of series elements. Figure 8.20 defines the number of applications of Kirchhoff's current law for each configuration of Fig. 8.19.

5. *Solve the resulting simultaneous linear equations for assumed branch currents.*

It is assumed that the use of the **determinants method** to solve for the currents I_1, I_2, and I_3 is understood and is a part of the student's mathematical background. If not, a detailed explanation of the procedure is provided in Appendix C. Calculators and computer software packages such as Mathcad can find the solutions quickly and accurately.

EXAMPLE 8.9 Apply the branch-current method to the network of Fig. 8.21.

Solution 1:

Step 1: Since there are three distinct branches (*cda, cba, ca*), three currents of arbitrary directions (I_1, I_2, I_3) are chosen, as indicated in Fig. 8.21. The current directions for I_1 and I_2 were chosen to match the "pressure" applied by sources E_1 and E_2, respectively. Since both I_1 and I_2 enter node *a*, I_3 is leaving.

Step 2: Polarities for each resistor are drawn to agree with assumed current directions, as indicated in Fig. 8.22.
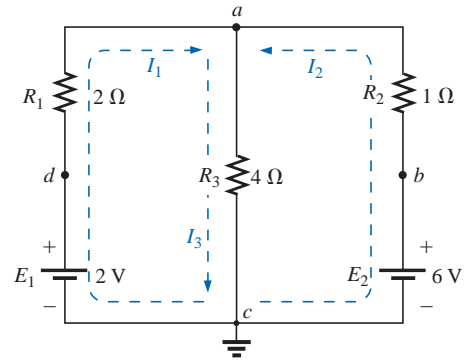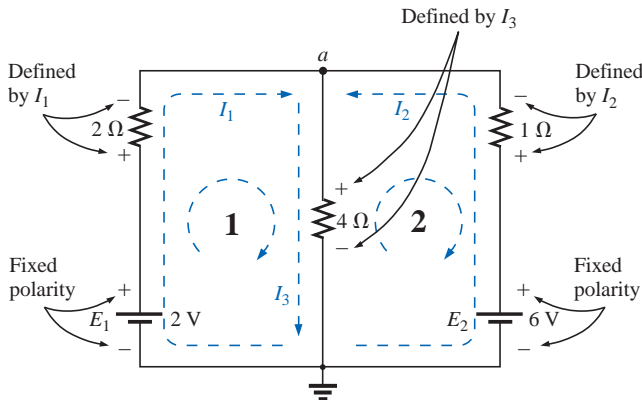

FIG. 8.21
Example 8.9.

FIG. 8.22
Inserting the polarities across the resistive elements as defined by the chosen branch currents.

Step 3: Kirchhoff's voltage law is applied around each closed loop (1 and 2) in the clockwise direction:

$$\text{loop 1:} \quad \Sigma_{\circlearrowright} V = +E_1 - V_{R_1} - V_{R_3} = 0$$

Rise in potential — Drop in potential

$$\text{loop 2:} \quad \Sigma_{\circlearrowright} V = +V_{R_3} + V_{R_2} - E_2 = 0$$

Rise in potential — Drop in potential

and

$$\text{loop 1:} \quad \Sigma_{\circlearrowright} V = +2\text{ V} - (2\ \Omega)I_1 - (4\ \Omega)I_3 = 0$$

Battery potential — Voltage drop across 2-Ω resistor — Voltage drop across 4-Ω resistor

$$\text{loop 2:} \quad \Sigma_{\circlearrowright} V = (4\ \Omega)I_3 + (1\ \Omega)I_2 - 6\text{ V} = 0$$

Step 4: Applying Kirchhoff's current law at node *a* (in a two-node network, the law is applied at only one node),

$$I_1 + I_2 = I_3$$

Step 5: There are three equations and three unknowns (units removed for clarity):

$$2 - 2I_1 - 4I_3 = 0 \qquad \text{Rewritten:} \quad 2I_1 + 0 + 4I_3 = 2$$
$$4I_3 + 1I_2 - \quad 6 = 0 \qquad \qquad \qquad 0 + I_2 + 4I_3 = 6$$
$$I_1 + \quad I_2 = I_3 \qquad \qquad \qquad I_1 + I_2 - \quad I_3 = 0$$

Using third-order determinants (Appendix C), we have

$$I_1 = \cfrac{\begin{vmatrix} 2 & 0 & 4 \\ 6 & 1 & 4 \\ 0 & 1 & -1 \end{vmatrix}}{D} = -1 \text{ A}$$

A negative sign in front of a branch current indicates only that the actual current is in the direction opposite to that assumed.

$$D = \begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}$$

$$I_2 = \cfrac{\begin{vmatrix} 2 & 2 & 4 \\ 0 & 6 & 4 \\ 1 & 0 & -1 \end{vmatrix}}{D} = 2 \text{ A}$$

$$I_3 = \cfrac{\begin{vmatrix} 2 & 0 & 2 \\ 0 & 1 & 6 \\ 1 & 1 & 0 \end{vmatrix}}{D} = 1 \text{ A}$$

Mathcad Solution: Once you understand the procedure for entering the parameters, you can use Mathcad to solve determinants such as
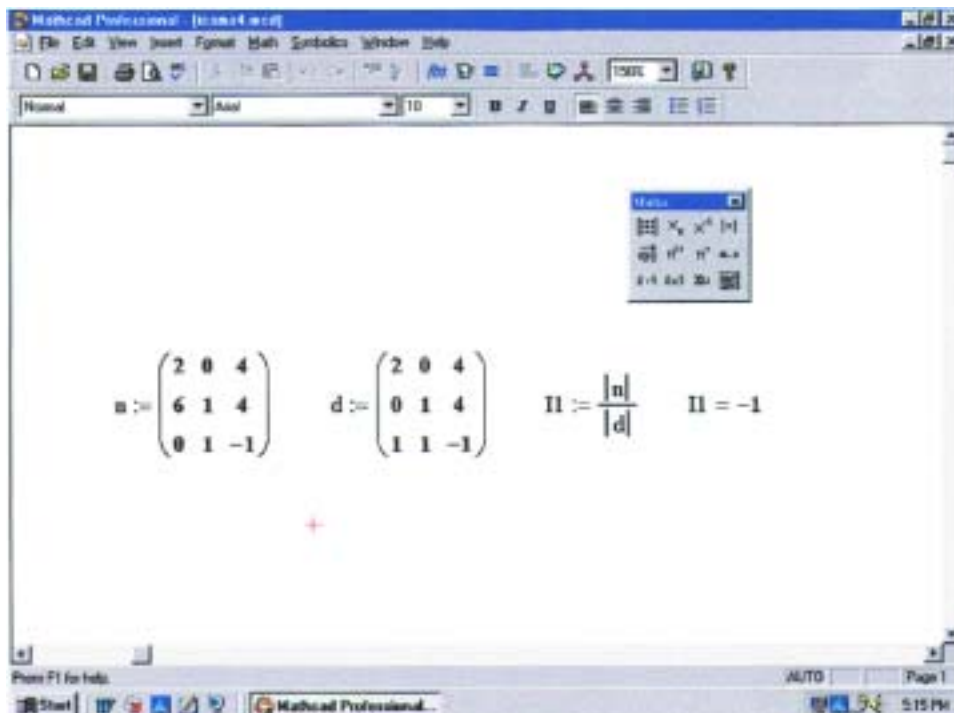

FIG. 8.23
Using Mathcad to verify the numerical calculations of Example 8.9.

appearing in Solution 1 in a very short time frame. The numerator is defined by n in the same manner described for earlier exercises. Then the sequence **View-Toolbars-Matrix** is applied to obtain the **Matrix** toolbar appearing in Fig. 8.23. Selecting the top left option called **Matrix** will result in the **Insert Matrix** dialog box in which **3 × 3** is selected. The 3 × 3 matrix will appear with a bracket to signal which parameter should be entered. Enter that parameter, and then use the left click of the mouse to select the next parameter you want to enter. When you have finished, move on to define the denominator d in the same manner. Then define the current of interest, select **Determinant** from the **Matrix** toolbar, and insert the numerator variable n. Follow with a division sign, and enter the **Determinant** of the denominator as shown in Fig. 8.23. Retype **I1** and select the equal sign; the correct result of **−1** will appear.

Once you have mastered the rather simple and direct process just described, the availability of Mathcad as a checking tool or solving mechanism will be deeply appreciated.

Solution 2: Instead of using third-order determinants as in Solution 1, we could reduce the three equations to two by substituting the third equation in the first and second equations:

$$2 - 2I_1 - 4\overbrace{(I_1 + I_2)}^{I_3} = 0 \qquad 2 - 2I_1 - 4I_1 - 4I_2 = 0$$

$$4\overbrace{(I_1 + I_2)}^{I_3} + I_2 - 6 = 0 \qquad 4I_1 + 4I_2 + I_2 - 6 = 0$$

or

$$-6I_1 - 4I_2 = -2$$
$$+4I_1 + 5I_2 = +6$$

Multiplying through by −1 in the top equation yields

$$6I_1 + 4I_2 = +2$$
$$4I_1 + 5I_2 = +6$$

and using determinants,

$$I_1 = \frac{\begin{vmatrix} 2 & 4 \\ 6 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & 4 \\ 4 & 5 \end{vmatrix}} = \frac{10 - 24}{30 - 16} = \frac{-14}{14} = \mathbf{-1A}$$

Using the TI-86 calculator:

det[[2,4][6,5]]/det[[6,4][4,5]] (ENTER) −1

CALC. 8.1

Note the det (determinant) obtained from a Math listing under a MATRX menu and the fact that each determinant must be determined individually. The first set of brackets within the overall determinant brackets of the first determinant defines the first row of the determinant, while the second set of brackets within the same determinant defines the second row. A comma separates the entries for each row. Obviously, the time to learn how to enter the parameters is minimal when you consider the savings in time and the accuracy obtained.

$$I_2 = \frac{\begin{vmatrix} 6 & 2 \\ 4 & 6 \end{vmatrix}}{14} = \frac{36 - 8}{14} = \frac{28}{14} = \textbf{2 A}$$

$$I_3 = I_1 + I_2 = -1 + 2 = \textbf{1 A}$$

It is now important that the impact of the results obtained be understood. The currents I_1, I_2, and I_3 are the actual currents in the branches in which they were defined. A negative sign in the solution simply reveals that the actual current has the opposite direction than initially defined—the magnitude is correct. Once the actual current directions and their magnitudes are inserted in the original network, the various voltages and power levels can be determined. For this example, the actual current directions and their magnitudes have been entered on the original network in Fig. 8.24. Note that the current through the series elements R_1 and E_1 is 1 A; the current through R_3, 1 A; and the current through the series elements R_2 and E_2, 2 A. Due to the minus sign in the solution, the direction of I_1 is opposite to that shown in Fig. 8.21. The voltage across any resistor can now be found using Ohm's law, and the power delivered by either source or to any one of the three resistors can be found using the appropriate power equation.
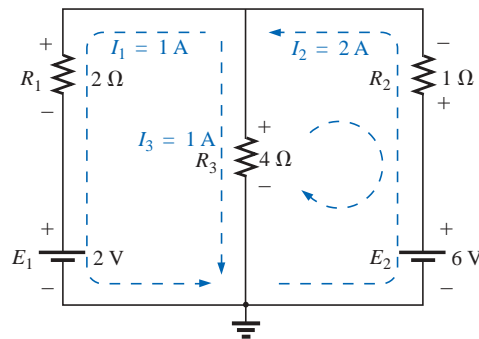

FIG. 8.24
Reviewing the results of the analysis of the network of Fig. 8.21.

Applying Kirchhoff's voltage law around the loop indicated in Fig. 8.24,

$$\Sigma_{\circlearrowright} V = +(4 \; \Omega)I_3 + (1 \; \Omega)I_2 - 6 \; V = 0$$

or
$$(4 \; \Omega)I_3 + (1 \; \Omega)I_2 = 6 \; V$$

and
$$(4 \; \Omega)(1 \; A) + (1 \; \Omega)(2 \; A) = 6 \; V$$
$$4 \; V + 2 \; V = 6 \; V$$
$$6 \; V = 6 \; V \quad \text{(checks)}$$

EXAMPLE 8.10 Apply branch-current analysis to the network of Fig. 8.25.

Solution: Again, the current directions were chosen to match the "pressure" of each battery. The polarities are then added and Kirchhoff's voltage law is applied around each closed loop in the clockwise direction. The result is as follows:

loop 1: $+15 \; V - (4 \; \Omega)I_1 + (10 \; \Omega)I_3 - 20 \; V = 0$

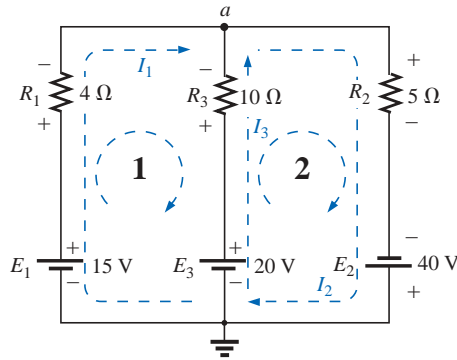loop 2: $+20 \; V - (10 \; \Omega)I_3 - (5 \; \Omega)I_2 + 40 \; V = 0$

FIG. 8.25
Example 8.10.

Applying Kirchhoff's current law at node a,

$$I_1 + I_3 = I_2$$

Substituting the third equation into the other two yields (with units removed for clarity)

$$15 - 4I_1 + 10I_3 - 20 = 0$$ Substituting for I_2 (since it occurs
$$20 - 10I_3 - 5(I_1 + I_3) + 40 = 0$$ only once in the two equations)

or

$$-4I_1 + 10I_3 = 5$$
$$-5I_1 - 15I_3 = -60$$

Multiplying the lower equation by -1, we have

$$-4I_1 + 10I_3 = 5$$
$$5I_1 + 15I_3 = 60$$

$$I_1 = \frac{\begin{vmatrix} 5 & 10 \\ 60 & 15 \end{vmatrix}}{\begin{vmatrix} -4 & 10 \\ 5 & 15 \end{vmatrix}} = \frac{75 - 600}{-60 - 50} = \frac{-525}{-110} = \textbf{4.773 A}$$

$$I_3 = \frac{\begin{vmatrix} -4 & 5 \\ 5 & 60 \end{vmatrix}}{-110} = \frac{-240 - 25}{-110} = \frac{-265}{-110} = \textbf{2.409 A}$$

$$I_2 = I_1 + I_3 = 4.773 + 2.409 = \textbf{7.182 A}$$

revealing that the assumed directions were the actual directions, with I_2 equal to the sum of I_1 and I_3.

8.7   MESH ANALYSIS (GENERAL APPROACH)

The second method of analysis to be described is called **mesh analysis.** The term *mesh* is derived from the similarities in appearance between the closed loops of a network and a wire mesh fence. Although this approach is on a more sophisticated plane than the branch-current method, it incorporates many of the ideas just developed. Of the two methods, mesh analysis is the one more frequently applied today. Branch-current analysis is introduced as a stepping stone to mesh analysis because branch currents are initially more "real" to the student than the **mesh (loop) currents** employed in mesh analysis. Essentially,

the mesh-analysis approach simply eliminates the need to substitute the results of Kirchhoff's current law into the equations derived from Kirchhoff's voltage law. It is now accomplished in the initial writing of the equations. The systematic approach outlined below should be followed when applying this method.

1. *Assign a distinct current in the clockwise direction to each independent, closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current. In fact, any direction can be chosen for each loop current with no loss in accuracy, as long as the remaining steps are followed properly. However, by choosing the clockwise direction as a standard, we can develop a shorthand method (Section 8.8) for writing the required equations that will save time and possibly prevent some common errors.*

This first step is accomplished most effectively by placing a loop current *within* each "window" of the network, as demonstrated in the previous section, to ensure that they are all independent. A variety of other loop currents can be assigned. In each case, however, be sure that the information carried by any one loop equation is not included in a combination of the other network equations. This is the crux of the terminology: *independent.* No matter how you choose your loop currents, the number of loop currents required is always equal to the number of windows of a planar (no-crossovers) network. On occasion a network may appear to be nonplanar. However, a redrawing of the network may reveal that it is, in fact, planar. Such may be the case in one or two problems at the end of the chapter.

Before continuing to the next step, let us ensure that the concept of a loop current is clear. For the network of Fig. 8.26, the loop current I_1 is the branch current of the branch containing the 2-Ω resistor and 2-V battery. The current through the 4-Ω resistor is not I_1, however, since there is also a loop current I_2 through it. Since they have opposite directions, $I_{4\Omega}$ equals the difference between the two, $I_1 - I_2$ or $I_2 - I_1$, depending on which you choose to be the defining direction. In other words, *a loop current is a branch current only when it is the only loop current assigned to that branch.*
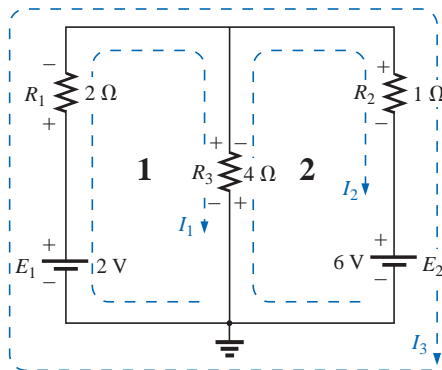
2. *Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop. Note the requirement that the polarities be placed within each loop. This requires, as shown in Fig. 8.26, that the 4-Ω resistor have two sets of polarities across it.*

3. *Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity and prepare us for the method to be introduced in the next section.*

   a. *If a resistor has two or more assumed currents through it, the total current through the resistor is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the* **same** *direction, minus the assumed currents through in the* **opposite** *direction.*

   b. *The polarity of a voltage source is unaffected by the direction of the assigned loop currents.*

4. *Solve the resulting simultaneous linear equations for the assumed loop currents.*

FIG. 8.26

Defining the mesh currents for a "two-window" network.

EXAMPLE 8.11 Consider the same basic network as in Example 8.9 of the preceding section, now appearing in Fig. 8.26.

Solution:

Step 1: Two loop currents (I_1 and I_2) are assigned in the clockwise direction in the windows of the network. A third loop (I_3) could have been included around the entire network, but the information carried by this loop is already included in the other two.

Step 2: Polarities are drawn within each window to agree with assumed current directions. Note that for this case, the polarities across the 4-Ω resistor are the opposite for each loop current.

Step 3: Kirchhoff's voltage law is applied around each loop in the clockwise direction. Keep in mind as this step is performed that the law is concerned only with the magnitude and polarity of the voltages around the closed loop and not with whether a voltage rise or drop is due to a battery or a resistive element. The voltage across each resistor is determined by $V = IR$, and for a resistor with more than one current through it, the current is the loop current of the loop being examined plus or minus the other loop currents as determined by their directions. If clockwise applications of Kirchhoff's voltage law are always chosen, the other loop currents will always be subtracted from the loop current of the loop being analyzed.

loop 1: $+E_1 - V_1 - V_3 = 0$ (clockwise starting at point a)

$$+2\text{ V} - (2\text{ Ω})I_1 - \overbrace{(4\text{ Ω})(I_1 - I_2)}^{\text{Voltage drop across}\\ \text{4-Ω resistor}} = 0$$

Total current through 4-Ω resistor

Subtracted since I_2 is opposite in direction to I_1.

loop 2: $-V_3 - V_2 - E_2 = 0$ (clockwise starting at point b)

$-(4\text{ Ω})(I_2 - I_1) - (1\text{ Ω})I_2 - 6\text{ V} = 0$

Step 4: The equations are then rewritten as follows (without units for clarity):

loop 1: $+2 - 2I_1 - 4I_1 + 4I_2 = 0$
loop 2: $-4I_2 + 4I_1 - 1I_2 - 6 = 0$

and

loop 1: $+2 - 6I_1 + 4I_2 = 0$
loop 2: $-5I_2 + 4I_1 - 6 = 0$

or

loop 1: $-6I_1 + 4I_2 = -2$
loop 2: $+4I_1 - 5I_2 = +6$

Applying determinants will result in

$$I_1 = -\mathbf{1\,A} \quad \text{and} \quad I_2 = -\mathbf{2\,A}$$

The minus signs indicate that the currents have a direction opposite to that indicated by the assumed loop current.

The actual current through the 2-V source and 2-Ω resistor is therefore 1 A in the other direction, and the current through the 6-V source and 1-Ω resistor is 2 A in the opposite direction indicated on the circuit. The current through the 4-Ω resistor is determined by the following equation from the original network:

loop 1: $I_{4\Omega} = I_1 - I_2 = -1\,\text{A} - (-2\,\text{A}) = -1\,\text{A} + 2\,\text{A}$
$= \mathbf{1\,A}$ (in the direction of I_1)

The outer loop (I_3) and *one* inner loop (either I_1 or I_2) would also have produced the correct results. This approach, however, will often lead to errors since the loop equations may be more difficult to write. The best method of picking these loop currents is to use the window approach.
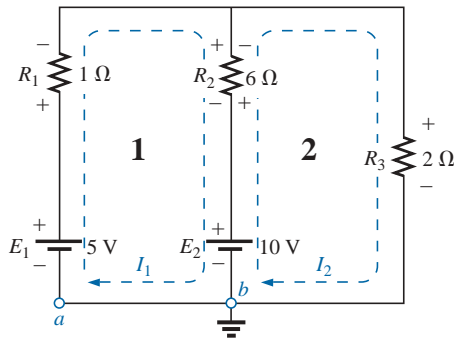
EXAMPLE 8.12 Find the current through each branch of the network of Fig. 8.27.

Solution:

Steps 1 and 2 are as indicated in the circuit. Note that the polarities of the 6-Ω resistor are different for each loop current.

Step 3: Kirchhoff's voltage law is applied around each closed loop in the clockwise direction:

loop 1: $+E_1 - V_1 - V_2 - E_2 = 0$ (clockwise starting at point *a*)
$+5\,\text{V} - (1\,\Omega)I_1 - (6\,\Omega)(I_1 - I_2) - 10\,\text{V} = 0$

I_2 flows through the 6-Ω resistor
in the direction opposite to I_1.

loop 2: $E_2 - V_2 - V_3 = 0$ (clockwise starting at point *b*)
$+10\,\text{V} - (6\,\Omega)(I_2 - I_1) - (2\,\Omega)I_2 = 0$

The equations are rewritten as

$$\left.\begin{array}{l} 5 - I_1 - 6I_1 + 6I_2 - 10 = 0 \\ 10 - 6I_2 + 6I_1 - 2I_2 = 0 \end{array}\right\} \begin{array}{l} -7I_1 + 6I_2 = 5 \\ +6I_1 - 8I_2 = -10 \end{array}$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} 5 & 6 \\ -10 & -8 \end{vmatrix}}{\begin{vmatrix} -7 & 6 \\ 6 & -8 \end{vmatrix}} = \frac{-40 + 60}{56 - 36} = \frac{20}{20} = \mathbf{1\,A}$$

$$I_2 = \frac{\begin{vmatrix} -7 & 5 \\ 6 & -10 \end{vmatrix}}{20} = \frac{70 - 30}{20} = \frac{40}{20} = \mathbf{2\,A}$$

Since I_1 and I_2 are positive and flow in opposite directions through the 6-Ω resistor and 10-V source, the total current in this branch is equal to the difference of the two currents in the direction of the larger:

$$I_2 > I_1 \quad (2\,\text{A} > 1\,\text{A})$$

Therefore,

$$I_{R_2} = I_2 - I_1 = 2\,\text{A} - 1\,\text{A} = \mathbf{1\,A} \quad \text{in the direction of } I_2$$

It is sometimes impractical to draw all the branches of a circuit at right angles to one another. The next example demonstrates how a portion of a network may appear due to various constraints. The method of analysis does not change with this change in configuration.

FIG. 8.27
Example 8.12.

EXAMPLE 8.13 Find the branch currents of the network of Fig. 8.28.

Solution:

Steps 1 and 2 are as indicated in the circuit.

Step 3: Kirchhoff's voltage law is applied around each closed loop:

loop 1: $-E_1 - I_1R_1 - E_2 - V_2 = 0$ (clockwise from point *a*)

$-6\,\text{V} - (2\,\Omega)I_1 - 4\,\text{V} - (4\,\Omega)(I_1 - I_2) = 0$

loop 2: $-V_2 + E_2 - V_3 - E_3 = 0$ (clockwise from point *b*)

$\underline{-(4\,\Omega)(I_2 - I_1) + 4\,\text{V} - (6\,\Omega)(I_2) - 3\,\text{V} = 0}$

which are rewritten as

$$\left.\begin{array}{c} -10 - 4I_1 - 2I_1 + 4I_2 = 0 \\ +1 + 4I_1 - 4I_2 - 6I_2 = 0 \end{array}\right\} \quad \begin{array}{c} -6I_1 + 4I_2 = +10 \\ +4I_1 - 10I_2 = -1 \end{array}$$

or, by multiplying the top equation by -1, we obtain

$$6I_1 - 4I_2 = -10$$
$$4I_1 - 10I_2 = -1$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} -10 & -4 \\ -1 & -10 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ 4 & -10 \end{vmatrix}} = \frac{100 - 4}{-60 + 16} = \frac{96}{-44} = \boldsymbol{-2.182\,\text{A}}$$

Using the TI-86 calculator:

> det[[−10,−4][−1,−10]]/det[[6,−4][4,−10]] (ENTER)   −2.182

CALC. 8.2

$$I_2 = \frac{\begin{vmatrix} 6 & -10 \\ 4 & -1 \end{vmatrix}}{-44} = \frac{-6 + 40}{-44} = \frac{34}{-44} = \boldsymbol{-0.773\,\text{A}}$$

The current in the 4-Ω resistor and 4-V source for loop 1 is

$$I_1 - I_2 = -2.182\,\text{A} - (-0.773\,\text{A})$$
$$= -2.182\,\text{A} + 0.773\,\text{A}$$
$$= \boldsymbol{-1.409\,\text{A}}$$

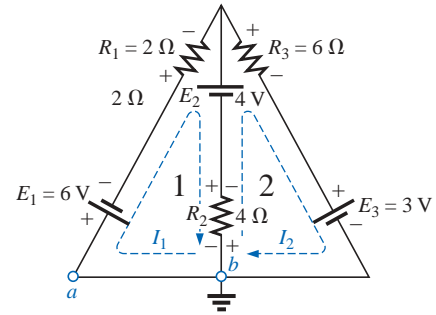revealing that it is 1.409 A in a direction opposite (due to the minus sign) to I_1 in loop 1.

FIG. 8.28
Example 8.13.

Supermesh Currents

On occasion there will be current sources in the network to which mesh analysis is to be applied. In such cases one can convert the current source to a voltage source (if a parallel resistor is present) and proceed as before or utilize a *supermesh* current and proceed as follows.

Start as before and assign a mesh current to each independent loop, including the current sources, as if they were resistors or voltage sources. Then mentally (redraw the network if necessary) remove the current sources (replace with open-circuit equivalents), and apply

Kirchhoff's voltage law to all the remaining independent paths of the network using the mesh currents just defined. Any resulting path, including two or more mesh currents, is said to be the path of a *supermesh* current. Then relate the chosen mesh currents of the network to the independent current sources of the network, and solve for the mesh currents. The next example will clarify the definition of a *supermesh* current and the procedure.

EXAMPLE 8.14 Using mesh analysis, determine the currents of the network of Fig. 8.29.
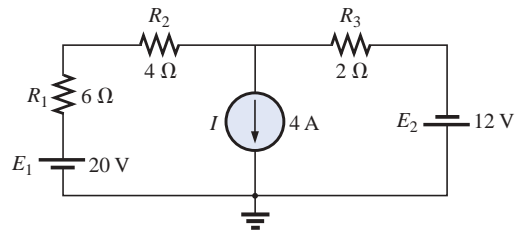
FIG. 8.29
Example 8.14.

Solution: First, the mesh currents for the network are defined, as shown in Fig. 8.30. Then the current source is mentally removed, as shown in Fig. 8.31, and Kirchhoff's voltage law is applied to the resulting network. The single path now including the effects of two mesh currents is referred to as the path of a *supermesh* current.
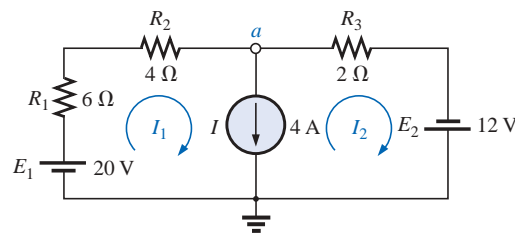

FIG. 8.30
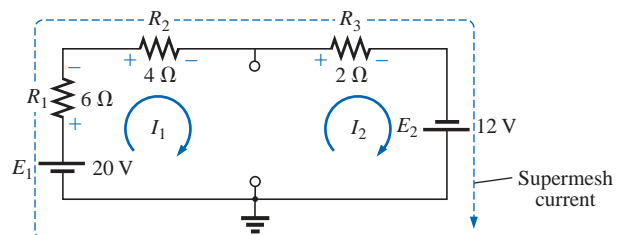*Defining the mesh currents for the network of Fig. 8.29.*

FIG. 8.31
Defining the supermesh current.

Applying Kirchhoff's law:

$$20\text{ V} - I_1(6\text{ }\Omega) - I_1(4\text{ }\Omega) - I_2(2\text{ }\Omega) + 12\text{ V} = 0$$

or
$$10I_1 + 2I_2 = 32$$

Node a is then used to relate the mesh currents and the current source using Kirchhoff's current law:

$$I_1 = I + I_2$$

The result is two equations and two unknowns:

$$10I_1 + 2I_2 = 32$$
$$I_1 - I_2 = 4$$

Applying determinants:

$$I_1 = \frac{\begin{vmatrix} 32 & 2 \\ 4 & -1 \end{vmatrix}}{\begin{vmatrix} 10 & 2 \\ 1 & -1 \end{vmatrix}} = \frac{(32)(-1) - (2)(4)}{(10)(-1) - (2)(1)} = \frac{40}{12} = \mathbf{3.33\ A}$$

and
$$I_2 = I_1 - I = 3.33\ A - 4\ A = \mathbf{-0.67\ A}$$

In the above analysis, it might appear that when the current source was removed, $I_1 = I_2$. However, the supermesh approach requires that we stick with the original definition of each mesh current and not alter those definitions when current sources are removed.

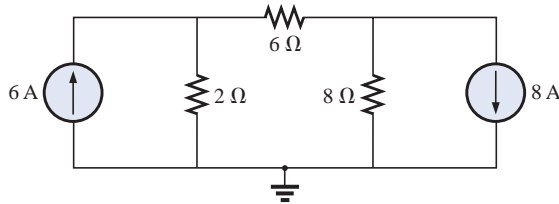**EXAMPLE 8.15** Using mesh analysis, determine the currents for the network of Fig. 8.32.

FIG. 8.32
Example 8.15.

Solution: The mesh currents are defined in Fig. 8.33. The current sources are removed, and the single supermesh path is defined in Fig. 8.34.

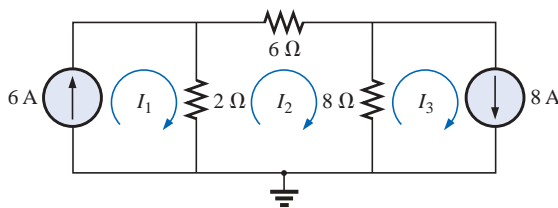**FIG. 8.33**
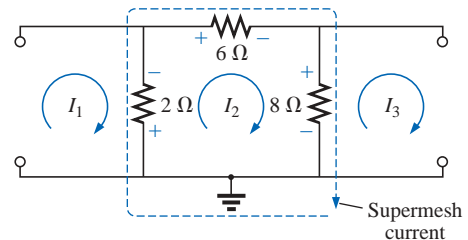*Defining the mesh currents for the network of Fig. 8.32.*

FIG. 8.34
Defining the supermesh current for the network of Fig. 8.32.

Applying Kirchhoff's voltage law around the supermesh path:

$$-V_{2\Omega} - V_{6\Omega} - V_{8\Omega} = 0$$
$$-(I_2 - I_1)2\ \Omega - I_2(6\ \Omega) - (I_2 - I_3)8\ \Omega = 0$$
$$-2I_2 + 2I_1 - 6I_2 - 8I_2 + 8I_3 = 0$$
$$2I_1 - 16I_2 + 8I_3 = 0$$

Introducing the relationship between the mesh currents and the current sources:

$$I_1 = 6 \text{ A}$$
$$I_3 = 8 \text{ A}$$

results in the following solutions:

$$2I_1 - 16I_2 + 8I_3 = 0$$
$$2(6 \text{ A}) - 16I_2 + 8(8 \text{ A}) = 0$$

and $\qquad I_2 = \dfrac{76 \text{ A}}{16} = \textbf{4.75 A}$

Then $\qquad I_{2\Omega}\!\downarrow = I_1 - I_2 = 6 \text{ A} - 4.75 \text{ A} = \textbf{1.25 A}$

and $\qquad I_{8\Omega}\!\uparrow = I_3 - I_2 = 8 \text{ A} - 4.75 \text{ A} = \textbf{3.25 A}$

Again, note that you must stick with your original definitions of the various mesh currents when applying Kirchhoff's voltage law around the resulting supermesh paths.
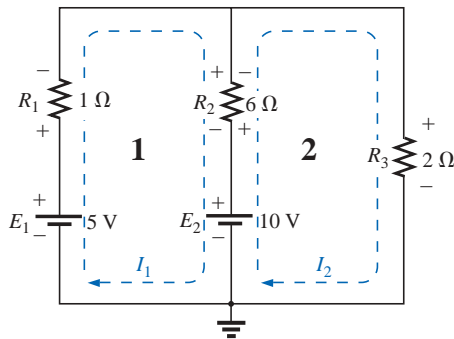
8.8   MESH ANALYSIS (FORMAT APPROACH)

Now that the basis for the mesh-analysis approach has been established, we will now examine a technique for writing the mesh equations more rapidly and usually with fewer errors. As an aid in introducing the procedure, the network of Example 8.12 (Fig. 8.27) has been redrawn in Fig. 8.35 with the assigned loop currents. (Note that each loop current has a clockwise direction.)

The equations obtained are

$$-7I_1 + 6I_2 = 5$$
$$6I_1 - 8I_2 = -10$$

which can also be written as

$$7I_1 - 6I_2 = -5$$
$$\underline{8I_2 - 6I_1 = 10}$$

and expanded as

$$\begin{array}{ccc} \textbf{Col. 1} & \textbf{Col. 2} & \textbf{Col. 3} \\ (1 + 6)I_1 - & 6I_2 & = (5 - 10) \\ (2 + 6)I_2 - & 6I_1 & = 10 \end{array}$$

Note in the above equations that column 1 is composed of a loop current times the sum of the resistors through which that loop current passes. Column 2 is the product of the resistors common to another loop current times that other loop current. Note that in each equation, this column is subtracted from column 1. Column 3 is the *algebraic* sum of the voltage sources through which the loop current of interest passes. A source is assigned a positive sign if the loop current passes from the negative to the positive terminal, and a negative value is assigned if the polarities are reversed. The comments above are correct only for a standard direction of loop current in each window, the one chosen being the clockwise direction.

The above statements can be extended to develop the following *format approach* to mesh analysis:

FIG. 8.35
Network of Fig. 8.27 redrawn with assigned loop currents.

1. *Assign a loop current to each independent, closed loop (as in the previous section) in a clockwise direction.*
2. *The number of required equations is equal to the number of chosen independent, closed loops. Column 1 of each equation is formed by summing the resistance values of those resistors through which the loop current of interest passes and multiplying the result by that loop current.*
3. *We must now consider the mutual terms, which, as noted in the examples above, are always subtracted from the first column. A* **mutual term** *is simply any resistive element having an additional loop current passing through it. It is possible to have more than one mutual term if the loop current of interest has an element in common with more than one other loop current. This will be demonstrated in an example to follow. Each term is the product of the mutual resistor and the other loop current passing through the same element.*
4. *The column to the right of the equality sign is the algebraic sum of the voltage sources through which the loop current of interest passes. Positive signs are assigned to those sources of voltage having a polarity such that the loop current passes from the negative to the positive terminal. A negative sign is assigned to those potentials for which the reverse is true.*
5. *Solve the resulting simultaneous equations for the desired loop currents.*

Before considering a few examples, be aware that since the column to the right of the equals sign is the algebraic sum of the voltage sources in that loop, *the format approach can be applied only to networks in which all current sources have been converted to their equivalent voltage source.*

EXAMPLE 8.16 Write the mesh equations for the network of Fig. 8.36, and find the current through the 7-Ω resistor.

Solution:

Step 1: As indicated in Fig. 8.36, each assigned loop current has a clockwise direction.

Steps 2 to 4:

$$I_1: \quad (8\ \Omega + 6\ \Omega + 2\ \Omega)I_1 - (2\ \Omega)I_2 = 4\ \text{V}$$
$$I_2: \quad \underline{(7\ \Omega + 2\ \Omega)I_2 - (2\ \Omega)I_1 = -9\ \text{V}}$$

and

$$16I_1 - 2I_2 = 4$$
$$\underline{9I_2 - 2I_1 = -9}$$

which, for determinants, are

$$16I_1 - 2I_2 = 4$$
$$-2I_1 + 9I_2 = -9$$

and

$$I_2 = I_{7\Omega} = \frac{\begin{vmatrix} 16 & 4 \\ -2 & -9 \end{vmatrix}}{\begin{vmatrix} 16 & -2 \\ -2 & 9 \end{vmatrix}} = \frac{-144 + 8}{144 - 4} = \frac{-136}{140}$$
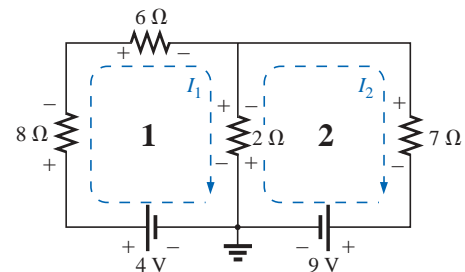
$$= -0.971\ \text{A}$$

FIG. 8.36

Example 8.16.

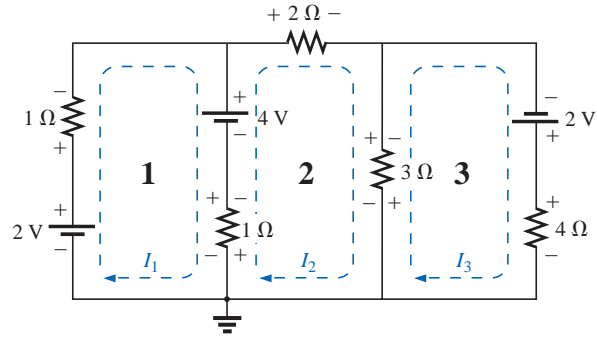**EXAMPLE 8.17** Write the mesh equations for the network of Fig. 8.37.

FIG. 8.37
Example 8.17.

Solution:

Step 1: Each window is assigned a loop current in the clockwise direction:

I_1 does not pass through an element mutual with I_3.

$$I_1: \qquad (1\ \Omega + 1\ \Omega)I_1 - (1\ \Omega)I_2 + 0 = 2\ \text{V} - 4\ \text{V}$$
$$I_2: \quad (1\ \Omega + 2\ \Omega + 3\ \Omega)I_2 - (1\ \Omega)I_1 - (3\ \Omega)I_3 = 4\ \text{V}$$
$$I_3: \qquad (3\ \Omega + 4\ \Omega)I_3 - (3\ \Omega)I_2 + 0 = 2\ \text{V}$$

I_3 does not pass through an element mutual with I_1.

Summing terms yields

$$2I_1 - I_2 + 0 = -2$$
$$6I_2 - I_1 - 3I_3 = 4$$
$$7I_3 - 3I_2 + 0 = 2$$

which are rewritten for determinants as

$$2I_1 - I_2 + 0 = -2$$
$$-I_1 + 6I_2 - 3I_3 = 4$$
$$0 - 3I_2 + 7I_3 = 2$$

Note that the coefficients of the *a* and *b* diagonals are equal. This *symmetry* about the *c*-axis will always be true for equations written using the format approach. It is a check on whether the equations were obtained correctly.

We will now consider a network with only one source of voltage to point out that mesh analysis can be used to advantage in other than multi-source networks.

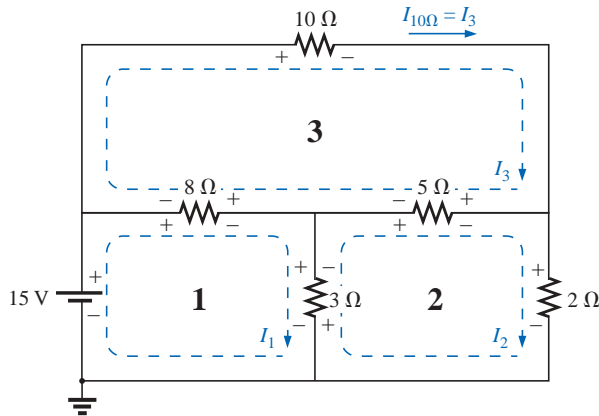**EXAMPLE 8.18** Find the current through the 10-Ω resistor of the network of Fig. 8.38.

FIG. 8.38
Example 8.18.

Solution 1:

$$I_1: \qquad (8\ \Omega + 3\ \Omega)I_1 - (8\ \Omega)I_3 - (3\ \Omega)I_2 = 15\ \text{V}$$
$$I_2: \quad (3\ \Omega + 5\ \Omega + 2\ \Omega)I_2 - (3\ \Omega)I_1 - (5\ \Omega)I_3 = 0$$
$$I_3: \quad (8\ \Omega + 10\ \Omega + 5\ \Omega)I_3 - (8\ \Omega)I_1 - (5\ \Omega)I_2 = 0$$

$$11I_1 - 8I_3 - 3I_2 = 15$$
$$10I_2 - 3I_1 - 5I_3 = 0$$
$$23I_3 - 8I_1 - 5I_2 = 0$$

or

$$11I_1 - 3I_2 - 8I_3 = 15$$
$$-3I_1 + 10I_2 - 5I_3 = 0$$
$$-8I_1 - 5I_2 + 23I_3 = 0$$

and

$$I_3 = I_{10\Omega} = \frac{\begin{vmatrix} 11 & -3 & 15 \\ -3 & 10 & 0 \\ -8 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 11 & -3 & -8 \\ -3 & 10 & -5 \\ -8 & -5 & 23 \end{vmatrix}} = \textbf{1.220 A}$$

Mathcad Solution: For this example, rather than take the time to develop the determinant form for each variable, we will apply Mathcad directly to the resulting equations. As shown in Fig. 8.39, a **Guess** value for each variable must first be defined. Such guessing helps the computer begin its iteration process as it searches for the solution. By providing a rough estimate of 1, the computer recognizes that the result will probably be a number with a magnitude less than 100 rather than have to worry about solutions that extend into the thousands or tens of thousands—the search has been narrowed considerably.

Next, as shown, the word **Given** must be entered to tell the computer that the defining equations will follow. Finally, each equation must be carefully entered and set equal to the constant on the right using the **Ctrl=** operation.

The results are then obtained with the **Find(I1,I2,I3)** expression and an equal sign. As shown, the results are available with an acceptable degree of accuracy even though entering the equations and performing the analysis took only a minute or two (with practice).
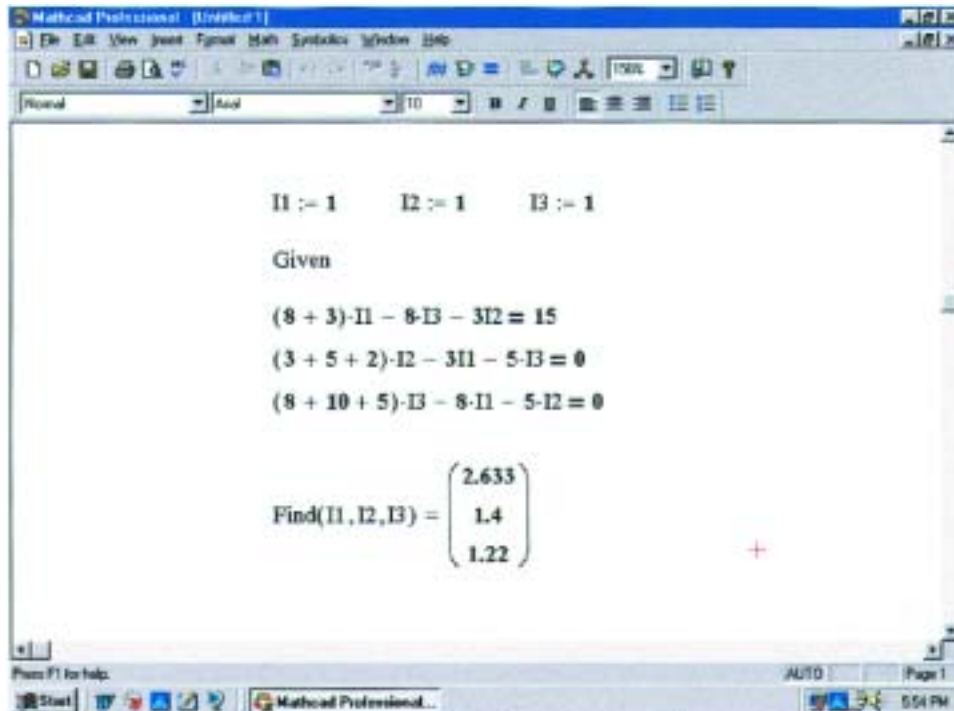
FIG. 8.39
Using Mathcad to verify the numerical calculations of Example 8.18.

Solution 2: Using the TI-86 calculator:

det[[11,−3,15][−3,10,0][−8,−5,0]]/det[[11,−3,−8][−3,10,−5][−8,−5,23]] (ENTER)   1.220

CALC. 8.3

This display certainly requires some care in entering the correct sequence of brackets in the required format, but it is still a rather neat, compact format.

8.9 NODAL ANALYSIS (GENERAL APPROACH)

Recall from the development of loop analysis that the general network equations were obtained by applying Kirchhoff's voltage law around each closed loop. We will now employ Kirchhoff's current law to develop a method referred to as **nodal analysis.**

A **node** is defined as a junction of two or more branches. If we now define one node of any network as a reference (that is, a point of zero potential or ground), the remaining nodes of the network will all have a fixed potential relative to this reference. For a network of N nodes, therefore, there will exist $(N - 1)$ nodes with a fixed potential relative to the assigned reference node. Equations relating these nodal voltages can be written by applying Kirchhoff's current law at each of the $(N - 1)$ nodes. To obtain the complete solution of a network, these nodal voltages are then evaluated in the same manner in which loop currents were found in loop analysis.

1. **Determine the number of nodes within the network.**
2. **Pick a reference node, and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.**
3. **Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law. In other words, for each node, don't be influenced by the direction that an unknown current for another node may have had. Each node is to be treated as a separate entity, independent of the application of Kirchhoff's current law to the other nodes.**
4. **Solve the resulting equations for the nodal voltages.**

A few examples will clarify the procedure defined by step 3. It will initially take some practice writing the equations for Kirchhoff's current law correctly, but in time the advantage of assuming that all the currents leave a node rather than identifying a specific direction for each branch will become obvious. (The same type of advantage is associated with assuming that all the mesh currents are clockwise when applying mesh analysis.)
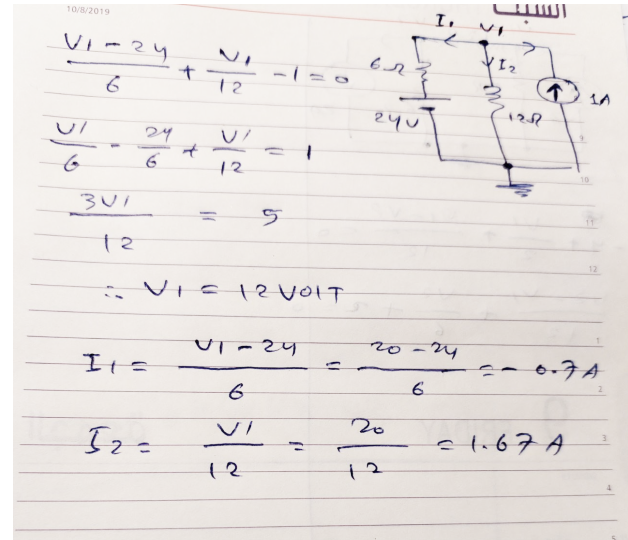
EXAMPLE 8.19 Apply nodal analysis to the network of Fig. 8.40.

Solution:

Steps 1 and 2: The network has two nodes, as shown in Fig. 8.41. The lower node is defined as the reference node at ground potential (zero volts), and the other node as V_1, the voltage from node 1 to ground.

Step 3: I_1 and I_2 are defined as leaving the node in Fig. 8.42, and Kirchhoff's current law is applied as follows:

$$I = I_1 + I_2$$

The current I_2 is related to the nodal voltage V_1 by Ohm's law:

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1}{R_2}$$

The current I_1 is also determined by Ohm's law as follows:

$$I_1 = \frac{V_{R_1}}{R_1}$$

with

$$V_{R_1} = V_1 - E$$

Substituting into the Kirchhoff's current law equation:

$$I = \frac{V_1 - E}{R_1} + \frac{V_1}{R_2}$$

and rearranging, we have

$$I = \frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{E}{R_1}$$

or

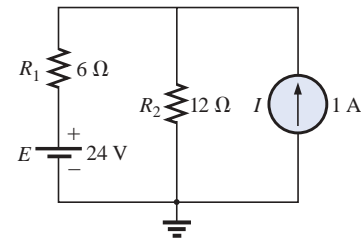$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_1} + I$$
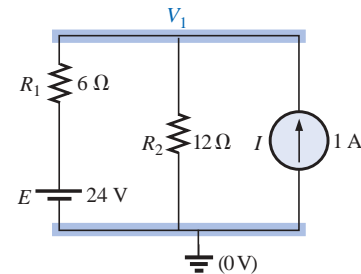

FIG. 8.40

Example 8.19.

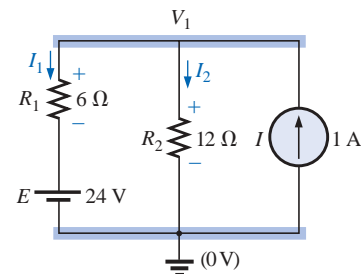**FIG. 8.41**

Network of Fig. 8.40 with assigned nodes.

FIG. 8.42

Applying Kirchhoff's current law to the node V_1.
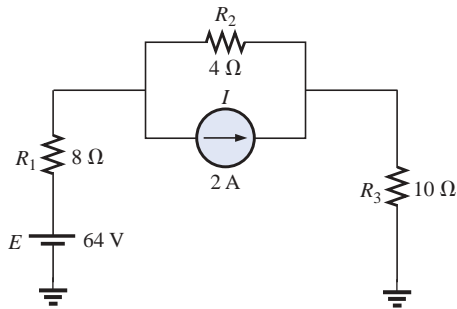
FIG. 8.43
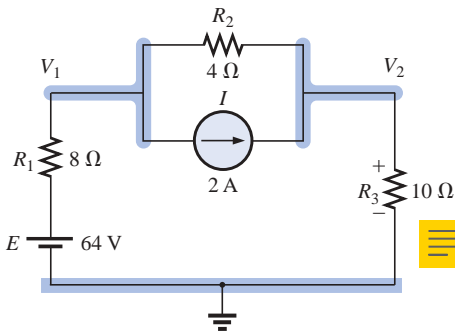*Example 8.20.*

FIG. 8.44
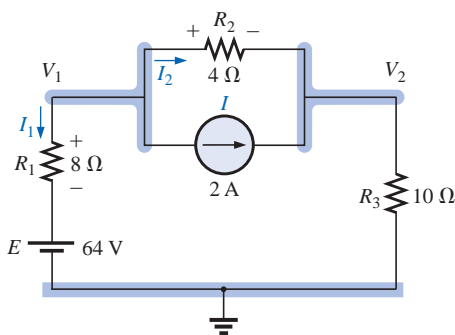*Defining the nodes for the network of Fig. 8.43.*

FIG. 8.45
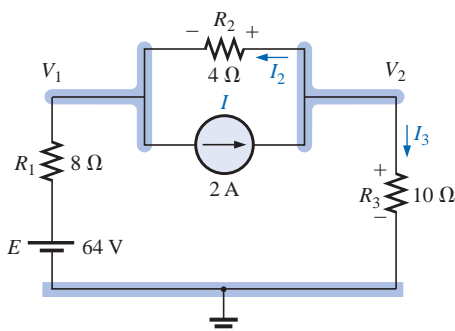*Applying Kirchhoff's current law to node V_1.*

FIG. 8.46
Applying Kirchhoff's current law to node V_2.

Substituting numerical values, we obtain

$$V_1\left(\frac{1}{6\,\Omega} + \frac{1}{12\,\Omega}\right) = \frac{24\text{ V}}{6\,\Omega} + 1\text{ A} = 4\text{ A} + 1\text{ A}$$

$$V_1\left(\frac{1}{4\,\Omega}\right) = 5\text{ A}$$

$$V_1 = \textbf{20 V}$$

The currents I_1 and I_2 can then be determined using the preceding equations:

$$I_1 = \frac{V_1 - E}{R_1} = \frac{20\text{ V} - 24\text{ V}}{6\,\Omega} = \frac{-4\text{ V}}{6\,\Omega}$$

$$= \mathbf{-0.667\text{ A}}$$

The minus sign indicates simply that the current I_1 has a direction opposite to that appearing in Fig. 8.42.

$$I_2 = \frac{V_1}{R_2} = \frac{20\text{ V}}{12\,\Omega} = \textbf{1.667 A}$$

EXAMPLE 8.20 Apply nodal analysis to the network of Fig. 8.43.

Solution 1:

Steps 1 and 2: The network has three nodes, as defined in Fig. 8.44, with the bottom node again defined as the reference node (at ground potential, or zero volts), and the other nodes as V_1 and V_2.

Step 3: For node V_1 the currents are defined as shown in Fig. 8.45, and Kirchhoff's current law is applied:

$$0 = I_1 + I_2 + I$$

with

$$I_1 = \frac{V_1 - E}{R_1}$$

and

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1 - V_2}{R_2}$$

so that

$$\frac{V_1 - E}{R_1} + \frac{V_1 - V_2}{R_2} + I = 0$$

or

$$\frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2} + I = 0$$

and

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) = -I + \frac{E}{R_1}$$

Substituting values:

$$V_1\left(\frac{1}{8\,\Omega} + \frac{1}{4\,\Omega}\right) - V_2\left(\frac{1}{4\,\Omega}\right) = -2\text{ A} + \frac{64\text{ V}}{8\,\Omega} = 6\text{ A}$$

For node V_2 the currents are defined as shown in Fig. 8.46, and Kirchhoff's current law is applied:

$$I = I_2 + I_3$$

with

$$I = \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3}$$

or
$$I = \frac{V_2}{R_2} - \frac{V_1}{R_2} + \frac{V_2}{R_3}$$

and
$$V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{R_2}\right) = I$$

Substituting values:

$$V_2\left(\frac{1}{4\ \Omega} + \frac{1}{10\ \Omega}\right) - V_1\left(\frac{1}{4\ \Omega}\right) = 2\ \text{A}$$

Step 4: The result is two equations and two unknowns:

$$V_1\left(\frac{1}{8\ \Omega} + \frac{1}{4\ \Omega}\right) - V_2\left(\frac{1}{4\ \Omega}\right) = 6\ \text{A}$$

$$-V_1\left(\frac{1}{4\ \Omega}\right) + V_2\left(\frac{1}{4\ \Omega} + \frac{1}{10\ \Omega}\right) = 2\ \text{A}$$

which become

$$0.375V_1 - 0.25V_2 = 6$$
$$-0.25V_1 + 0.35V_2 = 2$$

Using determinants,

$$V_1 = \textbf{37.818 V}$$
$$V_2 = \textbf{32.727 V}$$

Since E is greater than V_1, the current I_1 flows from ground to V_1 and is equal to

$$I_{R_1} = \frac{E - V_1}{R_1} = \frac{64\ \text{V} - 37.818\ \text{V}}{8\ \Omega} = \textbf{3.273 A}$$

The positive value for V_2 results in a current I_{R_3} from node V_2 to ground equal to

$$I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{V_2}{R_3} = \frac{32.727\ \text{V}}{10\ \Omega} = \textbf{3.273 A}$$

Since V_1 is greater than V_2, the current I_{R_2} flows from V_1 to V_2 and is equal to

$$I_{R_2} = \frac{V_1 - V_2}{R_2} = \frac{37.818\ \text{V} - 32.727\ \text{V}}{4\ \Omega} = \textbf{1.273 A}$$

Mathcad Solution: For this example, we will use Mathcad to work directly with the Kirchhoff's current law equations rather than taking the mathematical process down the line to more familiar forms. Simply define everything correctly, provide the **Guess** values, and insert **Given** where required. The process should be quite straightforward.

Note in Fig. 8.47 that the first equation comes from the fact that $I_1 + I_2 + I = 0$ while the second equation comes from $I_2 + I_3 = I$. Pay particular attention to the fact that the first equation is defined by Fig. 8.45 and the second by Fig. 8.46 because the direction of I_2 is different for each.

The results of $V_1 = 37.82$ V and $V_2 = 32.73$ V confirm the theoretical solution.

Mathcad Professional - [Untitled:2]

File Edit View Insert Format Math Symbolics Window Help

$I := 2$ $E := 64$ $R1 := 8$ $R2 := 4$ $R3 := 10$

$V1 := 1$ $V2 := 1$

Given

$$\frac{(V1 - E)}{R1} + \frac{(V1 - V2)}{R2} + I = 0$$

$$\frac{(V2 - V1)}{R2} + \frac{V2}{R3} = I$$

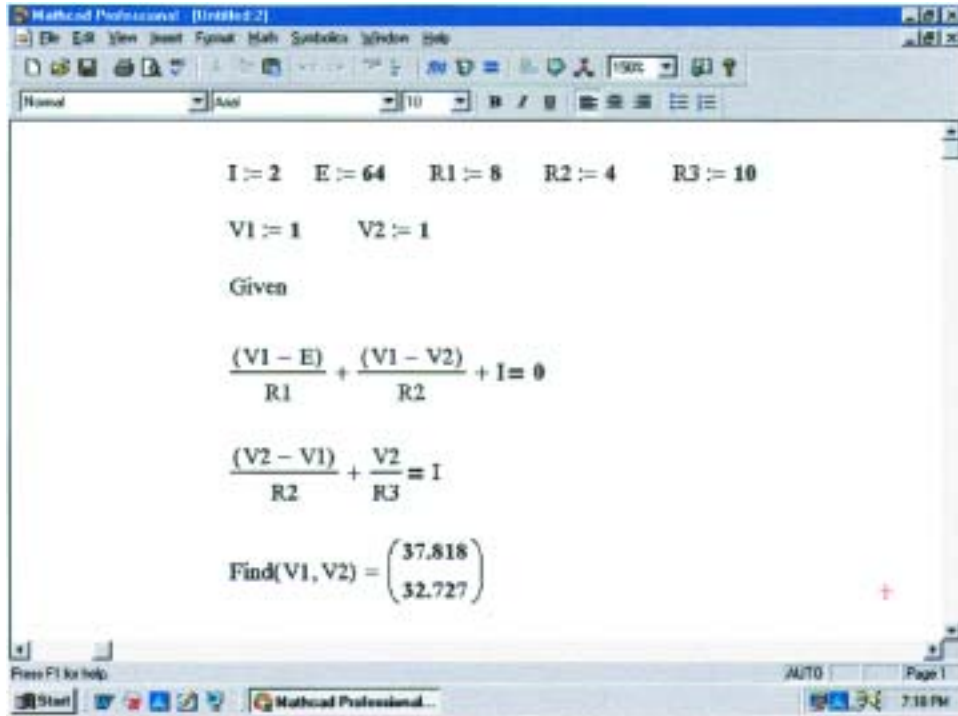$$\text{Find}(V1, V2) = \begin{pmatrix} 37.818 \\ 32.727 \end{pmatrix}$$

FIG. 8.47
Using Mathcad to verify the mathematical calculations of Example 8.20.

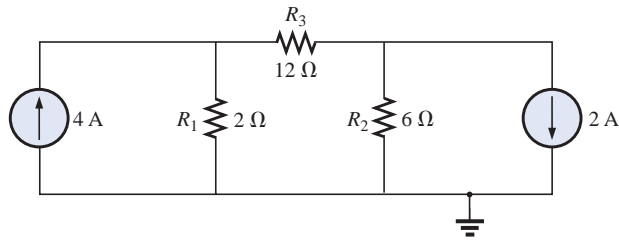**EXAMPLE 8.21** Determine the nodal voltages for the network of Fig. 8.48.

FIG. 8.48
Example 8.21.

Solution:
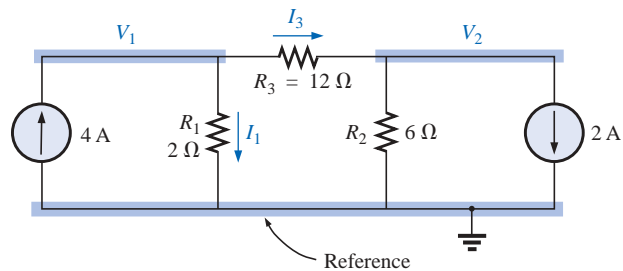
Steps 1 and 2: As indicated in Fig. 8.49.

FIG. 8.49
Defining the nodes and applying Kirchhoff's current law to the node V_1.

Step 3: Included in Fig. 8.49 for the node V_1. Applying Kirchhoff's current law:

$$4 \text{ A} = I_1 + I_3$$

and
$$4 \text{ A} = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3} = \frac{V_1}{2 \text{ }\Omega} + \frac{V_1 - V_2}{12 \text{ }\Omega}$$

Expanding and rearranging:

$$V_1 \left(\frac{1}{2 \text{ }\Omega} + \frac{1}{12 \text{ }\Omega} \right) - V_2 \left(\frac{1}{12 \text{ }\Omega} \right) = 4 \text{ A}$$

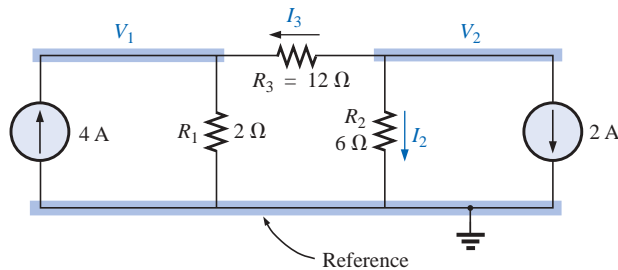For node V_2 the currents are defined as in Fig. 8.50.

FIG. 8.50

Applying Kirchhoff's current law to the node V_2.

Applying Kirchhoff's current law:

$$0 = I_3 + I_2 + 2 \text{ A}$$

and
$$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_2} + 2 \text{ A} = 0 \longrightarrow \frac{V_2 - V_1}{12 \text{ }\Omega} + \frac{V_2}{6 \text{ }\Omega} + 2 \text{ A} = 0$$

Expanding and rearranging:

$$V_2 \left(\frac{1}{12 \text{ }\Omega} + \frac{1}{6 \text{ }\Omega} \right) - V_1 \left(\frac{1}{12 \text{ }\Omega} \right) = -2 \text{ A}$$

resulting in two equations and two unknowns (numbered for later reference):

$$\left. \begin{array}{c} V_1 \left(\dfrac{1}{2 \text{ }\Omega} + \dfrac{1}{12 \text{ }\Omega} \right) - V_2 \left(\dfrac{1}{12 \text{ }\Omega} \right) = +4 \text{ A} \\[4mm] V_2 \left(\dfrac{1}{12 \text{ }\Omega} + \dfrac{1}{6 \text{ }\Omega} \right) - V_1 \left(\dfrac{1}{12 \text{ }\Omega} \right) = -2 \text{ A} \end{array} \right\} \qquad \textbf{(8.3)}$$

producing

$$\left. \begin{array}{l} \dfrac{7}{12} V_1 - \dfrac{1}{12} V_2 = +4 \\[4mm] -\dfrac{1}{12} V_1 + \dfrac{3}{12} V_2 = -2 \end{array} \right| \quad \begin{array}{l} 7V_1 - V_2 = 48 \\[4mm] -1V_1 + 3V_2 = -24 \end{array}$$

and
$$V_1 = \frac{\begin{vmatrix} 48 & -1 \\ -24 & 3 \end{vmatrix}}{\begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{120}{20} = \textbf{+6 V}$$

$$V_2 = \frac{\begin{vmatrix} 7 & 48 \\ -1 & -24 \end{vmatrix}}{20} = \frac{-120}{20} = \textbf{-6 V}$$

Since V_1 is greater than V_2, the current through R_3 passes from V_1 to V_2. Its value is

$$I_{R_3} = \frac{V_1 - V_2}{R_3} = \frac{6\text{ V} - (-6\text{ V})}{12\ \Omega} = \frac{12\text{ V}}{12\ \Omega} = \mathbf{1\ A}$$

The fact that V_1 is positive results in a current I_{R_1} from V_1 to ground equal to

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_1}{R_1} = \frac{6\text{ V}}{2\ \Omega} = \mathbf{3\ A}$$

Finally, since V_2 is negative, the current I_{R_2} flows from ground to V_2 and is equal to

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_2}{R_2} = \frac{6\text{ V}}{6\ \Omega} = \mathbf{1\ A}$$

Supernode

On occasion there will be independent voltage sources in the network to which nodal analysis is to be applied. In such cases we can convert the voltage source to a current source (if a series resistor is present) and proceed as before, or we can introduce the concept of a *supernode* and proceed as follows.

Start as before and assign a nodal voltage to each independent node of the network, including each independent voltage source as if it were a resistor or current source. Then mentally replace the independent voltage sources with short-circuit equivalents, and apply Kirchhoff's current law to the defined nodes of the network. Any node including the effect of elements tied only to *other* nodes is referred to as a *supernode* (since it has an additional number of terms). Finally, relate the defined nodes to the independent voltage sources of the network, and solve for the nodal voltages. The next example will clarify the definition of *supernode*.

EXAMPLE 8.22 Determine the nodal voltages V_1 and V_2 of Fig. 8.51 using the concept of a supernode.
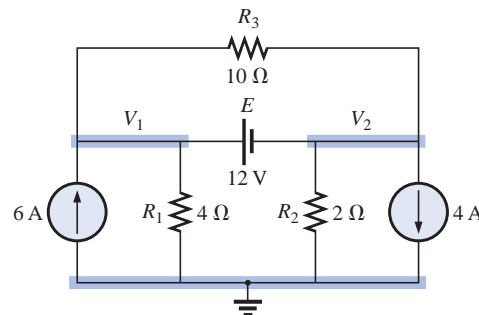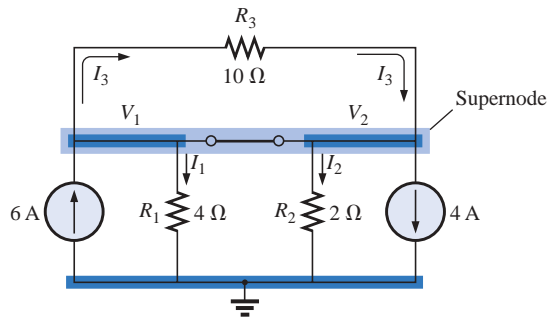

FIG. 8.51
Example 8.22.

Solution: Replacing the independent voltage source of 12 V with a short-circuit equivalent will result in the network of Fig. 8.52. Even though the mental application of a short-circuit equivalent is discussed above, it would be wise in the early stage of development to redraw the

FIG. 8.52
Defining the supernode for the network of Fig. 8.51.

network as shown in Fig. 8.52. The result is a single supernode for which Kirchhoff's current law must be applied. Be sure to leave the other defined nodes in place and use them to define the currents from that region of the network. In particular, note that the current I_3 will leave the supernode at V_1 and then enter the same supernode at V_2. It must therefore appear twice when applying Kirchhoff's current law, as shown below:

$$\Sigma\, I_i = \Sigma\, I_o$$

$$6\text{ A} + I_3 = I_1 + I_2 + 4\text{ A} + I_3$$

or

$$I_1 + I_2 = 6\text{ A} - 4\text{ A} = 2\text{ A}$$

Then

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = 2\text{ A}$$

and

$$\frac{V_1}{4\ \Omega} + \frac{V_2}{2\ \Omega} = 2\text{ A}$$

Relating the defined nodal voltages to the independent voltage source, we have

$$V_1 - V_2 = E = 12\text{ V}$$

which results in two equations and two unknowns:

$$0.25V_1 + 0.5V_2 = 2$$
$$V_1 - 1V_2 = 12$$

Substituting:

$$V_1 = V_2 + 12$$

$$0.25(V_2 + 12) + 0.5V_2 = 2$$

and

$$0.75V_2 = 2 - 3 = -1$$

so that

$$V_2 = \frac{-1}{0.75} = \mathbf{-1.333\text{ V}}$$

and

$$V_1 = V_2 + 12\text{ V} = -1.333\text{ V} + 12\text{ V} = \mathbf{+10.667\text{ V}}$$

The current of the network can then be determined as follows:

$$I_1\downarrow = \frac{V}{R_1} = \frac{10.667\text{ V}}{4\ \Omega} = \mathbf{2.667\text{ A}}$$

$$I_2\uparrow = \frac{V_2}{R_2} = \frac{1.333\text{ V}}{2\ \Omega} = \mathbf{0.667\text{ A}}$$

$$I_3\!\!\xrightarrow{} = \frac{V_1 - V_2}{10\ \Omega} = \frac{10.667\text{ V} - (-1.333\text{ V})}{10\ \Omega} = \frac{12\text{ V}}{10\ \Omega} = \mathbf{1.2\text{ A}}$$

A careful examination of the network at the beginning of the analysis would have revealed that the voltage across the resistor R_3 must be 12 V and I_3 must be equal to 1.2 A.

8.10 NODAL ANALYSIS (FORMAT APPROACH)

A close examination of Eq. (8.3) appearing in Example 8.21 reveals that the subscripted voltage at the node in which Kirchhoff's current law is applied is multiplied by the sum of the conductances attached to that node. Note also that the other nodal voltages within the same equation are multiplied by the negative of the conductance between the two nodes. The current sources are represented to the right of the equals sign with a positive sign if they supply current to the node and with a negative sign if they draw current from the node.

These conclusions can be expanded to include networks with any number of nodes. This will allow us to write nodal equations rapidly and in a form that is convenient for the use of determinants. A major requirement, however, is that *all voltage sources must first be converted to current sources before the procedure is applied.* Note the parallelism between the following four steps of application and those required for mesh analysis in Section 8.8:

1. *Choose a reference node and assign a subscripted voltage label to the (N − 1) remaining nodes of the network.*
2. *The number of equations required for a complete solution is equal to the number of subscripted voltages (N − 1). Column 1 of each equation is formed by summing the conductances tied to the node of interest and multiplying the result by that subscripted nodal voltage.*
3. *We must now consider the mutual terms that, as noted in the preceding example, are always subtracted from the first column. It is possible to have more than one mutual term if the nodal voltage of current interest has an element in common with more than one other nodal voltage. This will be demonstrated in an example to follow. Each mutual term is the product of the mutual conductance and the other nodal voltage tied to that conductance.*
4. *The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is assigned a positive sign if it supplies current to a node and a negative sign if it draws current from the node.*
5. *Solve the resulting simultaneous equations for the desired voltages.*

Let us now consider a few examples.

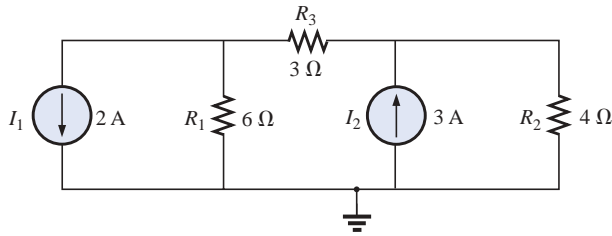**EXAMPLE 8.23** Write the nodal equations for the network of Fig. 8.53.

FIG. 8.53
Example 8.23.

Solution:

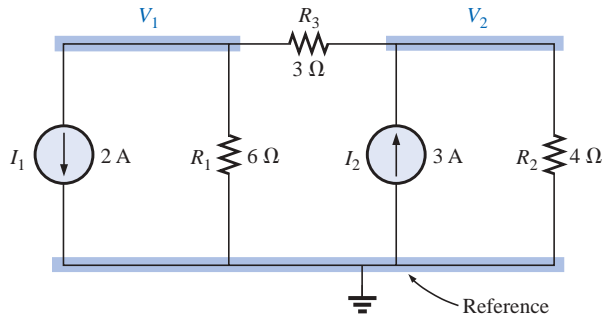*Step 1:* The figure is redrawn with assigned subscripted voltages in Fig. 8.54.

FIG. 8.54
Defining the nodes for the network of Fig. 8.53.

Steps 2 to 4:

Drawing current from node 1

$$V_1: \quad \underbrace{\left(\frac{1}{6\,\Omega} + \frac{1}{3\,\Omega} \right)}_{\substack{\text{Sum of} \\ \text{conductances} \\ \text{connected} \\ \text{to node 1}}} V_1 - \underbrace{\left(\frac{1}{3\,\Omega} \right)}_{\substack{\text{Mutual} \\ \text{conductance}}} V_2 = \overset{\downarrow}{-2\,\text{A}}$$

Supplying current to node 2

$$V_2: \quad \underbrace{\left(\frac{1}{4\,\Omega} + \frac{1}{3\,\Omega} \right)}_{\substack{\text{Sum of} \\ \text{conductances} \\ \text{connected} \\ \text{to node 2}}} V_2 - \underbrace{\left(\frac{1}{3\,\Omega} \right)}_{\substack{\text{Mutual} \\ \text{conductance}}} V_1 = \overset{\downarrow}{+3\,\text{A}}$$

and

$$\frac{1}{2}V_1 - \frac{1}{3}V_2 = -2$$

$$-\frac{1}{3}V_1 + \frac{7}{12}V_2 = 3$$

EXAMPLE 8.24 Find the voltage across the 3-Ω resistor of Fig. 8.55 by nodal analysis.
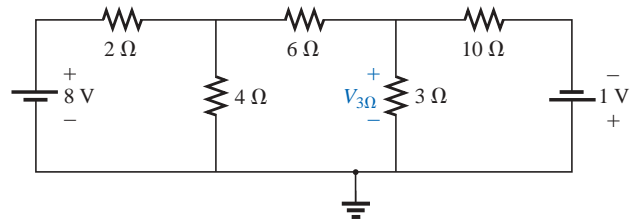

FIG. 8.55
Example 8.24.

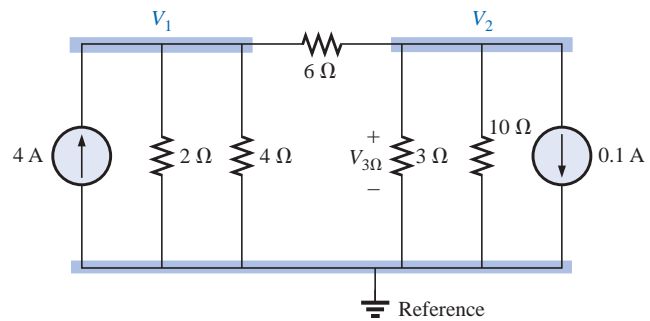**Solution:** Converting sources and choosing nodes (Fig. 8.56), we have

FIG. 8.56
Defining the nodes for the network of Fig. 8.55.

$$\left(\frac{1}{2\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{6\ \Omega}\right)V_1 - \left(\frac{1}{6\ \Omega}\right)V_2 = +4\ \text{A}$$

$$\left(\frac{1}{10\ \Omega} + \frac{1}{3\ \Omega} + \frac{1}{6\ \Omega}\right)V_2 - \left(\frac{1}{6\ \Omega}\right)V_1 = -0.1\ \text{A}$$

$$\frac{11}{12}V_1 - \frac{1}{6}V_2 = 4$$

$$-\frac{1}{6}V_1 + \frac{3}{5}V_2 = -0.1$$

resulting in

$$11V_1 - 2V_2 = +48$$
$$-5V_1 + 18V_2 = -3$$

and

$$V_2 = V_{3\Omega} = \frac{\begin{vmatrix} 11 & 48 \\ -5 & -3 \end{vmatrix}}{\begin{vmatrix} 11 & -2 \\ -5 & 18 \end{vmatrix}} = \frac{-33 + 240}{198 - 10} = \frac{207}{188} = \textbf{1.101 V}$$

As demonstrated for mesh analysis, nodal analysis can also be a very useful technique for solving networks with only one source.

EXAMPLE 8.25 Using nodal analysis, determine the potential across the 4-Ω resistor in Fig. 8.57.

Solution 1: The reference and four subscripted voltage levels were chosen as shown in Fig. 8.58. A moment of reflection should reveal that for any difference in potential between V_1 and V_3, the current through and the potential drop across each 5-Ω resistor will be the same. There-fore, V_4 is simply a midvoltage level between V_1 and V_3 and is known if V_1 and V_3 are available. We will therefore not include it in a nodal voltage and will redraw the network as shown in Fig. 8.59. Understand, however, that V_4 can be included if desired, although four nodal volt-ages will result rather than the three to be obtained in the solution of this problem.

$$V_1: \quad \left(\frac{1}{2\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{10\ \Omega}\right)V_1 - \left(\frac{1}{2\ \Omega}\right)V_2 - \left(\frac{1}{10\ \Omega}\right)V_3 = 0$$

$$V_2: \quad \left(\frac{1}{2\ \Omega} + \frac{1}{2\ \Omega}\right)V_2 - \left(\frac{1}{2\ \Omega}\right)V_1 - \left(\frac{1}{2\ \Omega}\right)V_3 = 3\ \text{A}$$

$$V_3: \quad \left(\frac{1}{10\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{4\ \Omega}\right)V_3 - \left(\frac{1}{2\ \Omega}\right)V_2 - \left(\frac{1}{10\ \Omega}\right)V_1 = 0$$

which are rewritten as

$$1.1V_1 - 0.5V_2 - 0.1V_3 = 0$$
$$V_2 - 0.5V_1 - 0.5V_3 = 3$$
$$0.85V_3 - 0.5V_2 - 0.1V_1 = 0$$

For determinants,

$$
\begin{array}{c}
\overset{c}{}\quad\overset{b}{}\quad\overset{a}{} \\
1.1V_1 - 0.5V_2 - 0.1V_3 = 0 \\
-0.5V_1 + 1V_2 - 0.5V_3 = 3 \\
-0.1V_1 - 0.5V_2 + 0.85V_3 = 0
\end{array}
$$

Before continuing, note the symmetry about the major diagonal in the equation above. Recall a similar result for mesh analysis. Exam-ples 8.23 and 8.24 also exhibit this property in the resulting equations. Keep this thought in mind as a check on future applications of nodal analysis.

$$V_3 = V_{4\Omega} = \frac{\begin{vmatrix} 1.1 & -0.5 & 0 \\ -0.5 & +1 & 3 \\ -0.1 & -0.5 & 0 \end{vmatrix}}{\begin{vmatrix} 1.1 & -0.5 & -0.1 \\ -0.5 & +1 & -0.5 \\ -0.1 & -0.5 & +0.85 \end{vmatrix}} = \mathbf{4.645\ V}$$

Mathcad Solution: By now the sequence of steps necessary to solve a series of equations using Mathcad should be quite familiar and less threatening than the first encounter. For this example, all the param-eters were entered in the three simultaneous equations, avoiding the
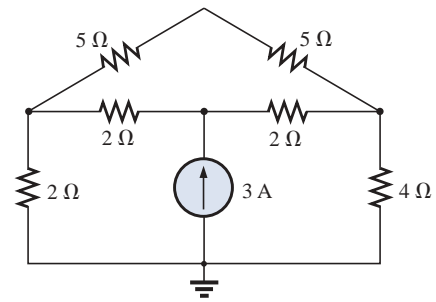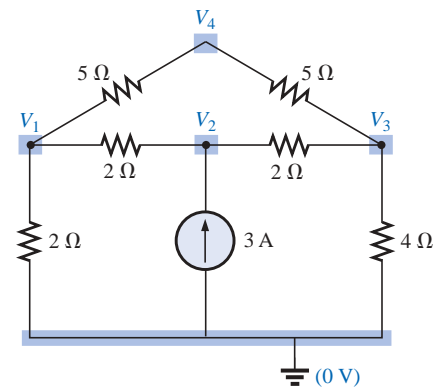

FIG. 8.57
Example 8.25.

FIG. 8.58
Defining the nodes for the network of Fig. 8.57.
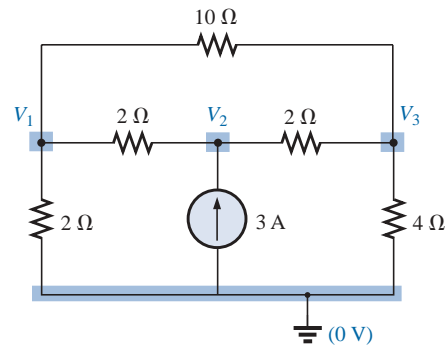

FIG. 8.59
Reducing the number of nodes for the network of Fig. 8.57 by combining the two 5-Ω resistors.

need to define each parameter of the network. Simply provide a **Guess** at the three nodal voltages, apply the word **Given,** and enter the three equations properly as shown in Fig. 8.60. It does take some practice to ensure that the bracket is moved to the proper location before making an entry, but this is simply part of the rules set up to maintain control of the operations to be performed. Finally, request the desired nodal voltages using the correct format. The numerical results will appear, again confirming our theoretical solutions.
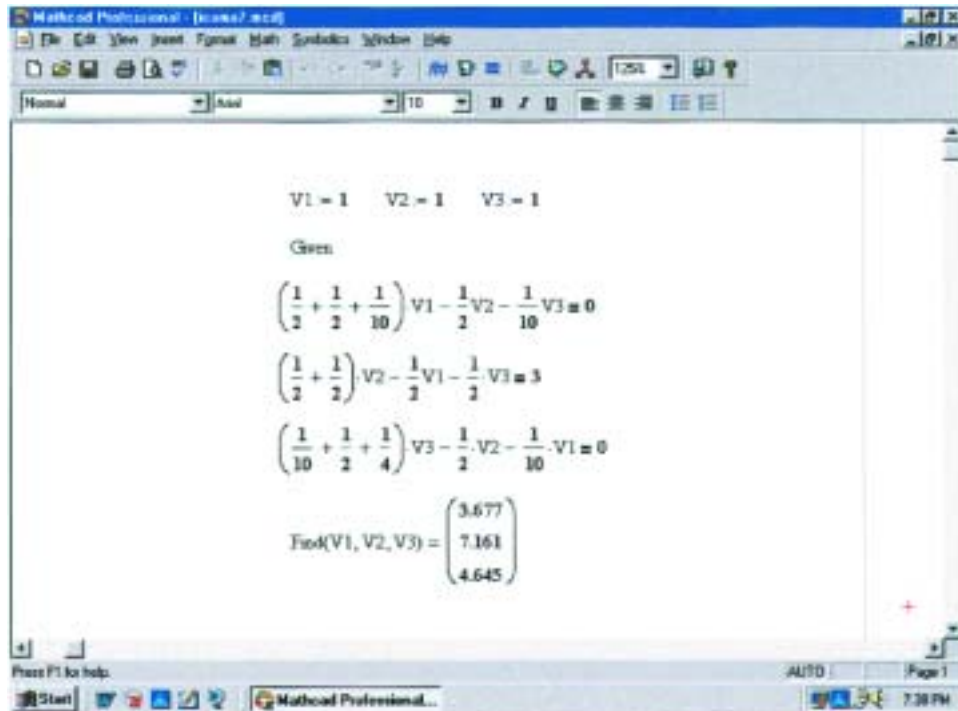

FIG. 8.60
Using Mathcad to verify the mathematical calculations of Example 8.25.

The next example has only one source applied to a ladder network.

EXAMPLE 8.26 Write the nodal equations and find the voltage across the 2-Ω resistor for the network of Fig. 8.61.
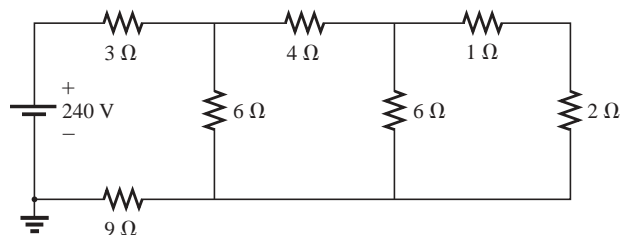

FIG. 8.61
Example 8.26.

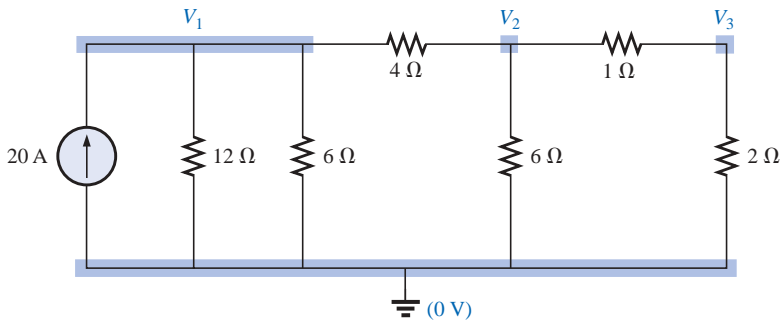*Solution:* The nodal voltages are chosen as shown in Fig. 8.62.

FIG. 8.62
Converting the voltage source to a current source and defining the nodes for the network of Fig. 8.61.

$$V_1: \quad \left(\frac{1}{12\,\Omega} + \frac{1}{6\,\Omega} + \frac{1}{4\,\Omega}\right)V_1 - \left(\frac{1}{4\,\Omega}\right)V_2 + \quad 0 \quad = 20 \text{ V}$$

$$V_2: \quad \left(\frac{1}{4\,\Omega} + \frac{1}{6\,\Omega} + \frac{1}{1\,\Omega}\right)V_2 - \left(\frac{1}{4\,\Omega}\right)V_1 - \left(\frac{1}{1\,\Omega}\right)V_3 = 0$$

$$V_3: \quad \left(\frac{1}{1\,\Omega} + \frac{1}{2\,\Omega}\right)V_3 - \left(\frac{1}{1\,\Omega}\right)V_2 + \quad 0 \quad = 0$$

and

$$0.5V_1 - 0.25V_2 + 0 \quad = 20$$

$$-0.25V_1 + \frac{17}{12}V_2 - 1V_3 = 0$$

$$0 - 1V_2 + 1.5V_3 = 0$$

Note the symmetry present about the major axis. Application of determinants reveals that

$$V_3 = V_{2\Omega} = \mathbf{10.667 \text{ V}}$$

8.11 BRIDGE NETWORKS

This section introduces the **bridge network,** a configuration that has a multitude of applications. In the chapters to follow, it will be employed in both dc and ac meters. In the electronics courses it will be encountered early in the discussion of rectifying circuits employed in converting a varying signal to one of a steady nature (such as dc). A number of other areas of application also require some knowledge of ac networks; these areas will be discussed later.

The bridge network may appear in one of the three forms as indicated in Fig. 8.63. The network of Fig. 8.63(c) is also called a *symmetrical lattice network* if $R_2 = R_3$ and $R_1 = R_4$. Figure 8.63(c) is an excellent example of how a planar network can be made to appear nonplanar. For the purposes of investigation, let us examine the network of Fig. 8.64 using mesh and nodal analysis.