7

Series-Parallel Networks

7.1 SERIES-PARALLEL NETWORKS

A firm understanding of the basic principles associated with series and parallel circuits is a sufficient background to begin an investigation of any single-source dc network having a combination of series and parallel elements or branches. Multisource networks are considered in detail in Chapters 8 and 9. In general,

series-parallel networks ***are networks that contain both series and parallel circuit configurations.***

One can become proficient in the analysis of series-parallel networks only through exposure, practice, and experience. In time the path to the desired unknown becomes more obvious as one recalls similar configurations and the frustration resulting from choosing the wrong approach. There are a few steps that can be helpful in getting started on the first few exercises, although the value of each will become apparent only with experience.

General Approach

1. Take a moment to study the problem "in total" and make a brief mental sketch of the overall approach you plan to use. The result may be time- and energy-saving shortcuts.
2. Next examine each region of the network independently before tying them together in series-parallel combinations. This will usually simplify the network and possibly reveal a direct approach toward obtaining one or more desired unknowns. It also eliminates many of the errors that might result due to the lack of a systematic approach.
3. Redraw the network as often as possible with the reduced branches and undisturbed unknown quantities to maintain clarity and provide the reduced networks for the trip back to unknown quantities from the source.

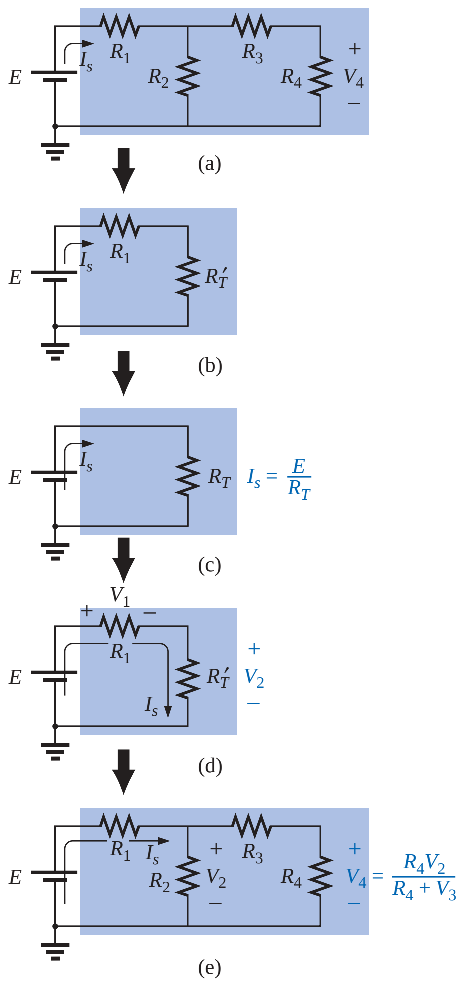

FIG. 7.1
Introducing the reduce and return approach.

4. When you have a solution, check that it is reasonable by considering the magnitudes of the energy source and the elements in the network. If it does not seem reasonable, either solve the circuit using another approach or check over your work very carefully.

Reduce and Return Approach

For many single-source, series-parallel networks, the analysis is one that works back to the source, determines the source current, and then finds its way to the desired unknown. In Fig. 7.1(a), for instance, the voltage V_4 is desired. The absence of a single series or parallel path to V_4 from the source immediately reveals that the methods introduced in the last two chapters cannot be applied here. First, series and parallel elements must be combined to establish the reduced circuit of Fig. 7.1(b). Then series elements are combined to form the simplest of configurations in Fig. 7.1(c). The source current can now be determined using Ohm's law, and we can proceed back through the network as shown in Fig. 7.1(d). The voltage V_2 can be determined and then the original network can be redrawn, as shown in Fig. 7.1(e). Since V_2 is now known, the voltage divider rule can be used to find the desired voltage V_4. Because of the similarities between the networks of Figs. 7.1(a) and 7.1(e), and between 7.1(b) and 7.1(d), the networks drawn during the reduction phase are often used for the return path.

Although all the details of the analysis were not described above, the general procedure for a number of series-parallel network problems employs the procedure described above: Work back for I_s and then follow the return path for the specific unknown. Not every problem will follow this path; some will have simpler, more direct solutions. However, the reduce and return approach will handle one type of problem that does surface over and over again.

Block Diagram Approach

The block diagram approach will be employed throughout to emphasize the fact that combinations of elements, not simply single resistive elements, can be in series or parallel. The approach will also reveal the number of seemingly different networks that have the same basic structure and therefore can involve similar analysis techniques.

Initially, there will be some concern about identifying series and parallel elements and branches and choosing the best procedure to follow toward a solution. However, as you progress through the examples and try a few problems, a common path toward most solutions will surface that can actually make the analysis of such systems an interesting, enjoyable experience.

In Fig. 7.2, blocks *B* and *C* are in parallel (points *b* and *c* in common), and the voltage source *E* is in series with block *A* (point *a* in common). The parallel combination of *B* and *C* is also in series with *A* and the voltage source *E* due to the common points *b* and *c*, respectively.

To ensure that the analysis to follow is as clear and uncluttered as possible, the following notation will be used for series and parallel combinations of elements. For series resistors R_1 and R_2, a comma will be inserted between their subscript notations, as shown here:

$$R_{1,2} = R_1 + R_2$$

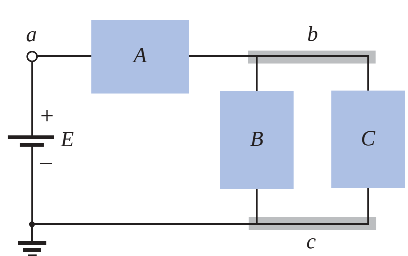

FIG. 7.2
Introducing the block diagram approach.

For parallel resistors R_1 and R_2, the parallel symbol will be inserted between their subscript notations, as follows:

$$R_{1\|2} = R_1 \,\|\, R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

EXAMPLE 7.1 If each block of Fig. 7.2 were a single resistive element, the network of Fig. 7.3 might result.

The parallel combination of R_B and R_C results in

$$R_{B\|C} = R_B \,\|\, R_C = \frac{(12\text{ k}\Omega)(6\text{ k}\Omega)}{12\text{ k}\Omega + 6\text{ k}\Omega} = 4\text{ k}\Omega$$

The equivalent resistance $R_{B\|C}$ is then in series with R_A, and the total resistance "seen" by the source is

$$\begin{aligned} R_T &= R_A + R_{B\|C} \\ &= 2\text{ k}\Omega + 4\text{ k}\Omega = \mathbf{6\text{ k}\Omega} \end{aligned}$$

The result is an equivalent network, as shown in Fig. 7.4, permitting the determination of the source current I_s.

$$I_s = \frac{E}{R_T} = \frac{54\text{ V}}{6\text{ k}\Omega} = \mathbf{9\text{ mA}}$$

and, since the source and R_A are in series,

$$I_A = I_s = 9\text{ mA}$$

We can then use the equivalent network of Fig. 7.5 to determine I_B and I_C using the current divider rule:

$$I_B = \frac{6\text{ k}\Omega(I_s)}{6\text{ k}\Omega + 12\text{ k}\Omega} = \frac{6}{18}I_s = \frac{1}{3}(9\text{ mA}) = \mathbf{3\text{ mA}}$$

$$I_C = \frac{12\text{ k}\Omega(I_s)}{12\text{ k}\Omega + 6\text{ k}\Omega} = \frac{12}{18}I_s = \frac{2}{3}(9\text{ mA}) = \mathbf{6\text{ mA}}$$

or, applying Kirchhoff's current law,

$$I_C = I_s - I_B = 9\text{ mA} - 3\text{ mA} = \mathbf{6\text{ mA}}$$

Note that in this solution, we worked back to the source to obtain the source current or total current supplied by the source. The remaining unknowns were then determined by working back through the network to find the other unknowns.

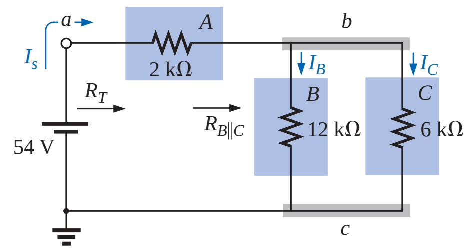

FIG. 7.3
Example 7.1.

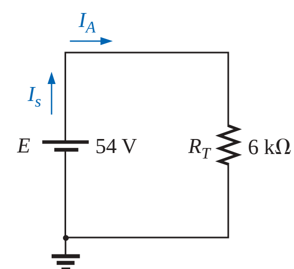

FIG. 7.4
Reduced equivalent of Fig. 7.3.

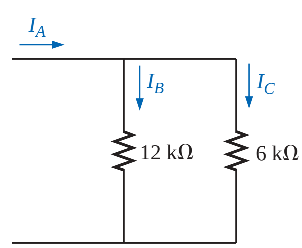

FIG. 7.5
Determining I_B and I_C for the network of Fig. 7.3.

EXAMPLE 7.2 It is also possible that the blocks A, B, and C of Fig. 7.2 contain the elements and configurations of Fig. 7.6. Working with each region:

A: $R_A = 4\ \Omega$

B: $R_B = R_2 \,\|\, R_3 = R_{2\|3} = \dfrac{R}{N} = \dfrac{4\ \Omega}{2} = 2\ \Omega$

C: $R_C = R_4 + R_5 = R_{4,5} = 0.5\ \Omega + 1.5\ \Omega = 2\ \Omega$

Blocks B and C are still in parallel, and

$$R_{B\|C} = \frac{R}{N} = \frac{2\ \Omega}{2} = 1\ \Omega$$

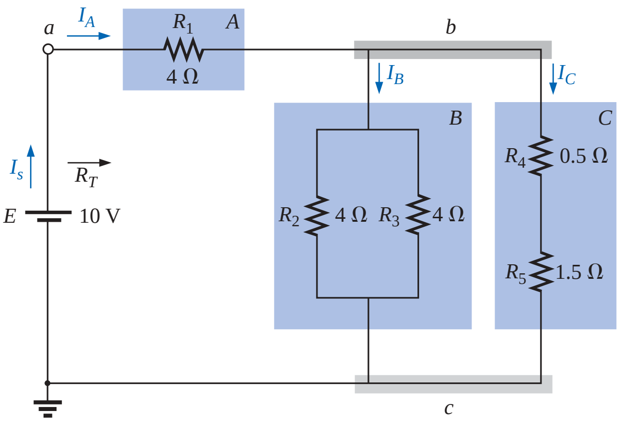

FIG. 7.6
Example 7.2.

with

$$R_T = R_A + R_{B\|C} = 4\,\Omega + 1\,\Omega = \mathbf{5\,\Omega}$$

(Note the similarity between this equation and that obtained for Example 7.1.)

and
$$I_s = \frac{E}{R_T} = \frac{10\text{ V}}{5\,\Omega} = \mathbf{2\text{ A}}$$

We can find the currents I_A, I_B, and I_C using the reduction of the network of Fig. 7.6 (recall Step 3) as found in Fig. 7.7. Note that I_A, I_B, and I_C are the same in Figs. 7.6 and 7.7 and therefore also appear in Fig. 7.7. In other words, the currents I_A, I_B, and I_C of Fig. 7.7 will have the same magnitude as the same currents of Fig. 7.6.

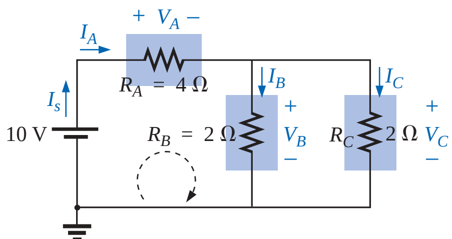

FIG. 7.7
Reduced equivalent of Fig. 7.6.

$$I_A = I_s = \mathbf{2\text{ A}}$$

and
$$I_B = I_C = \frac{I_A}{2} = \frac{I_s}{2} = \frac{2\text{ A}}{2} = \mathbf{1\text{ A}}$$

Returning to the network of Fig. 7.6, we have

$$I_{R_2} = I_{R_3} = \frac{I_B}{2} = \mathbf{0.5\text{ A}}$$

The voltages V_A, V_B, and V_C from either figure are

$$V_A = I_A R_A = (2\text{ A})(4\,\Omega) = \mathbf{8\text{ V}}$$
$$V_B = I_B R_B = (1\text{ A})(2\,\Omega) = \mathbf{2\text{ V}}$$
$$V_C = V_B = \mathbf{2\text{ V}}$$

Applying Kirchhoff's voltage law for the loop indicated in Fig. 7.7, we obtain

$$\Sigma_{\circlearrowright} V = E - V_A - V_B = 0$$
$$E = V_A + V_B = 8\text{ V} + 2\text{ V}$$

or
$$\underline{10\text{ V} = 10\text{ V} \quad \text{(checks)}}$$

EXAMPLE 7.3 Another possible variation of Fig. 7.2 appears in Fig. 7.8.

$$R_A = R_{1\|2} = \frac{(9\,\Omega)(6\,\Omega)}{9\,\Omega + 6\,\Omega} = \frac{54\,\Omega}{15} = 3.6\,\Omega$$

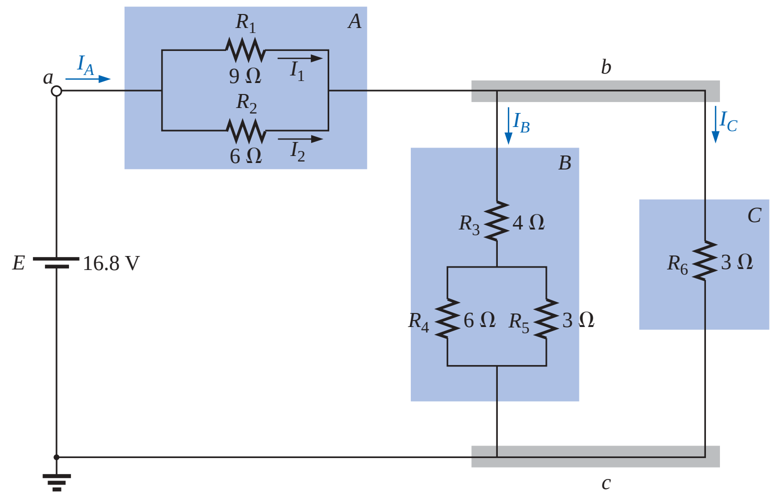

FIG. 7.8
Example 7.3.

$$R_B = R_3 + R_{4\|5} = 4\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 4\ \Omega + 2\ \Omega = 6\ \Omega$$

$$R_C = 3\ \Omega$$

The network of Fig. 7.8 can then be redrawn in reduced form, as shown in Fig. 7.9. Note the similarities between this circuit and the circuits of Figs. 7.3 and 7.7.

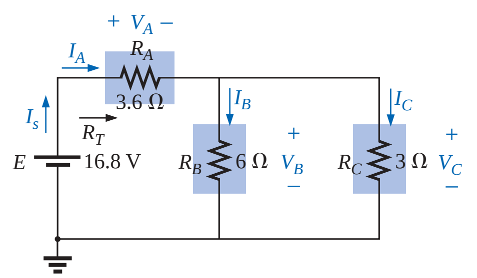

FIG. 7.9
Reduced equivalent of Fig. 7.8.

$$R_T = R_A + R_{B\|C} = 3.6\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega}$$

$$= 3.6\ \Omega + 2\ \Omega = \mathbf{5.6\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{16.8\text{ V}}{5.6\ \Omega} = \mathbf{3\ A}$$

$$I_A = I_s = \mathbf{3\ A}$$

Applying the current divider rule yields

$$I_B = \frac{R_C I_A}{R_C + R_B} = \frac{(3\ \Omega)(3\text{ A})}{3\ \Omega + 6\ \Omega} = \frac{9\text{ A}}{9} = \mathbf{1\ A}$$

By Kirchhoff's current law,

$$I_C = I_A - I_B = 3\text{ A} - 1\text{ A} = \mathbf{2\ A}$$

By Ohm's law,

$$V_A = I_A R_A = (3\text{ A})(3.6\ \Omega) = \mathbf{10.8\ V}$$

$$V_B = I_B R_B = V_C = I_C R_C = (2\text{ A})(3\ \Omega) = \mathbf{6\ V}$$

Returning to the original network (Fig. 7.8) and applying the current divider rule,

$$I_1 = \frac{R_2 I_A}{R_2 + R_1} = \frac{(6\ \Omega)(3\text{ A})}{6\ \Omega + 9\ \Omega} = \frac{18\text{ A}}{15} = \mathbf{1.2\ A}$$

By Kirchhoff's current law,

$$I_2 = I_A - I_1 = 3\text{ A} - 1.2\text{ A} = \mathbf{1.8\ A}$$

Figures 7.3, 7.6, and 7.8 are only a few of the infinite variety of configurations that the network can assume starting with the basic arrangement of Fig. 7.2. They were included in our discussion to emphasize the importance of considering each region of the network independently before finding the solution for the network as a whole.

The blocks of Fig. 7.2 can be arranged in a variety of ways. In fact, there is no limit on the number of series-parallel configurations that can appear within a given network. In reverse, the block diagram approach can be used effectively to reduce the apparent complexity of a system by identifying the major series and parallel components of the network. This approach will be demonstrated in the next few examples.

7.2 DESCRIPTIVE EXAMPLES

EXAMPLE 7.4 Find the current I_4 and the voltage V_2 for the network of Fig. 7.10.

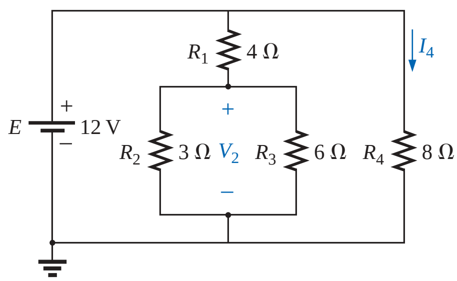

FIG. 7.10
Example 7.4.

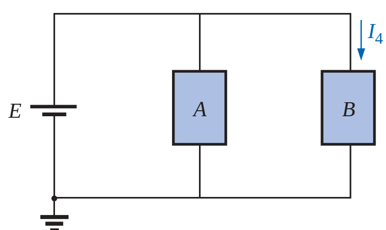

FIG. 7.11
Block diagram of Fig. 7.10.

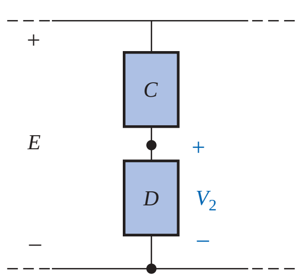

FIG. 7.12
Alternative block diagram for the first parallel branch of Fig. 7.10.

Solution: In this case, particular unknowns are requested instead of a complete solution. It would, therefore, be a waste of time to find all the currents and voltages of the network. The method employed should concentrate on obtaining only the unknowns requested. With the block diagram approach, the network has the basic structure of Fig. 7.11, clearly indicating that the three branches are in parallel and the voltage across A and B is the supply voltage. The current I_4 is now immediately obvious as simply the supply voltage divided by the resultant resistance for B. If desired, block A could be broken down further, as shown in Fig. 7.12, to identify C and D as series elements, with the voltage V_2 capable of being determined using the voltage divider rule once the resistance of C and D is reduced to a single value. This is an example of how a mental sketch of the approach might be made before applying laws, rules, and so on, to avoid dead ends and growing frustration.

Applying Ohm's law,

$$I_4 = \frac{E}{R_B} = \frac{E}{R_4} = \frac{12\text{ V}}{8\ \Omega} = \mathbf{1.5\ A}$$

Combining the resistors R_2 and R_3 of Fig. 7.10 will result in

$$R_D = R_2 \parallel R_3 = 3\ \Omega \parallel 6\ \Omega = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{9} = 2\ \Omega$$

and, applying the voltage divider rule,

$$V_2 = \frac{R_D E}{R_D + R_C} = \frac{(2\ \Omega)(12\text{ V})}{2\ \Omega + 4\ \Omega} = \frac{24\text{ V}}{6} = \mathbf{4\ V}$$

EXAMPLE 7.5 Find the indicated currents and voltages for the network of Fig. 7.13.

Solution: Again, only specific unknowns are requested. When the network is redrawn, it will be particularly important to note which unknowns are preserved and which will have to be determined using the original configuration. The block diagram of the network may appear as shown in Fig. 7.14, clearly revealing that A and B are in series. Note in this form the number of unknowns that have been preserved. The voltage V_1 will be the same across the three parallel branches of Fig. 7.13, and V_5 will be the same across R_4 and R_5. The unknown currents I_2 and I_4 are lost since they represent the currents through only one of the parallel branches. However, once V_1 and V_5 are known, the required currents can be found using Ohm's law.

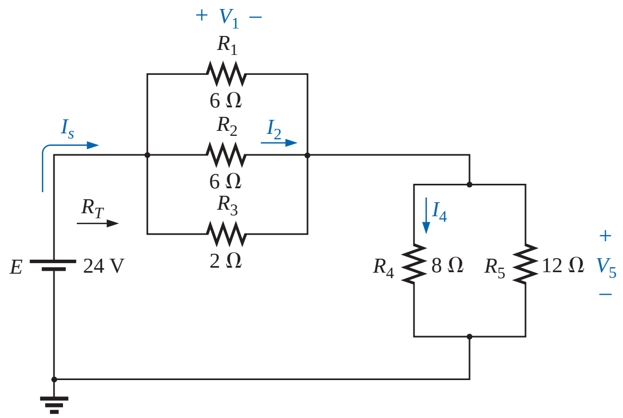

FIG. 7.13
Example 7.5.

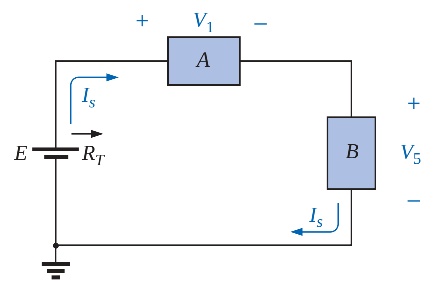

FIG. 7.14
Block diagram for Fig. 7.13.

$$R_{1\|2} = \frac{R}{N} = \frac{6\,\Omega}{2} = 3\,\Omega$$

$$R_A = R_{1\|2\|3} = \frac{(3\,\Omega)(2\,\Omega)}{3\,\Omega + 2\,\Omega} = \frac{6\,\Omega}{5} = 1.2\,\Omega$$

$$R_B = R_{4\|5} = \frac{(8\,\Omega)(12\,\Omega)}{8\,\Omega + 12\,\Omega} = \frac{96\,\Omega}{20} = 4.8\,\Omega$$

The reduced form of Fig. 7.13 will then appear as shown in Fig. 7.15, and

$$R_T = R_{1\|2\|3} + R_{4\|5} = 1.2\,\Omega + 4.8\,\Omega = \mathbf{6\,\Omega}$$

$$I_s = \frac{E}{R_T} = \frac{24\text{ V}}{6\,\Omega} = \mathbf{4\,A}$$

with
$$V_1 = I_s R_{1\|2\|3} = (4\text{ A})(1.2\,\Omega) = \mathbf{4.8\,V}$$

$$V_5 = I_s R_{4\|5} = (4\text{ A})(4.8\,\Omega) = \mathbf{19.2\,V}$$

Applying Ohm's law,

$$I_4 = \frac{V_5}{R_4} = \frac{19.2\text{ V}}{8\,\Omega} = \mathbf{2.4\,A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_1}{R_2} = \frac{4.8\text{ V}}{6\,\Omega} = \mathbf{0.8\,A}$$

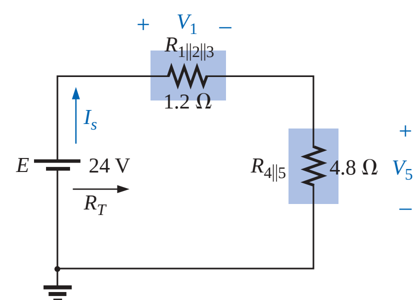

FIG. 7.15
Reduced form of Fig. 7.13.

The next example demonstrates that unknown voltages do not have to be across elements but can exist between any two points in a network. In addition, the importance of redrawing the network in a more familiar form is clearly revealed by the analysis to follow.

EXAMPLE 7.6

a. Find the voltages V_1, V_3, and V_{ab} for the network of Fig. 7.16.
b. Calculate the source current I_s.

Solutions: This is one of those situations where it might be best to redraw the network before beginning the analysis. Since combining both sources will not affect the unknowns, the network is redrawn as shown in Fig. 7.17, establishing a parallel network with the total source voltage across each parallel branch. The net source voltage is the difference between the two with the polarity of the larger.

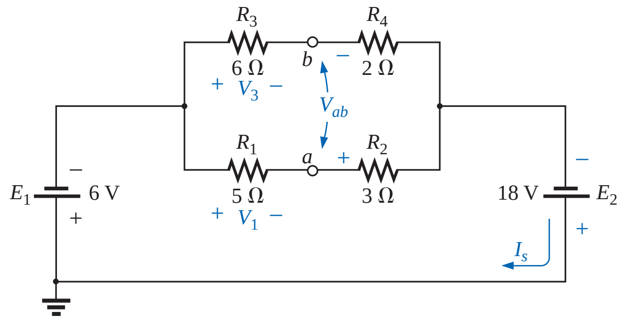

FIG. 7.16
Example 7.6.

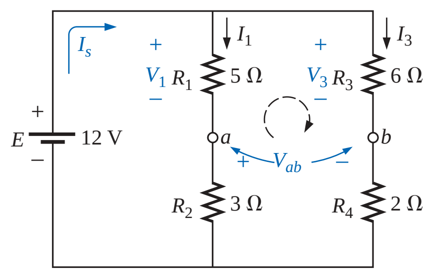

FIG. 7.17
Network of Fig. 7.16 redrawn.

a. Note the similarities with Fig. 7.12, permitting the use of the voltage divider rule to determine V_1 and V_3:

$$V_1 = \frac{R_1E}{R_1 + R_2} = \frac{(5\ \Omega)(12\ \text{V})}{5\ \Omega + 3\ \Omega} = \frac{60\ \text{V}}{8} = \mathbf{7.5\ V}$$

$$V_3 = \frac{R_3E}{R_3 + R_4} = \frac{(6\ \Omega)(12\ \text{V})}{6\ \Omega + 2\ \Omega} = \frac{72\ \text{V}}{8} = \mathbf{9\ V}$$

The open-circuit voltage V_{ab} is determined by applying Kirchhoff's voltage law around the indicated loop of Fig. 7.17 in the clockwise direction starting at terminal *a*.

$$+V_1 - V_3 + V_{ab} = 0$$

and $$V_{ab} = V_3 - V_1 = 9\ \text{V} - 7.5\ \text{V} = \mathbf{1.5\ V}$$

b. By Ohm's law,

$$I_1 = \frac{V_1}{R_1} = \frac{7.5\ \text{V}}{5\ \Omega} = 1.5\ \text{A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{9\ \text{V}}{6\ \Omega} = 1.5\ \text{A}$$

Applying Kirchhoff's current law,

$$I_s = I_1 + I_3 = 1.5\ \text{A} + 1.5\ \text{A} = \mathbf{3\ A}$$

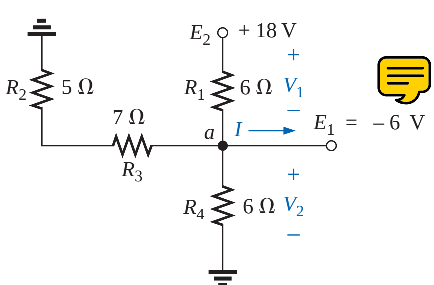

FIG. 7.18
Example 7.7.

EXAMPLE 7.7 For the network of Fig. 7.18, determine the voltages V_1 and V_2 and the current I.

Solution: It would indeed be difficult to analyze the network in the form of Fig. 7.18 with the symbolic notation for the sources and the reference or ground connection in the upper left-hand corner of the diagram. However, when the network is redrawn as shown in Fig. 7.19, the unknowns and the relationship between branches become significantly clearer. Note the common connection of the grounds and the replacing of the terminal notation by actual supplies.

It is now obvious that

$$V_2 = -E_1 = \mathbf{-6\ V}$$

The minus sign simply indicates that the chosen polarity for V_2 in Fig. 7.18 is opposite to that of the actual voltage. Applying Kirchhoff's voltage law to the loop indicated, we obtain

$$-E_1 + V_1 - E_2 = 0$$

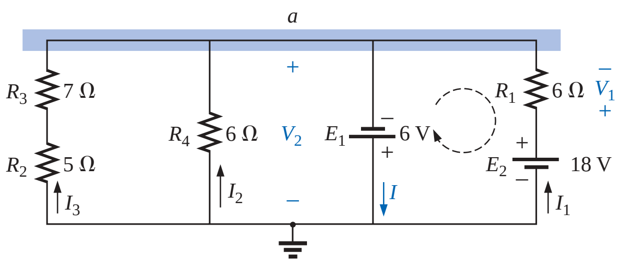

FIG. 7.19
Network of Fig. 7.18 redrawn.

and $$V_1 = E_2 + E_1 = 18\text{ V} + 6\text{ V} = \mathbf{24\text{ V}}$$

Applying Kirchhoff's current law to node *a* yields

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= \frac{V_1}{R_1} + \frac{E_1}{R_4} + \frac{E_1}{R_2 + R_3} \\ &= \frac{24\text{ V}}{6\ \Omega} + \frac{6\text{ V}}{6\ \Omega} + \frac{6\text{ V}}{12\ \Omega} \\ &= 4\text{ A} + 1\text{ A} + 0.5\text{ A} \\ I &= \mathbf{5.5\text{ A}} \end{aligned}$$

The next example is clear evidence of the fact that techniques learned in the current chapters will have far-reaching applications and will not be dropped for improved methods. Even though the **transistor** has not been introduced in this text, the dc levels of a transistor network can be examined using the basic rules and laws introduced in the early chapters of this text.

EXAMPLE 7.8 For the transistor configuration of Fig. 7.20, in which V_B and V_{BE} have been provided:

a. Determine the voltage V_E and the current I_E.
b. Calculate V_1.
c. Determine V_{BC} using the fact that the approximation $I_C = I_E$ is often applied to transistor networks.
d. Calculate V_{CE} using the information obtained in parts (a) through (c).

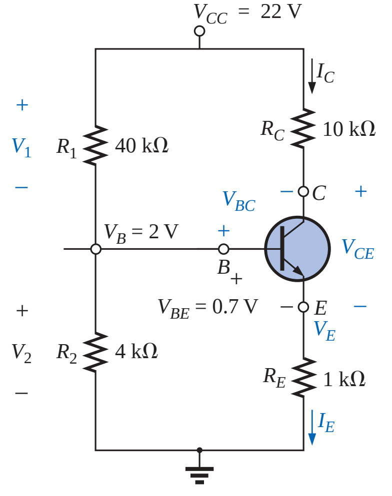

FIG. 7.20
Example 7.8.

Solutions:

a. From Fig. 7.20, we find

$$V_2 = V_B = 2\text{ V}$$

Writing Kirchhoff's voltage law around the lower loop yields

$$V_2 - V_{BE} - V_E = 0$$

or $$V_E = V_2 - V_{BE} = 2\text{ V} - 0.7\text{ V} = \mathbf{1.3\text{ V}}$$

and $$I_E = \frac{V_E}{R_E} = \frac{1.3\text{ V}}{1000\ \Omega} = \mathbf{1.3\text{ mA}}$$

b. Applying Kirchhoff's voltage law to the input side (left-hand region of the network) will result in

$$V_2 + V_1 - V_{CC} = 0$$