4

Ohm's Law, Power, and Energy

4.1 OHM'S LAW

Consider the following relationship:

$$\text{Effect} = \frac{\text{cause}}{\text{opposition}} \tag{4.1}$$

Every conversion of energy from one form to another can be related to this equation. In electric circuits, the *effect* we are trying to establish is the flow of charge, or *current*. The *potential difference*, or voltage, between two points is the *cause* ("pressure"), and the opposition is the *resistance* encountered.

An excellent analogy for the simplest of electrical circuits is the water in a hose connected to a pressure valve. Think of the electrons in the copper wire as the water in the hose, the pressure valve as the applied voltage, and the size of the hose as the factor that determines the resistance. If the pressure valve is closed, the water simply sits in the hose without motion, much like the electrons in a conductor without an applied voltage. When we open the pressure valve, water will flow through the hose much like the electrons in a copper wire when the voltage is applied. In other words, the absence of the "pressure" in one case and the voltage in the other will simply result in a system without motion or reaction. The rate at which the water will flow in the hose is a function of the size of the hose. A hose with a very small diameter will limit the rate at which water can flow through the hose, just as a copper wire with a small diameter will have a high resistance and will limit the current.

In summary, therefore, the absence of an applied "pressure" such as voltage in an electric circuit will result in no reaction in the system and no current in the electric circuit. Current is a reaction to the applied voltage and not the factor that gets the system in motion. To continue with the analogy, the more the pressure at the spigot, the more the rate

German (Erlangen, Cologne)
(1789–1854)
Physicist and Mathematician
Professor of Physics, University of Cologne

Courtesy of the Smithsonian Institution Photo No. 51,145

In 1827, developed one of the most important laws of electric circuits: *Ohm's law.* When the law was first introduced, the supporting documentation was considered lacking and foolish, causing him to lose his teaching position and search for a living doing odd jobs and some tutoring. It took some 22 years for his work to be recognized as a major contribution to the field. He was then awarded a chair at the University of Munich and received the Copley Medal of the Royal Society in 1841. His research also extended into the areas of molecular physics, acoustics, and telegraphic communication.

FIG. 4.1
Georg Simon Ohm.

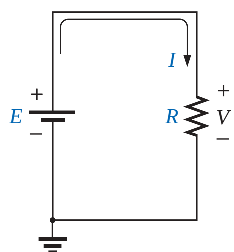


FIG. 4.2
Basic circuit.

of water flow through the hose, just as applying a higher voltage to the same circuit will result in a higher current.

Substituting the terms introduced above into Eq. (4.1) results in

$$\text{Current} = \frac{\text{potential difference}}{\text{resistance}}$$

and

$$I = \frac{E}{R} \quad \text{(amperes, A)} \tag{4.2}$$

Equation (4.2) is known as **Ohm's law** in honor of Georg Simon Ohm (Fig. 4.1). The law clearly reveals that for a fixed resistance, the greater the voltage (or pressure) across a resistor, the more the current, and the more the resistance for the same voltage, the less the current. In other words, the current is proportional to the applied voltage and inversely proportional to the resistance.

By simple mathematical manipulations, the voltage and resistance can be found in terms of the other two quantities:

$$E = IR \quad \text{(volts, V)} \tag{4.3}$$

and

$$R = \frac{E}{I} \quad \text{(ohms, } \Omega\text{)} \tag{4.4}$$

The three quantities of Eqs. (4.2) through (4.4) are defined by the simple circuit of Fig. 4.2. The current I of Eq. (4.2) results from applying a dc supply of E volts across a network having a resistance R. Equation (4.3) determines the voltage E required to establish a current I through a network with a total resistance R, and Equation (4.4) provides the resistance of a network that results in a current I due to an impressed voltage E.

Note in Fig. 4.2 that the voltage source "pressures" current in a direction that passes from the negative to the positive terminal of the battery. This will always be the case for single-source circuits. The effect of more than one source in the network will be examined in the chapters to follow. The symbol for the voltage of the battery (a source of electrical energy) is the uppercase letter E, whereas the loss in potential energy across the resistor is given the symbol V. The polarity of the voltage drop across the resistor is as defined by the applied source because the two terminals of the battery are connected directly across the resistive element.

EXAMPLE 4.1 Determine the current resulting from the application of a 9-V battery across a network with a resistance of 2.2 Ω.

Solution: Eq. (4.2):

$$I = \frac{E}{R} = \frac{9\text{ V}}{2.2\ \Omega} = \mathbf{4.09\ A}$$

EXAMPLE 4.2 Calculate the resistance of a 60-W bulb if a current of 500 mA results from an applied voltage of 120 V.

Solution: Eq. (4.4):

$$R = \frac{E}{I} = \frac{120\text{ V}}{500 \times 10^{-3}\text{ A}} = \mathbf{240\ \Omega}$$

For an isolated resistive element, the polarity of the voltage drop is as shown in Fig. 4.3(a) for the indicated current direction. A reversal in current will reverse the polarity, as shown in Fig. 4.3(b). In general, the flow of charge is from a high (+) to a low (−) potential. Polarities as established by current direction will become increasingly important in the analysis to follow.

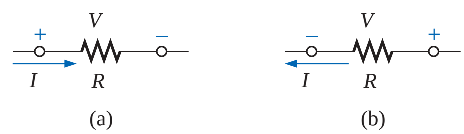


FIG. 4.3
Defining polarities.

EXAMPLE 4.3 Calculate the current through the 2-kΩ resistor of Fig. 4.4 if the voltage drop across it is 16 V.

Solution:

$$I = \frac{V}{R} = \frac{16\text{ V}}{2 \times 10^{3}\ \Omega} = \mathbf{8\ mA}$$

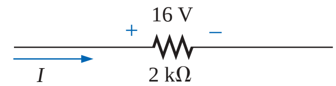


FIG. 4.4
Example 4.3.

EXAMPLE 4.4 Calculate the voltage that must be applied across the soldering iron of Fig. 4.5 to establish a current of 1.5 A through the iron if its internal resistance is 80 Ω.

Solution:

$$E = IR = (1.5\text{ A})(80\ \Omega) = \mathbf{120\ V}$$

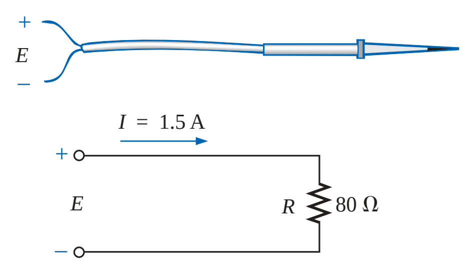


FIG. 4.5
Example 4.4.

In a number of the examples in this chapter, such as Example 4.4 above, the voltage applied is actually that obtained from an ac outlet in the home, office, or laboratory. This approach was used to provide an opportunity for the student to relate to real-world situations as soon as possible and to demonstrate that a number of the equations derived in this chapter are applicable to ac networks also. Chapter 13 will provide a direct relationship between ac and dc voltages that permits the mathematical substitutions used in this chapter. In other words, don't be concerned about the fact that some of the voltages and currents appearing in the examples of this chapter are actually ac voltages, because the equations for dc networks have exactly the same format, and all the solutions will be correct.

4.2 PLOTTING OHM'S LAW

Graphs, characteristics, plots, and the like, play an important role in every technical field as a mode through which the broad picture of the behavior or response of a system can be conveniently displayed. It is therefore critical to develop the skills necessary both to read data and to plot them in such a manner that they can be interpreted easily.

For most sets of characteristics of electronic devices, the current is represented by the vertical axis (ordinate), and the voltage by the horizontal axis (abscissa), as shown in Fig. 4.6. First note that the vertical axis is in

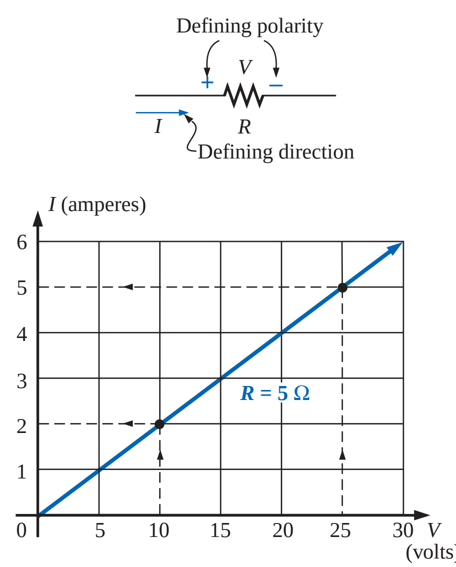


FIG. 4.6
Plotting Ohm's law.

amperes and the horizontal axis is in volts. For some plots, I may be in milliamperes (mA), microamperes (μA), or whatever is appropriate for the range of interest. The same is true for the levels of voltage on the horizontal axis. Note also that the chosen parameters require that the spacing between numerical values of the vertical axis be different from that of the horizontal axis. The linear (straight-line) graph reveals that the resistance is not changing with current or voltage level; rather, it is a fixed quantity throughout. The current direction and the voltage polarity appearing at the top of Fig. 4.6 are the defined direction and polarity for the provided plot. If the current direction is opposite to the defined direction, the region below the horizontal axis is the region of interest for the current I. If the voltage polarity is opposite to that defined, the region to the left of the current axis is the region of interest. For the standard fixed resistor, the first quadrant, or region, of Fig. 4.6 is the only region of interest. However, you will encounter many devices in your electronics courses that will use the other quadrants of a graph.

Once a graph such as Fig. 4.6 is developed, the current or voltage at any level can be found from the other quantity by simply using the resulting plot. For instance, at $V = 25$ V, if a vertical line is drawn on Fig. 4.6 to the curve as shown, the resulting current can be found by drawing a horizontal line over to the current axis, where a result of 5 A is obtained. Similarly, at $V = 10$ V, a vertical line to the plot and a horizontal line to the current axis will result in a current of 2 A, as determined by Ohm's law.

If the resistance of a plot is unknown, it can be determined at any point on the plot since a straight line indicates a fixed resistance. At any point on the plot, find the resulting current and voltage, and simply substitute into the following equation:

$$R_{dc} = \frac{V}{I} \tag{4.5}$$

To test Eq. (4.5), consider a point on the plot where $V = 20$ V and $I = 4$ A. The resulting resistance is $R_{dc} = V/I = 20\text{ V}/4\text{ A} = 5\ \Omega$. For comparison purposes, a 1-Ω and 10-Ω resistor were plotted on the graph of Fig. 4.7. Note that the less the resistance, the steeper the slope (closer to the vertical axis) of the curve.

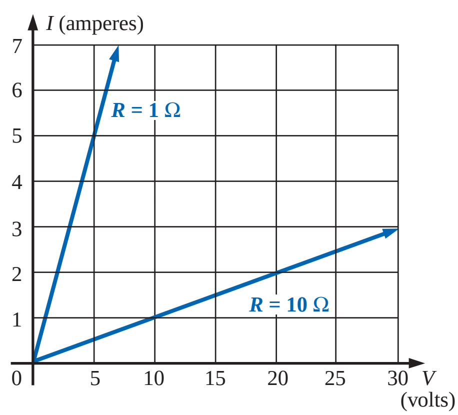


FIG. 4.7
Demonstrating on an I-V plot that the less the resistance, the steeper is the slope.

If we write Ohm's law in the following manner and relate it to the basic straight-line equation

$$\begin{array}{ccccccc} I & = & \frac{1}{R} & \cdot & E & + & 0 \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ y & = & m & \cdot & x & + & b \end{array}$$

we find that the slope is equal to 1 divided by the resistance value, as indicated by the following:

$$m = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta I}{\Delta V} = \frac{1}{R} \tag{4.6}$$

where Δ signifies a small, finite change in the variable.

Equation (4.6) clearly reveals that the greater the resistance, the less the slope. If written in the following form, Equation (4.6) can be used to determine the resistance from the linear curve:

$$R = \frac{\Delta V}{\Delta I} \quad \text{(ohms)} \tag{4.7}$$

The equation states that by choosing a particular ΔV (or ΔI), one can obtain the corresponding ΔI (or ΔV, respectively) from the graph, as shown in Fig. 4.8, and then determine the resistance. It the plot is a straight line, Equation (4.7) will provide the same result no matter where the equation is applied. However, if the plot curves at all, the resistance will change.

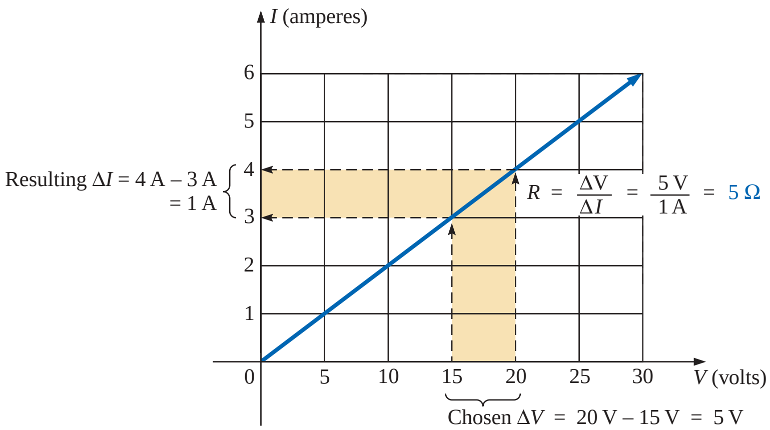


FIG. 4.8
Applying Eq. (4.6).

EXAMPLE 4.5 Determine the resistance associated with the curve of Fig. 4.9 using Eqs. (4.5) and (4.7), and compare results.

Solution: At $V = 6$ V, $I = 3$ mA, and

$$R_{dc} = \frac{V}{I} = \frac{6\text{ V}}{3\text{ mA}} = \mathbf{2\ k\Omega}$$

For the interval between 6 V and 8 V,

$$R = \frac{\Delta V}{\Delta I} = \frac{2\text{ V}}{1\text{ mA}} = \mathbf{2\ k\Omega}$$

The results are equivalent.

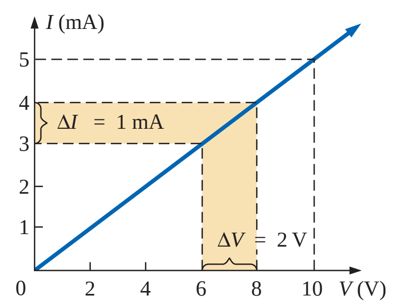


FIG. 4.9
Example 4.5.

Before leaving the subject, let us first investigate the characteristics of a very important semiconductor device called the **diode,** which will be examined in detail in basic electronics courses. This device will ideally act like a low-resistance path to current in one direction and a high-resistance path to current in the reverse direction, much like a switch that will pass current in only one direction. A typical set of characteristics appears in Fig. 4.10. Without any mathematical calculations, the closeness of the characteristic to the voltage axis for negative values of applied voltage indicates that this is the low-conductance (high resistance, switch opened) region. Note that this region extends to approximately 0.7 V positive. However, for values of applied voltage greater than 0.7 V, the vertical rise in the characteristics indicates a high-conductivity (low resistance, switch closed) region. Application of Ohm's law will now verify the above conclusions.

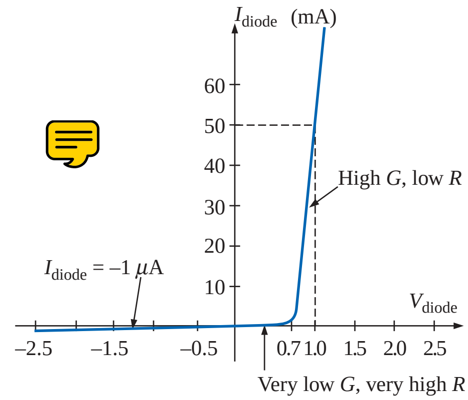


FIG. 4.10
Semiconductor diode characteristic.

At $V = +1$ V,

$$R_{diode} = \frac{V}{I} = \frac{1\text{ V}}{50\text{ mA}} = \frac{1\text{ V}}{50 \times 10^{-3}\text{ A}}$$
$$= 20\ \Omega$$

(a relatively low value for most applications)

At $V = -1$ V,

$$R_{diode} = \frac{V}{I} = \frac{1\text{ V}}{1\ \mu\text{A}}$$
$$= 1\text{ M}\Omega$$

(which is often represented by an open-circuit equivalent)

4.3 POWER

Power is an indication of how much work (the conversion of energy from one form to another) can be done in a specified amount of time, that is, a *rate* of doing work. For instance, a large motor has more power than a small motor because it can convert more electrical energy into mechanical energy in the same period of time. Since converted energy is measured in *joules* (J) and time in seconds (s), power is measured in joules/second (J/s). The electrical unit of measurement for power is the watt (W), defined by

$$1\text{ watt (W)} = 1\text{ joule/second (J/s)} \tag{4.8}$$

In equation form, power is determined by

$$P = \frac{W}{t} \quad \text{(watts, W, or joules/second, J/s)} \tag{4.9}$$

with the energy W measured in joules and the time t in seconds.

Throughout the text, the abbreviation for energy (W) can be distinguished from that for the watt (W) by the fact that one is in italics while the other is in roman. In fact, all variables in the dc section appear in italics while the units appear in roman.

The unit of measurement, the watt, is derived from the surname of James Watt (Fig. 4.11), who was instrumental in establishing the standards for power measurements. He introduced the **horsepower** (hp) as a measure of the average power of a strong dray horse over a full working day. It is approximately 50% more than can be expected from the average horse. The horsepower and watt are related in the following manner:

$$1 \text{ horsepower} \cong 746 \text{ watts}$$

The power delivered to, or absorbed by, an electrical device or system can be found in terms of the current and voltage by first substituting Eq. (2.7) into Eq. (4.9):

$$P = \frac{W}{t} = \frac{QV}{t} = V\frac{Q}{t}$$

But

$$I = \frac{Q}{t}$$

so that

$$P = VI \quad \text{(watts)} \tag{4.10}$$

By direct substitution of Ohm's law, the equation for power can be obtained in two other forms:

$$P = VI = V\left(\frac{V}{R}\right)$$

and

$$P = \frac{V^2}{R} \quad \text{(watts)} \tag{4.11}$$

or

$$P = VI = (IR)I$$

and

$$P = I^2R \quad \text{(watts)} \tag{4.12}$$

The result is that the power absorbed by the resistor of Fig. 4.12 can be found directly depending on the information available. In other words, if the current and resistance are known, it pays to use Eq. (4.12) directly, and if V and I are known, use of Eq. (4.10) is appropriate. It saves having to apply Ohm's law before determining the power.

Power can be delivered or absorbed as defined by the polarity of the voltage and the direction of the current. For all dc voltage sources, power is being *delivered* by the source if the current has the direction appearing in Fig. 4.13(a). Note that the current has the same direction as established by the source in a single-source network. If the current direction and polarity are as shown in Fig. 4.13(b) due to a multisource network, the battery is absorbing power much as when a battery is being charged.

For resistive elements, all the power delivered is dissipated in the form of heat because the voltage polarity is defined by the current direction (and vice versa), and current will always enter the terminal of higher potential corresponding with the absorbing state of Fig. 4.13(b). A reversal of the current direction in Fig. 4.12 will also reverse the polarity of the voltage across the resistor and match the conditions of Fig. 4.13(b).

Scottish (Greenock, Birmingham)
(1736–1819)
Instrument Maker and Inventor
Elected Fellow of the Royal Society of London in 1785

Courtesy of the Smithsonian Institution Photo No. 30,391

In 1757, at the age of 21, used his innovative talents to design mathematical instruments such as the *quadrant, compass,* and various *scales*. In 1765, introduced the use of a separate *condenser* to increase the efficiency of steam engines. In the years to follow he received a number of important patents on improved engine design, including a rotary motion for the steam engine (versus the reciprocating action) and a double-action engine, in which the piston pulled as well as pushed in its cyclic motion. Introduced the term **horsepower** as the average power of a strong dray (small cart) horse over a full working day.

FIG. 4.11
James Watt.

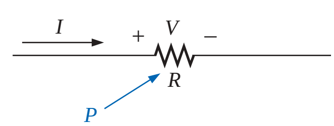
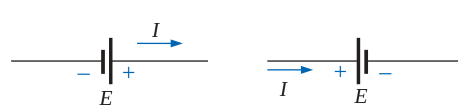


FIG. 4.12
Defining the power to a resistive element.



(a) (b)

FIG. 4.13
Battery power: (a) supplied; (b) absorbed.

The magnitude of the power delivered or absorbed by a battery is given by

$$P = EI \quad \text{(watts)} \tag{4.13}$$

with E the battery terminal voltage and I the current through the source.

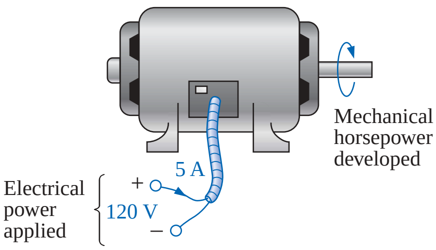


FIG. 4.14
Example 4.6.

EXAMPLE 4.6 Find the power delivered to the dc motor of Fig. 4.14.

Solution:

$$P = VI = (120\text{ V})(5\text{ A}) = 600\text{ W} = \mathbf{0.6\ kW}$$

EXAMPLE 4.7 What is the power dissipated by a 5-Ω resistor if the current is 4 A?

Solution:

$$P = I^2R = (4\text{ A})^2(5\ \Omega) = \mathbf{80\ W}$$

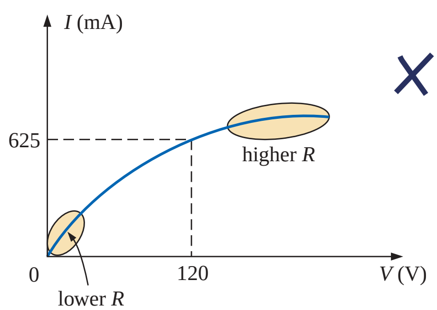


FIG. 4.15
The nonlinear I-V characteristics of a 75-W light bulb.

EXAMPLE 4.8 The *I-V* characteristics of a light bulb are provided in Fig. 4.15. Note the nonlinearity of the curve, indicating a wide range in resistance of the bulb with applied voltage as defined by the discussion of Section 4.2. If the rated voltage is 120 V, find the wattage rating of the bulb. Also calculate the resistance of the bulb under rated conditions.

Solution: At 120 V,

$$I = 0.625\text{ A}$$

and

$$P = VI = (120\text{ V})(0.625\text{ A}) = \mathbf{75\ W}$$

At 120 V,

$$R = \frac{V}{I} = \frac{120\text{ V}}{0.625\text{ A}} = \mathbf{192\ \Omega}$$

Sometimes the power is given and the current or voltage must be determined. Through algebraic manipulations, an equation for each variable is derived as follows:

$$P = I^2R \Rightarrow I^2 = \frac{P}{R}$$

and

$$I = \sqrt{\frac{P}{R}} \quad \text{(amperes)} \tag{4.14}$$

$$P = \frac{V^2}{R} \Rightarrow V^2 = PR$$

and

$$V = \sqrt{PR} \quad \text{(volts)} \tag{4.15}$$

EXAMPLE 4.9 Determine the current through a 5-kΩ resistor when the power dissipated by the element is 20 mW.

Solution: Eq. (4.14):

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{20 \times 10^{-3}\,\text{W}}{5 \times 10^{3}\,\Omega}} = \sqrt{4 \times 10^{-6}} = 2 \times 10^{-3}\,\text{A}$$
$$= \mathbf{2\ mA}$$

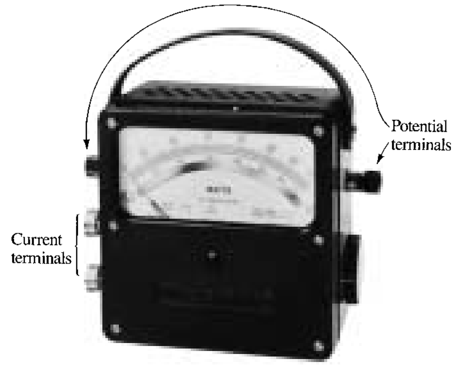
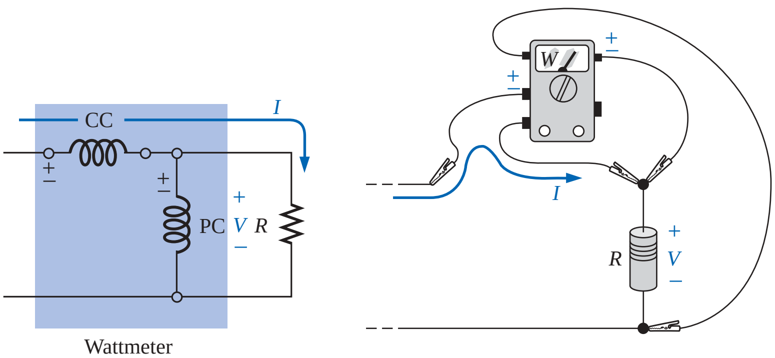


FIG. 4.16
Wattmeter. (Courtesy of Electrical Instrument Service, Inc.)

4.4 WATTMETERS

As one might expect, there are instruments that can measure the power delivered by a source and to a dissipative element. One such instrument, the **wattmeter,** appears in Fig. 4.16. Since power is a function of both the current and the voltage levels, four terminals must be connected as shown in Fig. 4.17 to measure the power to the resistor *R*.



(a) (b)

FIG. 4.17
Wattmeter connections.

If the current coils (CC) and potential coils (PC) of the wattmeter are connected as shown in Fig. 4.17, there will be an up-scale reading on the wattmeter. A reversal of either coil will result in a below-zero indication. Three voltage terminals may be available on the voltage side to permit a choice of voltage levels. On most wattmeters, the current terminals are physically larger than the voltage terminals to provide safety and to ensure a solid connection.

4.5 EFFICIENCY

A flowchart for the energy levels associated with any system that converts energy from one form to another is provided in Fig. 4.18. Take particular note of the fact that the output energy level must always be less than the applied energy due to losses and storage within the system. The best one can hope for is that W_o and W_i are relatively close in magnitude.

Conservation of energy requires that

Energy input = energy output + energy lost or stored in the system

Dividing both sides of the relationship by *t* gives

$$\frac{W_{\text{in}}}{t} = \frac{W_{\text{out}}}{t} + \frac{W_{\text{lost or stored by the system}}}{t}$$

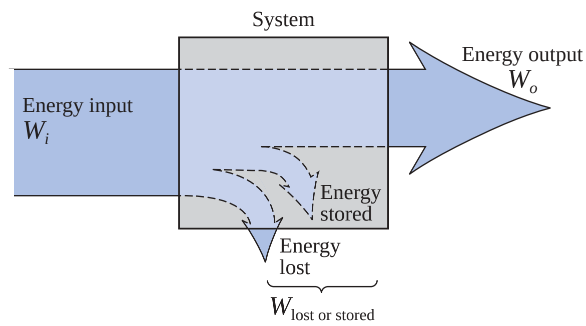


FIG. 4.18
Energy flow through a system.

Since $P = W/t$, we have the following:

$$P_i = P_o + P_{\text{lost or stored}} \quad \text{(W)} \qquad \textbf{(4.16)}$$

The **efficiency** (η) of the system is then determined by the following equation:

$$\text{Efficiency} = \frac{\text{power output}}{\text{power input}}$$

and

$$\eta = \frac{P_o}{P_i} \quad \text{(decimal number)} \qquad \textbf{(4.17)}$$

where η (lowercase Greek letter eta) is a decimal number. Expressed as a percentage,

$$\eta\% = \frac{P_o}{P_i} \times 100\% \quad \text{(percent)} \qquad \textbf{(4.18)}$$

In terms of the input and output energy, the efficiency in percent is given by

$$\eta\% = \frac{W_o}{W_i} \times 100\% \quad \text{(percent)} \qquad \textbf{(4.19)}$$

The maximum possible efficiency is 100%, which occurs when $P_o = P_i$, or when the power lost or stored in the system is zero. Obviously, the greater the internal losses of the system in generating the necessary output power or energy, the lower the net efficiency.

EXAMPLE 4.10 A 2-hp motor operates at an efficiency of 75%. What is the power input in watts? If the applied voltage is 220 V, what is the input current?

Solution:

$$\eta\% = \frac{P_o}{P_i} \times 100\%$$

$$0.75 = \frac{(2\text{ hp})(746\text{ W/hp})}{P_i}$$

and $$P_i = \frac{1492\text{ W}}{0.75} = \mathbf{1989.33\text{ W}}$$

$$P_i = EI \quad \text{or} \quad I = \frac{P_i}{E} = \frac{1989.33\text{ W}}{220\text{ V}} = \mathbf{9.04\text{ A}}$$

EXAMPLE 4.11 What is the output in horsepower of a motor with an efficiency of 80% and an input current of 8 A at 120 V?

Solution:

$$\eta\% = \frac{P_o}{P_i} \times 100\%$$

$$0.80 = \frac{P_o}{(120\text{ V})(8\text{ A})}$$

and $$P_o = (0.80)(120\text{ V})(8\text{ A}) = 768\text{ W}$$

with $$768\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \mathbf{1.029\text{ hp}}$$

EXAMPLE 4.12 If $\eta = 0.85$, determine the output energy level if the applied energy is 50 J.

Solution:

$$\eta = \frac{W_o}{W_i} \Rightarrow W_o = \eta W_i$$

$$= (0.85)(50\text{ J})$$
$$= \mathbf{42.5\text{ J}}$$

The very basic components of a generating (voltage) system are depicted in Fig. 4.19. The source of mechanical power is a structure such as a paddlewheel that is turned by water rushing over the dam. The gear train will then ensure that the rotating member of the generator is turning at rated speed. The output voltage must then be fed through a transmission system to the load. For each component of the system, an

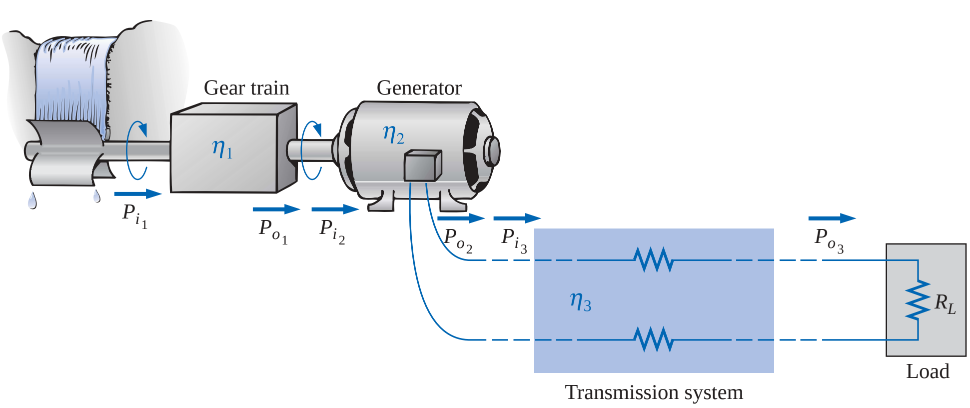


FIG. 4.19
Basic components of a generating system.

input and output power have been indicated. The efficiency of each system is given by

$$\eta_1 = \frac{P_{o_1}}{P_{i_1}} \qquad \eta_2 = \frac{P_{o_2}}{P_{i_2}} \qquad \eta_3 = \frac{P_{o_3}}{P_{i_3}}$$

If we form the product of these three efficiencies,

$$\eta_1 \cdot \eta_2 \cdot \eta_3 = \frac{P_{o_1}}{P_{i_1}} \cdot \frac{P_{o_2}}{P_{i_2}} \cdot \frac{P_{o_3}}{P_{i_3}}$$

and substitute the fact that $P_{i_2} = P_{o_1}$ and $P_{i_3} = P_{o_2}$, we find that the quantities indicated above will cancel, resulting in P_{o_3}/P_{i_1}, which is a measure of the efficiency of the entire system. In general, for the representative cascaded system of Fig. 4.20,

$$\boxed{\eta_{\text{total}} = \eta_1 \cdot \eta_2 \cdot \eta_3 \cdots \eta_n} \qquad \textbf{(4.20)}$$

FIG. 4.20
Cascaded system.

EXAMPLE 4.13 Find the overall efficiency of the system of Fig. 4.19 if $\eta_1 = 90\%$, $\eta_2 = 85\%$, and $\eta_3 = 95\%$.

Solution:

$$\eta_T = \eta_1 \cdot \eta_2 \cdot \eta_3 = (0.90)(0.85)(0.95) = 0.727, \text{ or } \mathbf{72.7\%}$$

EXAMPLE 4.14 If the efficiency η_1 drops to 40%, find the new overall efficiency and compare the result with that obtained in Example 4.13.

Solution:

$$\eta_T = \eta_1 \cdot \eta_2 \cdot \eta_3 = (0.40)(0.85)(0.95) = 0.323, \text{ or } \mathbf{32.3\%}$$

Certainly 32.3% is noticeably less than 72.7%. The total efficiency of a cascaded system is therefore determined primarily by the lowest efficiency (weakest link) and is less than (or equal to if the remaining efficiencies are 100%) the least efficient link of the system.

4.6 ENERGY

For power, which is the rate of doing work, to produce an energy conversion of any form, it must be *used over a period of time.* For example, a motor may have the horsepower to run a heavy load, but unless the motor is *used* over a period of time, there will be no energy conversion. In addition, the longer the motor is used to drive the load, the greater will be the energy expended.

The **energy** (W) lost or gained by any system is therefore determined by

$$\boxed{W = Pt} \qquad \text{(wattseconds, Ws, or joules)} \qquad \textbf{(4.21)}$$

Since power is measured in watts (or joules per second) and time in seconds, the unit of energy is the *wattsecond* or *joule* (note Fig. 4.21) as indicated above. The wattsecond, however, is too small a quantity for most practical purposes, so the *watthour* (Wh) and *kilowatthour* (kWh) were defined, as follows:

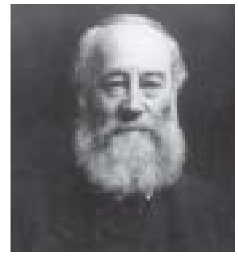
$$\text{Energy (Wh)} = \text{power (W)} \times \text{time (h)} \tag{4.22}$$

$$\text{Energy (kWh)} = \frac{\text{power (W)} \times \text{time (h)}}{1000} \tag{4.23}$$

Note that the energy in kilowatthours is simply the energy in watthours divided by 1000. To develop some sense for the kilowatthour energy level, consider that 1 kWh is the energy dissipated by a 100-W bulb in 10 h.

The **kilowatthour meter** is an instrument for measuring the energy supplied to the residential or commercial user of electricity. It is normally connected directly to the lines at a point just prior to entering the power distribution panel of the building. A typical set of dials is shown in Fig. 4.22(a) with a photograph of an analog kilowatthour meter. As indicated, each power of ten below a dial is in kilowatthours. The more rapidly the aluminum disc rotates, the greater the energy demand. The dials are connected through a set of gears to the rotation of this disc. A solid-state digital meter with an extended range of capabilities appears in Fig. 4.22(b).

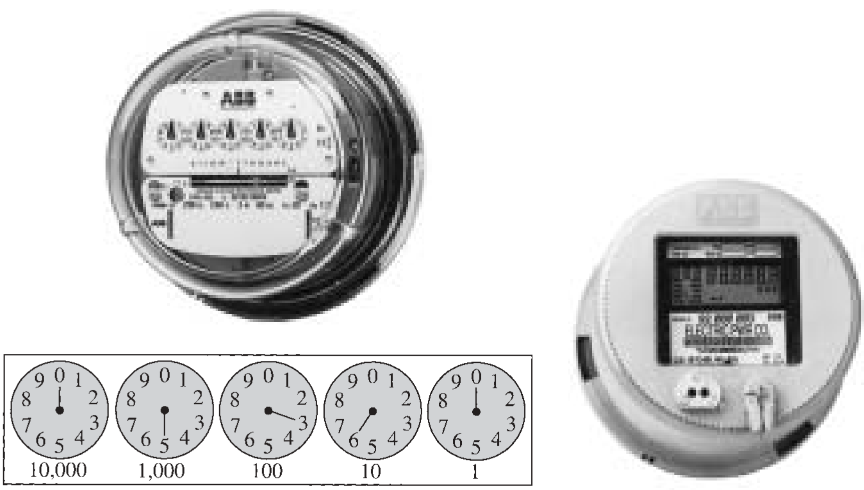
British (Salford, Manchester)
(1818–89)
Physicist
Honorary Doctorates from the Universities of Dublin and Oxford



Bettmann Archive Photo Number 076800P

Contributed to the important fundamental *law of conservation of energy* by establishing that various forms of energy, whether electrical, mechanical, or heat, are in the same family and can be exchanged from one form to another. In 1841 introduced *Joule's law*, which stated that the heat developed by electric current in a wire is proportional to the product of the current squared and the resistance of the wire (I^2R). He further determined that the heat emitted was equivalent to the power absorbed and therefore heat is a form of energy.

FIG. 4.21
James Prescott Joule.



(a) (b)

FIG. 4.22
Kilowatthour meters: (a) analog; (b) digital. (Courtesy of ABB Electric Metering Systems.)

EXAMPLE 4.15 For the dial positions of Fig. 4.22(a), calculate the electricity bill if the previous reading was 4650 kWh and the average cost is 9¢ per kilowatthour.

Solution:

$$5360 \text{ kWh} - 4650 \text{ kWh} = 710 \text{ kWh used}$$

$$710 \cancel{\text{kWh}}\left(\frac{9¢}{\cancel{\text{kWh}}}\right) = \mathbf{\$63.90}$$

EXAMPLE 4.16 How much energy (in kilowatthours) is required to light a 60-W bulb continuously for 1 year (365 days)?

Solution:

$$W = \frac{Pt}{1000} = \frac{(60\text{ W})(24\text{ h/day})(365\text{ days})}{1000} = \frac{525{,}600\text{ Wh}}{1000}$$
$$= \mathbf{525.60\ kWh}$$

EXAMPLE 4.17 How long can a 205-W television set be on before using more than 4 kWh of energy?

Solution:

$$W = \frac{Pt}{1000} \Rightarrow t\text{ (hours)} = \frac{(W)(1000)}{P}$$
$$= \frac{(4\text{ kWh})(1000)}{205\text{ W}} = \mathbf{19.51\ h}$$

EXAMPLE 4.18 What is the cost of using a 5-hp motor for 2 h if the rate is 9¢ per kilowatthour?

Solution:

$$W\text{ (kilowatthours)} = \frac{Pt}{1000} = \frac{(5\text{ hp} \times 746\text{ W/hp})(2\text{ h})}{1000} = 7.46\text{ kWh}$$
$$\text{Cost} = (7.46\text{ kWh})(9¢/\text{kWh}) = \mathbf{67.14¢}$$

EXAMPLE 4.19 What is the total cost of using all of the following at 9¢ per kilowatthour?

A 1200-W toaster for 30 min
Six 50-W bulbs for 4 h
A 400-W washing machine for 45 min
A 4800-W electric clothes dryer for 20 min

Solution:

$$W = \frac{(1200\text{ W})(\frac{1}{2}\text{ h}) + (6)(50\text{ W})(4\text{ h}) + (400\text{ W})(\frac{3}{4}\text{ h}) + (4800\text{ W})(\frac{1}{3}\text{ h})}{1000}$$
$$= \frac{600\text{ Wh} + 1200\text{ Wh} + 300\text{ Wh} + 1600\text{ Wh}}{1000} = \frac{3700\text{ Wh}}{1000}$$
$$W = 3.7\text{ kWh}$$
$$\text{Cost} = (3.7\text{ kWh})(9¢/\text{kWh}) = \mathbf{33.3¢}$$

The chart in Fig. 4.23 shows the average cost per kilowatthour compared to the kilowatthours used per customer. Note that the cost today is above the level of 1926 and the average customer uses more than 20 times as much electrical energy in a year. Keep in mind that the chart of Fig. 4.23 is the average cost across the nation. Some states have average rates close to 5¢ per kilowatthour, whereas others approach 12¢ per kilowatthour.

Table 4.1 lists some common household appliances with their typical wattage ratings. It might prove interesting for the reader to calculate the cost of operating some of these appliances over a period of time using the chart in Fig. 4.23 to find the cost per kilowatthour.

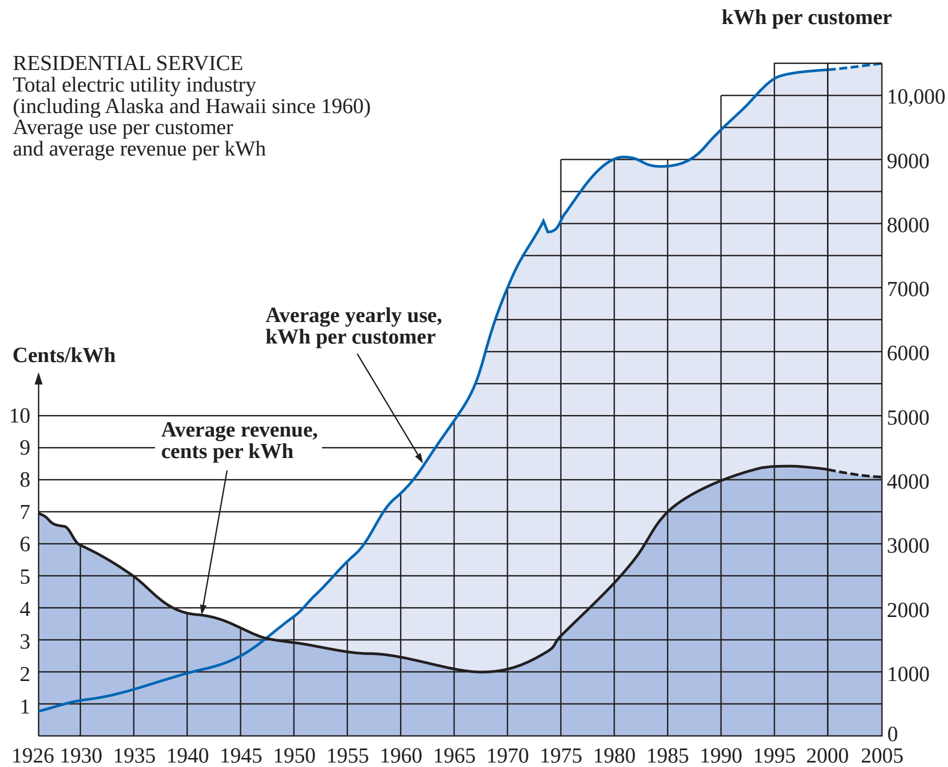


FIG. 4.23

Cost per kWh and average kWh per customer versus time. (Courtesy of Edison Electric Institute.)

TABLE 4.1

Typical wattage ratings of some common household items.

Appliance	Wattage Rating	Appliance	Wattage Rating
Air conditioner	860	Lap-top computer:	
Blow dryer	1,300	Sleep	< 1 W (Typically 0.3 W to 0.5 W)
Cassette player/recorder	5	Normal	10–20 W
Cellular phone:		High	25–35 W
Standby	$\cong$ 35 mW	Microwave oven	1,200
Talk	$\cong$ 4.3 W	Pager	1–2 mW
Clock	2	Phonograph	75
Clothes dryer (electric)	4,800	Projector	1,200
Coffee maker	900	Radio	70
Dishwasher	1,200	Range (self-cleaning)	12,200
Fan:		Refrigerator (automatic defrost)	1,800
Portable	90	Shaver	15
Window	200	Stereo equipment	110
Heater	1,322	Sun lamp	280
Heating equipment:		Toaster	1,200
Furnace fan	320	Trash compactor	400
Oil-burner motor	230	TV (color)	200
Iron, dry or steam	1,100	Videocassette recorder	110
		Washing machine	500
		Water heater	4,500

Courtesy of General Electric Co.