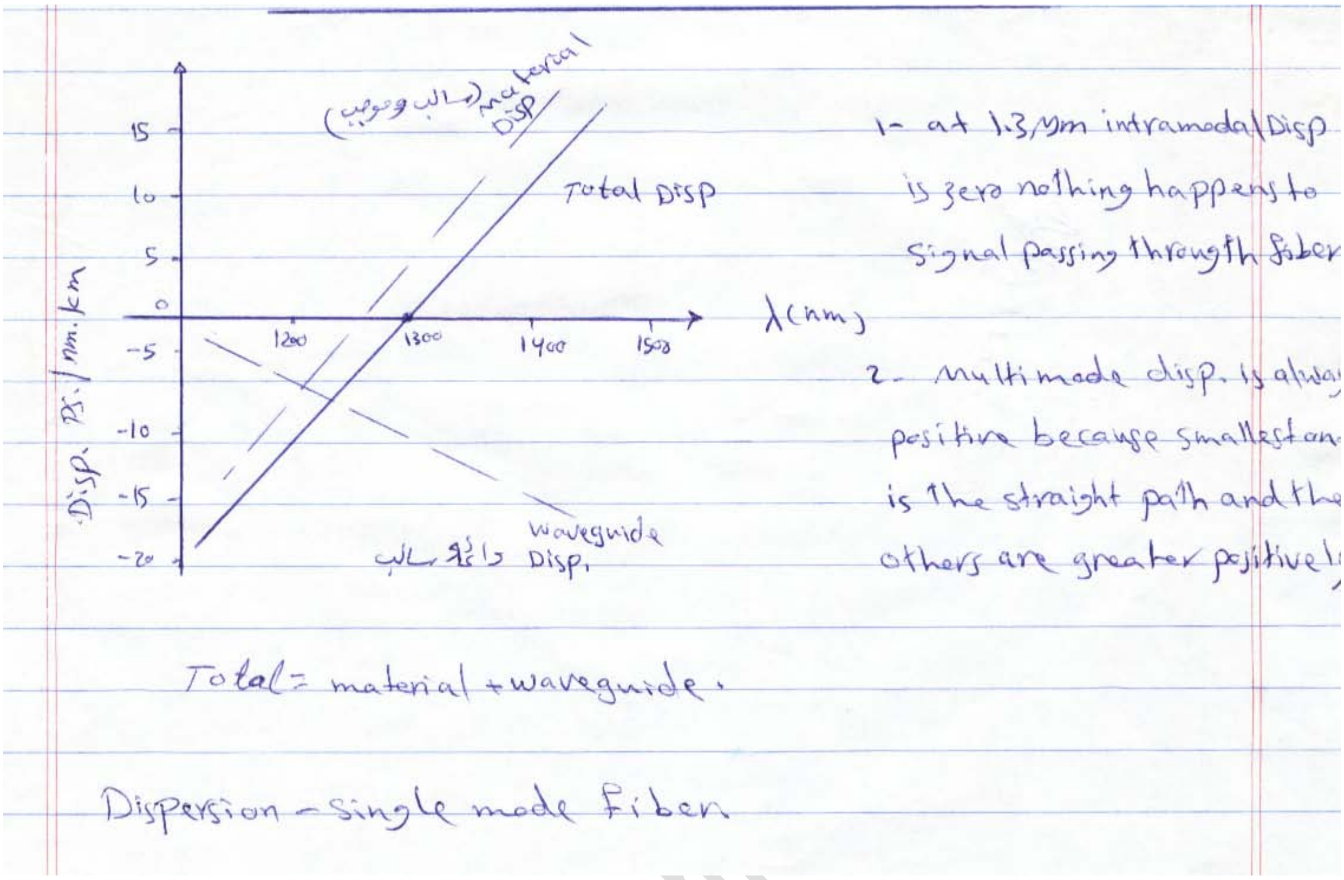
$$B_{opt} \times L = \frac{0.2}{\sigma_T} = \frac{0.2}{32.4 \times 10^{-9}}$$

$$= 6.2 \text{ MHz.km.}$$

Fiber limitations



Type	units	effect in	typical value	mechanism
material	ps/nm.Km	Step index + GRI + mono-mode	-20 ps/nm.km To -5 ps/nm.km	Index of refraction of material is a function of wavelength.
waveguide	ps/nm.Km	monomode	-5 ps/nm.Km	waveguide propagation constant is a function of wavelength.
multimode	ns/Km	Step index GRI	20 ps/km To 50 ps/km	delta path difference for each mode is translated to delta velocity for each path.



LED: $\Delta\lambda/\lambda \approx 0.04$ (Light-Emitting Diode)

FP-LD: $\Delta\lambda/\lambda \approx 0.004$ (Fabry-Perot Laser Diode)

DFB-LD: $\Delta\lambda/\lambda \approx 0.0004$ (Distributed FeedBack Laser Diode)