5

Series Circuits

5.1 INTRODUCTION

Two types of current are readily available to the consumer today. One is *direct current* (dc), in which ideally the flow of charge (current) does not change in magnitude (or direction) with time. The other is *sinusoidal alternating current* (ac), in which the flow of charge is continually changing in magnitude (and direction) with time. The next few chapters are an introduction to circuit analysis purely from a dc approach. The methods and concepts will be discussed in detail for direct current; when possible, a short discussion will suffice to cover any variations we might encounter when we consider ac in the later chapters.

The battery of Fig. 5.1, by virtue of the potential difference between its terminals, has the ability to cause (or "pressure") charge to flow through the simple circuit. The positive terminal attracts the electrons through the wire at the same rate at which electrons are supplied by the negative terminal. As long as the battery is connected in the circuit and maintains its terminal characteristics, the current (dc) through the circuit will not change in magnitude or direction.

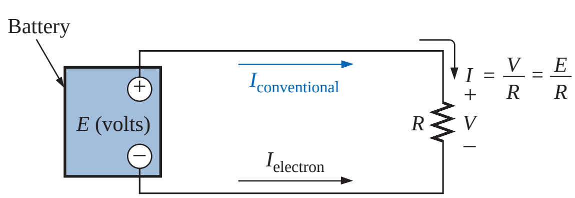

FIG. 5.1
Introducing the basic components of an electric circuit.

If we consider the wire to be an ideal conductor (that is, having no opposition to flow), the potential difference V across the resistor will equal the applied voltage of the battery: V (volts) $= E$ (volts).

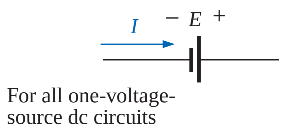

FIG. 5.2
Defining the direction of conventional flow for single-source dc circuits.

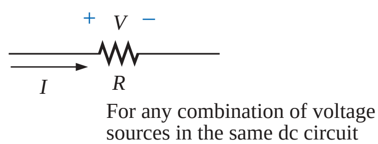

FIG. 5.3
Defining the polarity resulting from a conventional current I through a resistive element.

The current is limited only by the resistor R. The higher the resistance, the less the current, and conversely, as determined by Ohm's law.

By convention (as discussed in Chapter 2), the direction of **conventional current flow** $I_{conventional}$ as shown in Fig. 5.1 is opposite to that of **electron flow** ($I_{electron}$). Also, the uniform flow of charge dictates that the direct current I be the same everywhere in the circuit. By following the direction of conventional flow, we notice that there is a rise in potential across the battery ($-$ to $+$), and a drop in potential across the resistor ($+$ to $-$). For single-voltage-source dc circuits, conventional flow always passes from a low potential to a high potential when passing through a voltage source, as shown in Fig. 5.2. However, conventional flow always passes from a high to a low potential when passing through a resistor for any number of voltage sources in the same circuit, as shown in Fig. 5.3.

The circuit of Fig. 5.1 is the simplest possible configuration. This chapter and the chapters to follow will add elements to the system in a very specific manner to introduce a range of concepts that will form a major part of the foundation required to analyze the most complex system. Be aware that the laws, rules, and so on, introduced in Chapters 5 and 6 will be used throughout your studies of electrical, electronic, or computer systems. They will not be dropped for a more advanced set as you progress to more sophisticated material. It is therefore critical that the concepts be understood thoroughly and that the various procedures and methods be applied with confidence.

5.2 SERIES CIRCUITS

A **circuit** consists of any number of elements joined at terminal points, providing at least one closed path through which charge can flow. The circuit of Fig. 5.4(a) has three elements joined at three terminal points (a, b, and c) to provide a closed path for the current I.

Two elements are in series if

1. ***They have only one terminal in common (i.e., one lead of one is connected to only one lead of the other).***
2. ***The common point between the two elements is not connected to another current-carrying element.***

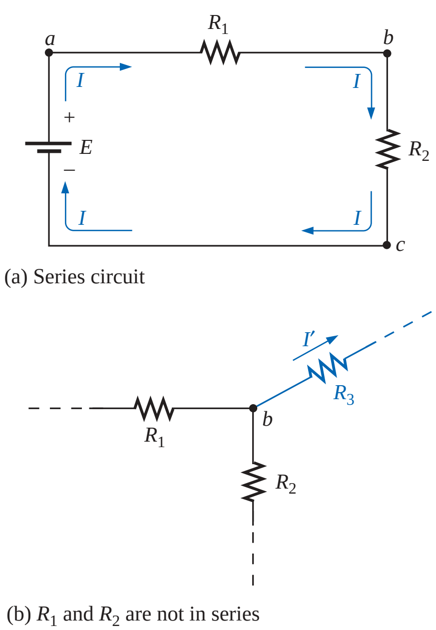

FIG. 5.4
(a) Series circuit; (b) situation in which R_1 and R_2 are not in series.

In Fig. 5.4(a), the resistors R_1 and R_2 are in series because they have *only* point b in common. The other ends of the resistors are connected elsewhere in the circuit. For the same reason, the battery E and resistor R_1 are in series (terminal a in common), and the resistor R_2 and the battery E are in series (terminal c in common). Since all the elements are in series, the network is called a **series circuit.** Two common examples of series connections include the tying of small pieces of rope together to form a longer rope and the connecting of pipes to get water from one point to another.

If the circuit of Fig. 5.4(a) is modified such that a current-carrying resistor R_3 is introduced, as shown in Fig. 5.4(b), the resistors R_1 and R_2 are no longer in series due to a violation of number 2 of the above definition of series elements.

The current is the same through series elements.

For the circuit of Fig. 5.4(a), therefore, the current I through each resistor is the same as that through the battery. The fact that the current is the same through series elements is often used as a path to determine whether two elements are in series or to confirm a conclusion.

A **branch** of a circuit is any portion of the circuit that has one or more elements in series. In Fig. 5.4(a), the resistor R_1 forms one branch of the circuit, the resistor R_2 another, and the battery E a third.

The total resistance of a series circuit is the sum of the resistance levels.

In Fig. 5.4(a), for example, the total resistance (R_T) is equal to $R_1 + R_2$. Note that the total resistance is actually the resistance "seen" by the battery as it "looks" into the series combination of elements as shown in Fig. 5.5.

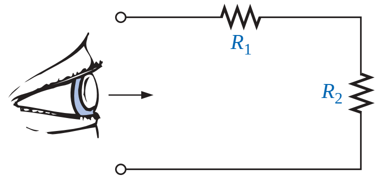

FIG. 5.5
Resistance "seen" by source.

In general, to find the total resistance of N resistors in series, the following equation is applied:

$$\boxed{R_T = R_1 + R_2 + R_3 + \cdots + R_N} \quad \text{(ohms, } \Omega) \quad \textbf{(5.1)}$$

Once the total resistance is known, the circuit of Fig. 5.4(a) can be redrawn as shown in Fig. 5.6, clearly revealing that the only resistance the source "sees" is the total resistance. It is totally unaware of how the elements are connected to establish R_T. Once R_T is known, the current drawn from the source can be determined using Ohm's law, as follows:

$$\boxed{I_s = \frac{E}{R_T}} \quad \text{(amperes, A)} \quad \textbf{(5.2)}$$

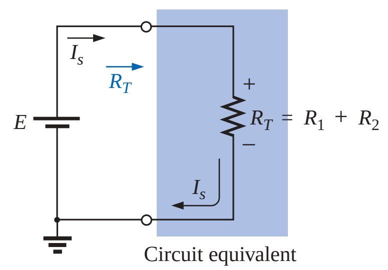

FIG. 5.6
Replacing the series resistors R_1 and R_2 of Fig. 5.5 with the total resistance.

Since E is fixed, the magnitude of the source current will be totally dependent on the magnitude of R_T. A larger R_T will result in a relatively small value of I_s, while lesser values of R_T will result in increased current levels.

The fact that the current is the same through each element of Fig. 5.4(a) permits a direct calculation of the voltage across each resistor using Ohm's law; that is,

$$\boxed{V_1 = IR_1,\ V_2 = IR_2,\ V_3 = IR_3,\ \ldots,\ V_N = IR_N} \quad \text{(volts, V)} \quad \textbf{(5.3)}$$

The power delivered to each resistor can then be determined using any one of three equations as listed below for R_1:

$$\boxed{P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}} \quad \text{(watts, W)} \quad \textbf{(5.4)}$$

The power delivered by the source is

$$\boxed{P_{\text{del}} = EI} \quad \text{(watts, W)} \quad \textbf{(5.5)}$$

The total power delivered to a resistive circuit is equal to the total power dissipated by the resistive elements.

That is,

$$\boxed{P_{\text{del}} = P_1 + P_2 + P_3 + \cdots + P_N} \quad \textbf{(5.6)}$$

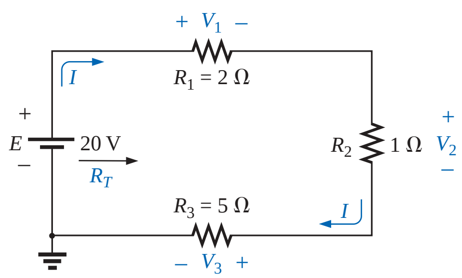

FIG. 5.7
Example 5.1.

EXAMPLE 5.1

a. Find the total resistance for the series circuit of Fig. 5.7.
b. Calculate the source current I_s.
c. Determine the voltages V_1, V_2, and V_3.
d. Calculate the power dissipated by R_1, R_2, and R_3.
e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d).

Solutions:

a. $R_T = R_1 + R_2 + R_3 = 2\ \Omega + 1\ \Omega + 5\ \Omega = \mathbf{8\ \Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{20\ \text{V}}{8\ \Omega} = \mathbf{2.5\ A}$

c. $V_1 = IR_1 = (2.5\ \text{A})(2\ \Omega) = \mathbf{5\ V}$
$V_2 = IR_2 = (2.5\ \text{A})(1\ \Omega) = \mathbf{2.5\ V}$
$V_3 = IR_3 = (2.5\ \text{A})(5\ \Omega) = \mathbf{12.5\ V}$

d. $P_1 = V_1I_1 = (5\ \text{V})(2.5\ \text{A}) = \mathbf{12.5\ W}$
$P_2 = I_2^2R_2 = (2.5\ \text{A})^2(1\ \Omega) = \mathbf{6.25\ W}$
$P_3 = V_3^2/R_3 = (12.5\ \text{V})^2/5\ \Omega = \mathbf{31.25\ W}$

e. $P_{\text{del}} = EI = (20\ \text{V})(2.5\ \text{A}) = \mathbf{50\ W}$
$P_{\text{del}} = P_1 + P_2 + P_3$
$50\ \text{W} = 12.5\ \text{W} + 6.25\ \text{W} + 31.25\ \text{W}$
$\underline{50\ \text{W} = 50\ \text{W} \quad \text{(checks)}}$

To find the total resistance of N resistors of the same value in series, simply multiply the value of *one* of the resistors by the number in series; that is,

$$\boxed{R_T = NR} \tag{5.7}$$

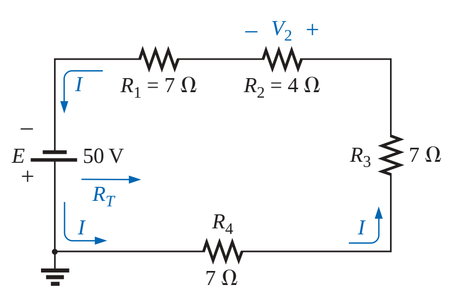

FIG. 5.8
Example 5.2.

EXAMPLE 5.2 Determine R_T, I, and V_2 for the circuit of Fig. 5.8.

Solution: Note the current direction as established by the battery and the polarity of the voltage drops across R_2 as determined by the current direction. Since $R_1 = R_3 = R_4$,

$$R_T = NR_1 + R_2 = (3)(7\ \Omega) + 4\ \Omega = 21\ \Omega + 4\ \Omega = \mathbf{25\ \Omega}$$

$$I = \frac{E}{R_T} = \frac{50\ \text{V}}{25\ \Omega} = \mathbf{2\ A}$$

$$V_2 = IR_2 = (2\ \text{A})(4\ \Omega) = \mathbf{8\ V}$$

Examples 5.1 and 5.2 are straightforward substitution-type problems that are relatively easy to solve with some practice. Example 5.3, however, is evidence of another type of problem that requires a firm grasp of the fundamental equations and an ability to identify which equation to use first. The best preparation for this type of exercise is simply to work through as many problems of this kind as possible.

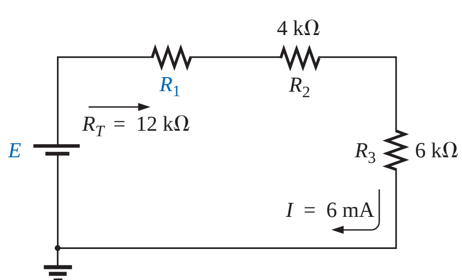

FIG. 5.9
Example 5.3.

EXAMPLE 5.3 Given R_T and I, calculate R_1 and E for the circuit of Fig. 5.9.

Solution:

$$R_T = R_1 + R_2 + R_3$$
$$12\ \text{k}\Omega = R_1 + 4\ \text{k}\Omega + 6\ \text{k}\Omega$$
$$R_1 = 12\ \text{k}\Omega - 10\ \text{k}\Omega = \mathbf{2\ k\Omega}$$
$$E = IR_T = (6 \times 10^{-3}\ \text{A})(12 \times 10^{3}\ \Omega) = \mathbf{72\ V}$$

5.3 VOLTAGE SOURCES IN SERIES

Voltage sources can be connected in series, as shown in Fig. 5.10, to increase or decrease the total voltage applied to a system. The net voltage is determined simply by summing the sources with the same polarity and subtracting the total of the sources with the opposite "pressure." The net polarity is the polarity of the larger sum.

In Fig. 5.10(a), for example, the sources are all "pressuring" current to the right, so the net voltage is

$$E_T = E_1 + E_2 + E_3 = 10\ \text{V} + 6\ \text{V} + 2\ \text{V} = 18\ \text{V}$$

as shown in the figure. In Fig. 5.10(b), however, the greater "pressure" is to the left, with a net voltage of

$$E_T = E_2 + E_3 - E_1 = 9\ \text{V} + 3\ \text{V} - 4\ \text{V} = 8\ \text{V}$$

and the polarity shown in the figure.

5.4 KIRCHHOFF'S VOLTAGE LAW

Note Fig. 5.11.

Kirchhoff's voltage law ***(KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.***

A **closed loop** is any continuous path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit. In Fig. 5.12, by following the current, we can trace a continuous path that leaves point *a* through R_1 and returns through E without leaving the circuit. Therefore, *abcda* is a closed loop. For us to be able to apply Kirchhoff's voltage law, the summation of potential rises and drops must be made in one direction around the closed loop.

For uniformity, the clockwise (CW) direction will be used throughout the text for all applications of Kirchhoff's voltage law. Be aware, however, that the same result will be obtained if the counterclockwise (CCW) direction is chosen and the law applied correctly.

A plus sign is assigned to a potential rise ($-$ to $+$), and a minus sign to a potential drop ($+$ to $-$). If we follow the current in Fig. 5.12 from point *a*, we first encounter a potential drop V_1 ($+$ to $-$) across R_1 and then another potential drop V_2 across R_2. Continuing through the voltage source, we have a potential rise E ($-$ to $+$) before returning to point *a*. In symbolic form, where Σ represents summation, $\circlearrowright$ the closed loop, and V the potential drops and rises, we have

$$\boxed{\Sigma_{\circlearrowright}\, V = 0} \quad \text{(Kirchhoff's voltage law in symbolic form)} \qquad \textbf{(5.8)}$$

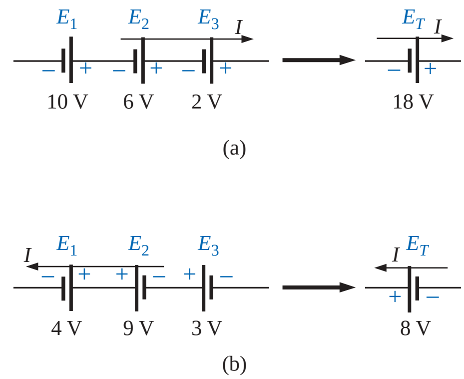

FIG. 5.10
Reducing series dc voltage sources to a single source.

German (Königsberg, Berlin)
(1824–87)
Physicist
Professor of Physics, University of Heidelberg

Courtesy of the Smithsonian Institution Photo No. 58,283

Although a contributor to a number of areas in the physics domain, he is best known for his work in the electrical area with his definition of the relationships between the currents and voltages of a network in 1847. Did extensive research with German chemist Robert Bunsen (developed the *Bunsen burner*), resulting in the discovery of the important elements of *cesium* and *rubidium*.

FIG. 5.11
Gustav Robert Kirchhoff.

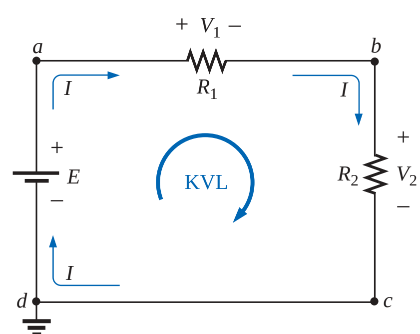

FIG. 5.12
Applying Kirchhoff's voltage law to a series dc circuit.

which for the circuit of Fig. 5.12 yields (clockwise direction, following the current I and starting at point d):

$$+E - V_1 - V_2 = 0$$

or $$E = V_1 + V_2$$

revealing that

the applied voltage of a series circuit equals the sum of the voltage drops across the series elements.

Kirchhoff's voltage law can also be stated in the following form:

$$\boxed{\Sigma_{\circlearrowright} V_{\text{rises}} = \Sigma_{\circlearrowright} V_{\text{drops}}} \qquad \textbf{(5.9)}$$

which in words states that the sum of the rises around a closed loop must equal the sum of the drops in potential. The text will emphasize the use of Eq. (5.8), however.

If the loop were taken in the counterclockwise direction starting at point a, the following would result:

$$\Sigma_{\circlearrowleft} V = 0$$

$$-E + V_2 + V_1 = 0$$

or, as before, $$E = V_1 + V_2$$

The application of Kirchhoff's voltage law need not follow a path that includes current-carrying elements.

For example, in Fig. 5.13 there is a difference in potential between points a and b, even though the two points are not connected by a current-carrying element. Application of Kirchhoff's voltage law around the closed loop will result in a difference in potential of 4 V between the two points. That is, using the clockwise direction:

$$+12\text{ V} - V_x - 8\text{ V} = 0$$

and $$V_x = \mathbf{4\text{ V}}$$

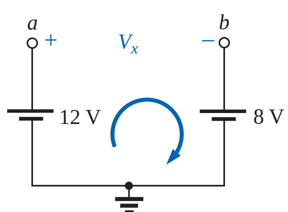

FIG. 5.13
Demonstration that a voltage can exist between two points not connected by a current-carrying conductor.

EXAMPLE 5.4 Determine the unknown voltages for the networks of Fig. 5.14.

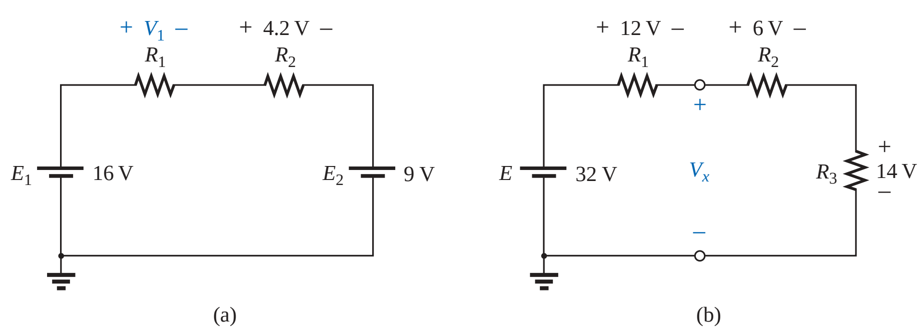

FIG. 5.14
Example 5.4.

Solution: When applying Kirchhoff's voltage law, be sure to concentrate on the polarities of the voltage rise or drop rather than on the type of element. In other words, do not treat a voltage drop across a resistive element differently from a voltage drop across a source. If the polarity dictates that a drop has occurred, that is the important fact when applying the law. In Fig. 5.14(a), for instance, if we choose the clockwise direction, we will find that there is a drop across the resistors R_1 and R_2 and a drop across the source E_2. All will therefore have a minus sign when Kirchhoff's voltage law is applied.

Application of Kirchhoff's voltage law to the circuit of Fig. 5.14(a) in the clockwise direction will result in

$$+E_1 - V_1 - V_2 - E_2 = 0$$

and
$$V_1 = E_1 - V_2 - E_2 = 16\text{ V} - 4.2\text{ V} - 9\text{ V}$$
$$= \mathbf{2.8\text{ V}}$$

The result clearly indicates that there was no need to know the values of the resistors or the current to determine the unknown voltage. Sufficient information was carried by the other voltage levels to permit a determination of the unknown.

In Fig. 5.14(b) the unknown voltage is not across a current-carrying element. However, as indicated in the paragraphs above, Kirchhoff's voltage law is not limited to current-carrying elements. In this case there are two possible paths for finding the unknown. Using the clockwise path, including the voltage source E, will result in

$$+E - V_1 - V_x = 0$$

and
$$V_x = E - V_1 = 32\text{ V} - 12\text{ V}$$
$$= \mathbf{20\text{ V}}$$

Using the clockwise direction for the other loop involving R_2 and R_3 will result in

$$+V_x - V_2 - V_3 = 0$$

and
$$V_x = V_2 + V_3 = 6\text{ V} + 14\text{ V}$$
$$= \mathbf{20\text{ V}}$$

matching the result above.

EXAMPLE 5.5 Find V_1 and V_2 for the network of Fig. 5.15.

Solution: For path 1, starting at point a in a clockwise direction:

$$+25\text{ V} - V_1 + 15\text{ V} = 0$$

and
$$V_1 = \mathbf{40\text{ V}}$$

For path 2, starting at point a in a clockwise direction:

$$-V_2 - 20\text{ V} = 0$$

and
$$V_2 = \mathbf{-20\text{ V}}$$

The minus sign simply indicates that the actual polarities of the potential difference are opposite the assumed polarity indicated in Fig. 5.15.

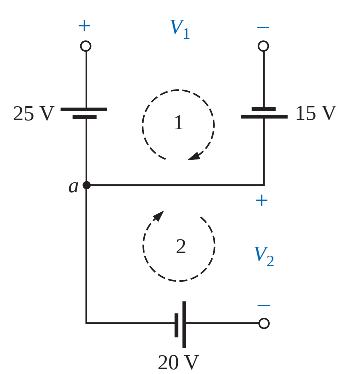

FIG. 5.15
Example 5.5.

The next example will emphasize the fact that when we are applying Kirchhoff's voltage law, it is the polarities of the voltage rise or drop that are the important parameters, and not the type of element involved.

EXAMPLE 5.6 Using Kirchhoff's voltage law, determine the unknown voltages for the network of Fig. 5.16.

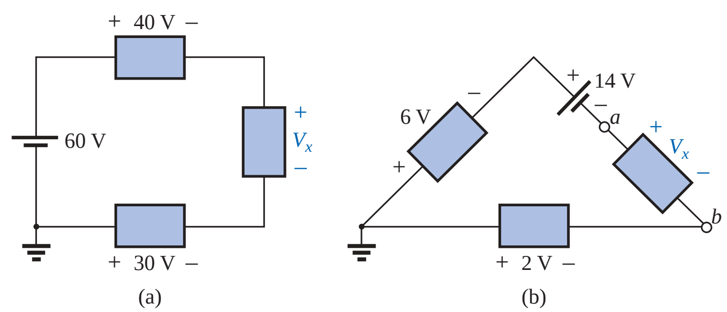

FIG. 5.16
Example 5.6.

Solution: Note in each circuit that there are various polarities across the unknown elements since they can contain any mixture of components. Applying Kirchhoff's voltage law to the network of Fig. 5.16(a) in the clockwise direction will result in

$$60 \text{ V} - 40 \text{ V} - V_x + 30 \text{ V} = 0$$

and
$$V_x = 60 \text{ V} + 30 \text{ V} - 40 \text{ V} = 90 \text{ V} - 40 \text{ V} = \mathbf{50\ V}$$

In Fig. 5.16(b) the polarity of the unknown voltage is not provided. In such cases, make an assumption about the polarity, and apply Kirchhoff's voltage law as before. If the result has a plus sign, the assumed polarity was correct. If it has a minus sign, the magnitude is correct, but the assumed polarity has to be reversed. In this case if we assume *a* to be positive and *b* to be negative, an application of Kirchhoff's voltage law in the clockwise direction will result in

$$-6 \text{ V} - 14 \text{ V} - V_x + 2 \text{ V} = 0$$

and
$$V_x = -20 \text{ V} + 2 \text{ V} = \mathbf{-18\ V}$$

Since the result is negative, we know that *a* should be negative and *b* should be positive, but the magnitude of 18 V is correct.

EXAMPLE 5.7 For the circuit of Fig. 5.17:

a. Find R_T.
b. Find *I*.
c. Find V_1 and V_2.
d. Find the power to the 4-Ω and 6-Ω resistors.
e. Find the power delivered by the battery, and compare it to that dissipated by the 4-Ω and 6-Ω resistors combined.
f. Verify Kirchhoff's voltage law (clockwise direction).

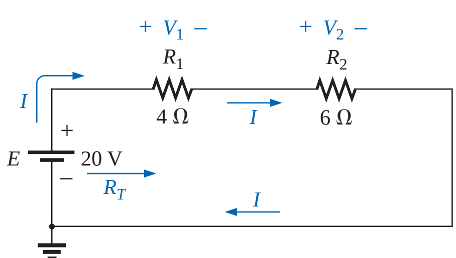

FIG. 5.17
Example 5.7.

Solutions:

a. $R_T = R_1 + R_2 = 4\ \Omega + 6\ \Omega = \mathbf{10\ \Omega}$

b. $I = \dfrac{E}{R_T} = \dfrac{20 \text{ V}}{10\ \Omega} = \mathbf{2\ A}$

c. $V_1 = IR_1 = (2\text{ A})(4\ \Omega) = \mathbf{8\ V}$
$V_2 = IR_2 = (2\text{ A})(6\ \Omega) = \mathbf{12\ V}$

d. $P_{4\Omega} = \dfrac{V_1^2}{R_1} = \dfrac{(8\text{ V})^2}{4} = \dfrac{64}{4} = \mathbf{16\ W}$

$P_{6\Omega} = I^2R_2 = (2\text{ A})^2(6\ \Omega) = (4)(6) = \mathbf{24\ W}$

e. $P_E = EI = (20\text{ V})(2\text{ A}) = \mathbf{40\ W}$
$P_E = P_{4\Omega} + P_{6\Omega}$
$40\text{ W} = 16\text{ W} + 24\text{ W}$
$40\text{ W} = 40\text{ W}$ (checks)

f. $\Sigma_{\circlearrowright} V = +E - V_1 - V_2 = 0$
$E = V_1 + V_2$
$20\text{ V} = 8\text{ V} + 12\text{ V}$
$20\text{ V} = 20\text{ V}$ (checks)

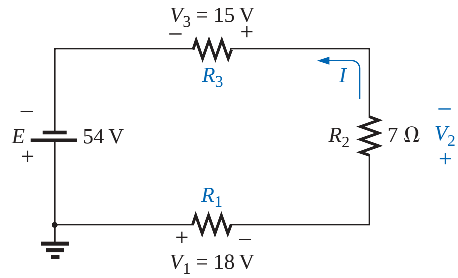

FIG. 5.18
Example 5.8.

EXAMPLE 5.8 For the circuit of Fig. 5.18:
a. Determine V_2 using Kirchhoff's voltage law.
b. Determine I.
c. Find R_1 and R_3.

Solutions:
a. Kirchhoff's voltage law (clockwise direction):

$$-E + V_3 + V_2 + V_1 = 0$$

or
$$E = V_1 + V_2 + V_3$$

and $V_2 = E - V_1 - V_3 = 54\text{ V} - 18\text{ V} - 15\text{ V} = \mathbf{21\ V}$

b. $I = \dfrac{V_2}{R_2} = \dfrac{21\text{ V}}{7\ \Omega} = \mathbf{3\ A}$

c. $R_1 = \dfrac{V_1}{I} = \dfrac{18\text{ V}}{3\text{ A}} = \mathbf{6\ \Omega}$

$R_3 = \dfrac{V_3}{I} = \dfrac{15\text{ V}}{3\text{ A}} = \mathbf{5\ \Omega}$

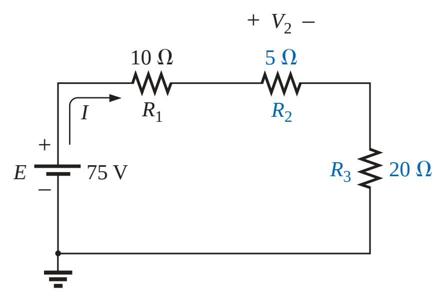

FIG. 5.19
Series dc circuit with elements to be interchanged.

5.5 INTERCHANGING SERIES ELEMENTS

The elements of a series circuit can be interchanged without affecting the total resistance, current, or power to each element. For instance, the network of Fig. 5.19 can be redrawn as shown in Fig. 5.20 without affecting I or V_2. The total resistance R_T is 35 Ω in both cases, and $I = 70\text{ V}/35\ \Omega = 2\text{ A}$. The voltage $V_2 = IR_2 = (2\text{ A})(5\ \Omega) = 10\text{ V}$ for both configurations.

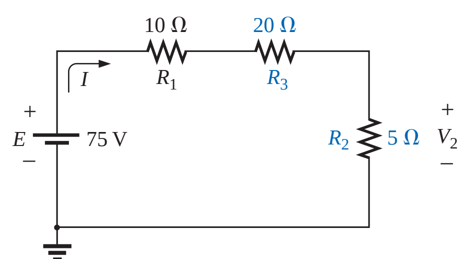

FIG. 5.20
Circuit of Fig. 5.19 with R_2 and R_3 interchanged.

EXAMPLE 5.9 Determine I and the voltage across the 7-Ω resistor for the network of Fig. 5.21.

Solution: The network is redrawn in Fig. 5.22.

$$R_T = (2)(4\ \Omega) + 7\ \Omega = 15\ \Omega$$

$$I = \frac{E}{R_T} = \frac{37.5\text{ V}}{15\ \Omega} = \mathbf{2.5\ A}$$

$$V_{7\Omega} = IR = (2.5\text{ A})(7\ \Omega) = \mathbf{17.5\ V}$$

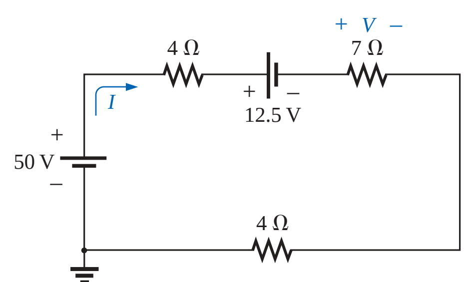

FIG. 5.21
Example 5.9.