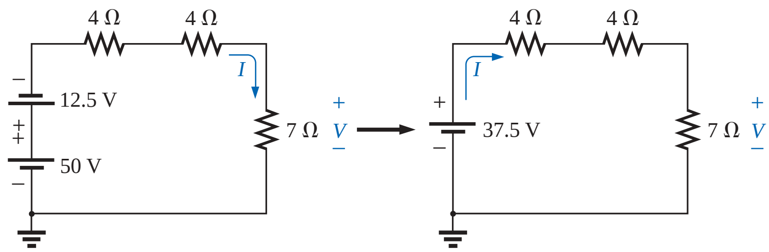


**FIG. 5.22**
*Redrawing the circuit of Fig. 5.21.*

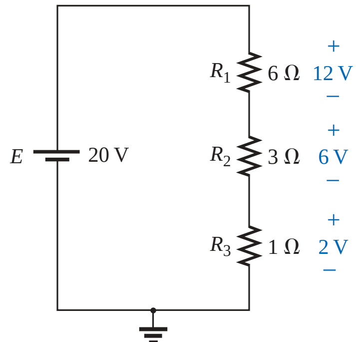


**FIG. 5.23**
*Revealing how the voltage will divide across series resistive elements.*

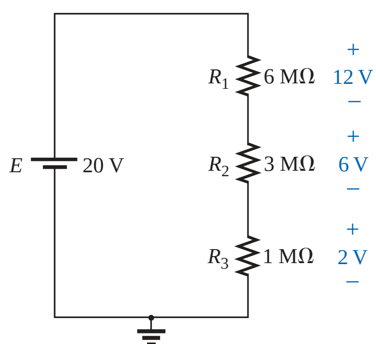


**FIG. 5.24**
*The ratio of the resistive values determines the voltage division of a series dc circuit.*

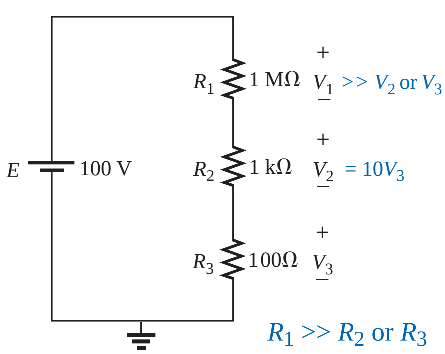


**FIG. 5.25**
*The largest of the series resistive elements will capture the major share of the applied voltage.*

## 5.6 VOLTAGE DIVIDER RULE

In a series circuit,

***the voltage across the resistive elements will divide as the magnitude of the resistance levels.***

For example, the voltages across the resistive elements of Fig. 5.23 are provided. The largest resistor of 6 Ω captures the bulk of the applied voltage, while the smallest resistor $R_3$ has the least. Note in addition that, since the resistance level of $R_1$ is 6 times that of $R_3$, the voltage across $R_1$ is 6 times that of $R_3$. The fact that the resistance level of $R_2$ is 3 times that of $R_1$ results in three times the voltage across $R_2$. Finally, since $R_1$ is twice $R_2$, the voltage across $R_1$ is twice that of $R_2$. In general, therefore, the voltage across series resistors will have the same ratio as their resistance levels.

It is particularly interesting to note that, if the resistance levels of all the resistors of Fig. 5.23 are increased by the same amount, as shown in Fig. 5.24, the voltage levels will all remain the same. In other words, even though the resistance levels were increased by a factor of 1 million, the voltage ratios remain the same. Clearly, therefore, it is the ratio of resistor values that counts when it comes to voltage division and not the relative magnitude of all the resistors. The current level of the network will be severely affected by the change in resistance level from Fig. 5.23 to Fig. 5.24, but the voltage levels will remain the same.

Based on the above, a first glance at the series network of Fig. 5.25 should suggest that the major part of the applied voltage will appear across the 1-MΩ resistor and very little across the 100-Ω resistor. In fact, 1 MΩ = (1000)1 kΩ = (10,000)100 Ω, revealing that $V_1 = 1000V_2 = 10{,}000V_3$.

Solving for the current and then the three voltage levels will result in

$$I = \frac{E}{R_T} = \frac{100\text{ V}}{1{,}001{,}100\ \Omega} \cong 99.89\ \mu\text{A}$$

and

$$V_1 = IR_1 = (99.89\ \mu\text{A})(1\text{ M}\Omega) = \mathbf{99.89\ V}$$
$$V_2 = IR_2 = (99.89\ \mu\text{A})(1\text{ k}\Omega) = \mathbf{99.89\ mV} = 0.09989\text{ V}$$
$$V_3 = IR_3 = (99.89\ \mu\text{A})(100\ \Omega) = \mathbf{9.989\ mV} = 0.009989\text{ V}$$

clearly substantiating the above conclusions. For the future, therefore, use this approach to estimate the share of the input voltage across series elements to act as a check against the actual calculations or to simply obtain an estimate with a minimum of effort.

In the above discussion the current was determined before the voltages of the network were determined. There is, however, a method referred to as the **voltage divider rule** (VDR) that permits determining the voltage levels without first finding the current. The rule can be derived by analyzing the network of Fig. 5.26.

$$R_T = R_1 + R_2$$

and
$$I = \frac{E}{R_T}$$

Applying Ohm's law:

$$V_1 = IR_1 = \left(\frac{E}{R_T}\right)R_1 = \frac{R_1E}{R_T}$$

with
$$V_2 = IR_2 = \left(\frac{E}{R_T}\right)R_2 = \frac{R_2E}{R_T}$$

Note that the format for $V_1$ and $V_2$ is

$$\boxed{V_x = \frac{R_xE}{R_T}} \qquad \text{(voltage divider rule)} \qquad \textbf{(5.10)}$$

where $V_x$ is the voltage across $R_x$, $E$ is the impressed voltage across the series elements, and $R_T$ is the total resistance of the series circuit.

In words, the **voltage divider rule** states that

***the voltage across a resistor in a series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements.***

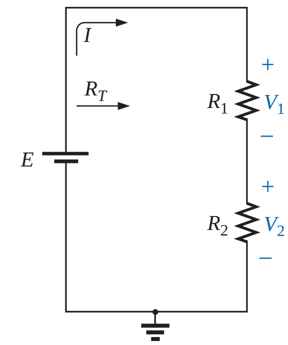


**FIG. 5.26**
*Developing the voltage divider rule.*

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**EXAMPLE 5.10** Determine the voltage $V_1$ for the network of Fig. 5.27.

***Solution:*** Eq. (5.10):

$$V_1 = \frac{R_1E}{R_T} = \frac{R_1E}{R_1 + R_2} = \frac{(20\ \Omega)(64\ \text{V})}{20\ \Omega + 60\ \Omega} = \frac{1280\ \text{V}}{80} = \mathbf{16\ V}$$

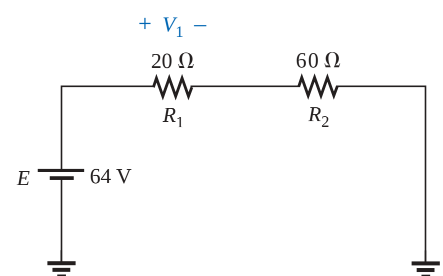


**FIG. 5.27**
*Example 5.10.*

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**EXAMPLE 5.11** Using the voltage divider rule, determine the voltages $V_1$ and $V_3$ for the series circuit of Fig. 5.28.

***Solution:***

$$V_1 = \frac{R_1E}{R_T} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{2\ \text{k}\Omega + 5\ \text{k}\Omega + 8\ \text{k}\Omega} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega}$$

$$= \frac{(2 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega} = \frac{90\ \text{V}}{15} = \mathbf{6\ V}$$

$$V_3 = \frac{R_3E}{R_T} = \frac{(8\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega} = \frac{(8 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega}$$

$$= \frac{360\ \text{V}}{15} = \mathbf{24\ V}$$

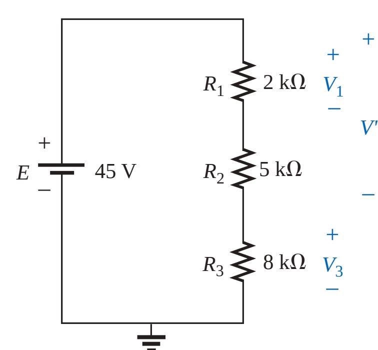


**FIG. 5.28**
*Example 5.11.*

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The rule can be extended to the voltage across two or more series elements if the resistance in the numerator of Eq. (5.10) is expanded to include the total resistance of the series elements that the voltage is to be found across ($R'$); that is,

$$\boxed{V' = \frac{R'E}{R_T}} \quad \text{(volts)} \tag{5.11}$$

**EXAMPLE 5.12** Determine the voltage $V'$ in Fig. 5.28 across resistors $R_1$ and $R_2$.

***Solution:***

$$V' = \frac{R'E}{R_T} = \frac{(2\text{ k}\Omega + 5\text{ k}\Omega)(45\text{ V})}{15\text{ k}\Omega} = \frac{(7\text{ k}\Omega)(45\text{ V})}{15\text{ k}\Omega} = \mathbf{21\text{ V}}$$

There is also no need for the voltage $E$ in the equation to be the source voltage of the network. For example, if $V$ is the total voltage across a number of series elements such as those shown in Fig. 5.29, then

$$V_{2\Omega} = \frac{(2\ \Omega)(27\text{ V})}{4\ \Omega + 2\ \Omega + 3\ \Omega} = \frac{54\text{ V}}{9} = \mathbf{6\text{ V}}$$

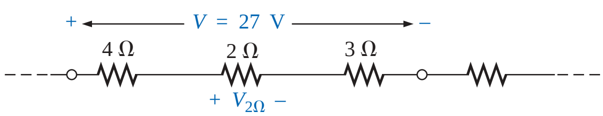


**FIG. 5.29**
*The total voltage across series elements need not be an independent voltage source.*

**EXAMPLE 5.13** Design the voltage divider of Fig. 5.30 such that $V_{R_1} = 4V_{R_2}$.

***Solution:*** The total resistance is defined by

$$R_T = \frac{E}{I} = \frac{20\text{ V}}{4\text{ mA}} = 5\text{ k}\Omega$$

Since $V_{R_1} = 4V_{R_2}$,

$$R_1 = 4R_2$$

Thus
$$R_T = R_1 + R_2 = 4R_2 + R_2 = 5R_2$$

and
$$5R_2 = 5\text{ k}\Omega$$
$$R_2 = \mathbf{1\text{ k}\Omega}$$

and
$$R_1 = 4R_2 = \mathbf{4\text{ k}\Omega}$$

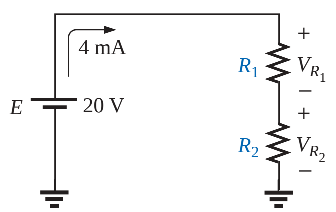


**FIG. 5.30**
*Example 5.13.*

## 5.7 NOTATION

Notation will play an increasingly important role in the analysis to follow. It is important, therefore, that we begin to examine the notation used throughout the industry.