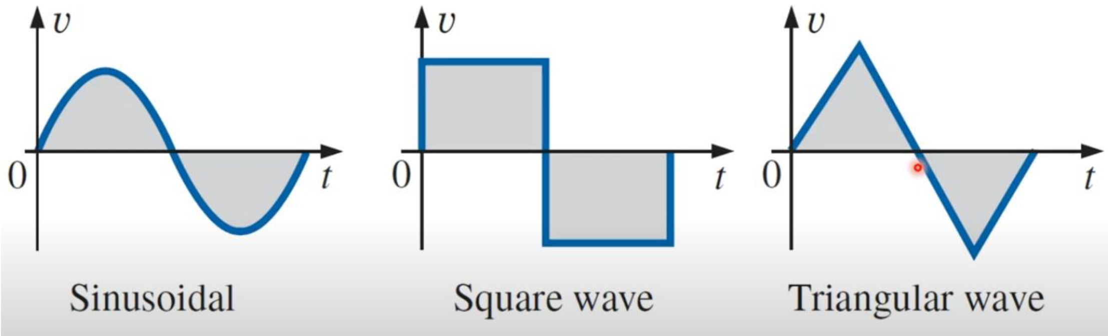
AC Circuits

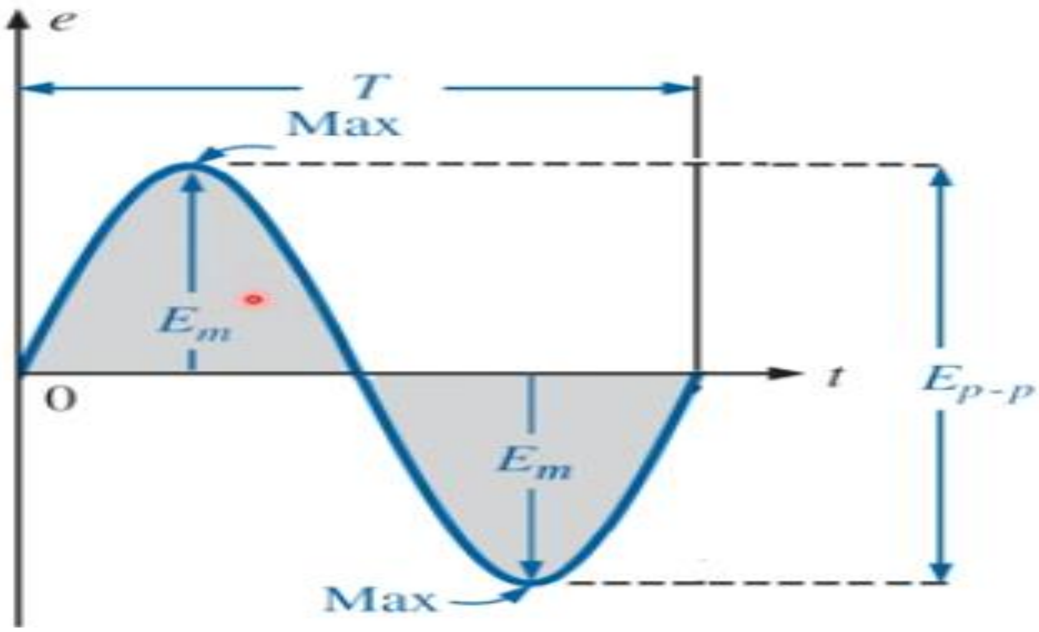
Sinusoidal Alternating waveforms

The analysis of networks in which the magnitude of the source of e.m.f varies in a set manure



The term alternating indicates only that the waveform alternates between two prescribed levels, the term sinusoidal, square, triangular must also be applied. The pattern of particular interest is the sinusoidal Ac voltage.

Definitions:



Wave from: The path followed by a quantity, such as the electromotive force in the figure above, which is plotted as a function of some variable such as time and angle in degrees or radians.

Instantaneous value: The wave size at any moment in time, denoted by the letters (Em1 and Em2) as in the figure above.

Peak amplitude: The maximum value of a waveform, as measured from its average or mean value, denoted by uppercase letters (such as Em).

Peak value: The maximum instantaneous value of a function as measured from the zero-volt level.

Peak-to-peak value: Denoted by E_{P-P} or V_{P-P} the full voltage between positive and negative peaks of the waveform that is, the sum of the meglitinide of the positive and negative peaks.

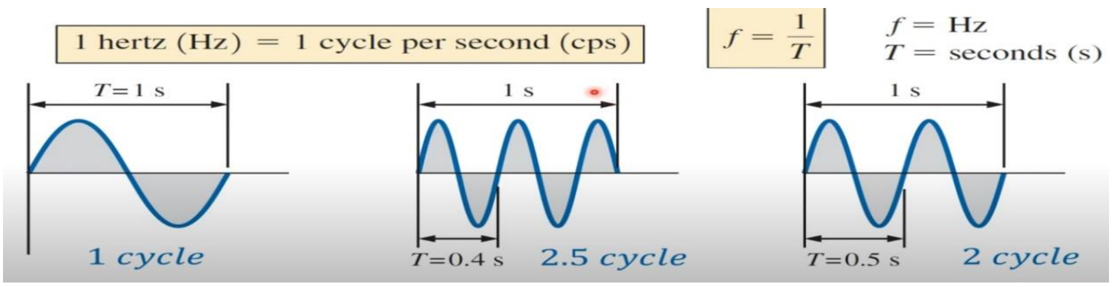
Periodic waveform: A waveform that continually repeats the same time interval

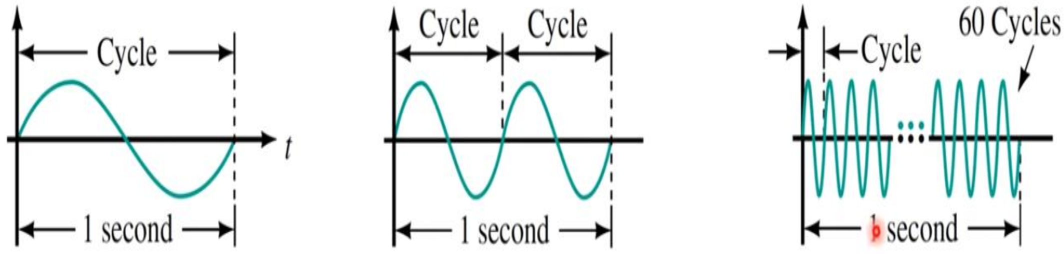
Period(T): The time of aperiodic waveform.

Cycle: The portion of a waveform contained period of time.

Frequency(f): The number of cycles that occur in 1 second. The unit of frequency in cycle/second (cps) or Hertz (Hz).

Where





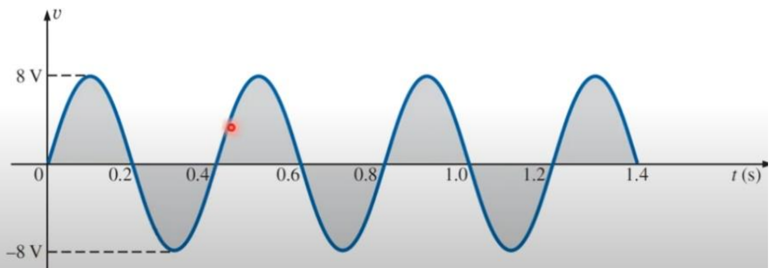
(a) 1 cycle per second = 1 Hz (b) 2 cycles per second = 2 Hz (c) 60 cycles per second = 60 Hz

EXAMPLE 1: For the sinusoidal waveform in Figure shown.

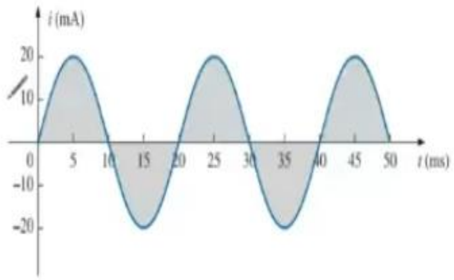
a. What is the peak value?
b. What is the instantaneous value at 0.3 s and 0.6 s?
c. What is the peak-to-peak value of the waveform?
d. What is the period of the waveform?
e. How many cycles are shown?
f. What is the frequency of the waveform?

Solutions:

a. **8 V.**
b. At 0.3 s, **8 V; ** at 0.6 s, **0 V.**
c. **16 V.**
d. **0.4 s.**
e. **3.5 cycles.**
f. **2.5 cps,** or **2.5 Hz.**



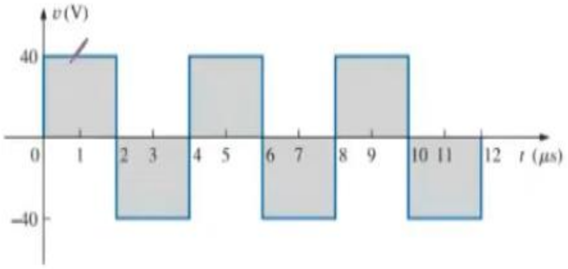
1. For the sinusoidal waveform in Fig. 13.80:
   a. What is the peak value?
   b. What is the instantaneous value at 15 ms and at 20 ms?
   c. What is the peak-to-peak value of the waveform?
   d. What is the period of the waveform?
   e. How many cycles are shown?



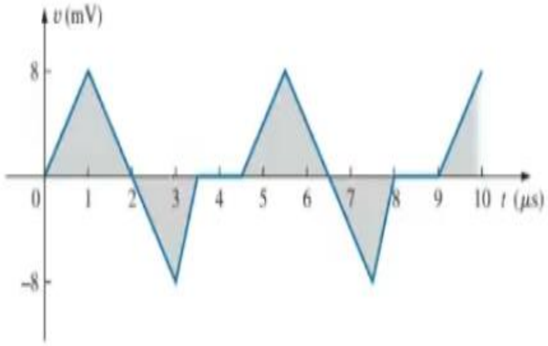
Sol:

a. **20 mA**
b. 15 ms: **−20 mA**, 20 ms: **0 mA**
c. **40 mA**
d. **20 ms**
e. **2.5 cycles**

2. For the square-wave signal in Fig. 13.81:
   a. What is the peak value?
   b. What is the instantaneous value at 5 μs and at 11 μs?
   c. What is the peak-to-peak value of the waveform?
   d. What is the period of the waveform?
   e. How many cycles are shown?



3. For the periodic waveform in Fig. 13.82:
   a. What is the peak value?
   b. What is the instantaneous value at 3 μs and at 9 μs?
   c. What is the peak-to-peak value of the waveform?
   d. What is the period of the waveform?
   e. How many cycles are shown?



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4. Find the period of a periodic waveform whose frequency is
   a. 25 Hz.
   b. 40 MHz.
   c. 25 kHz.
   d. 1 Hz.

Sol:

a. $T = \frac{1}{f} = \frac{1}{25\text{ Hz}} = \mathbf{40\text{ ms}}$

b. $T = \frac{1}{f} = \frac{1}{40\text{ mHz}} = \mathbf{25\text{ ns}}$

c. $T = \frac{1}{f} = \frac{1}{25\text{ kHz}} = \mathbf{40\ \mu s}$

d. $T = \frac{1}{f} = \frac{1}{1\text{ Hz}} = \mathbf{1\text{ s}}$

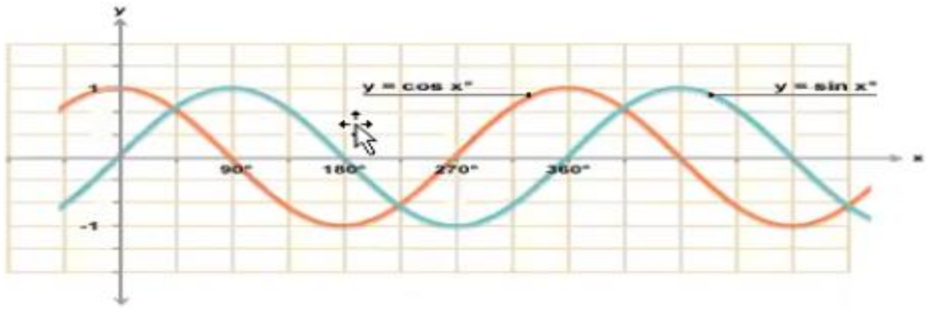
6. If a periodic waveform has a frequency of 20 Hz, how long (in seconds) will it take to complete five cycles?

Sol:

$$T = \frac{1}{20\text{ Hz}} = 0.05\text{ s},\ 5(0.05\text{ s}) = \mathbf{0.25\text{ s}}$$

<u>The Sine wave:</u> The basic mathematical format for sinusoidal wave form is ($E_m \sin \theta$)

The cosine wave is said to **lead** the sine wave by 90° and the sine wave is side to **lag** the cosine wave by 90°



sine wave with cosine wave

The unit of measurement for the horizontal axis can be **time**, **degrees**, or **radians**.

$$\theta = \omega t$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$ The angular frequency in radians/s

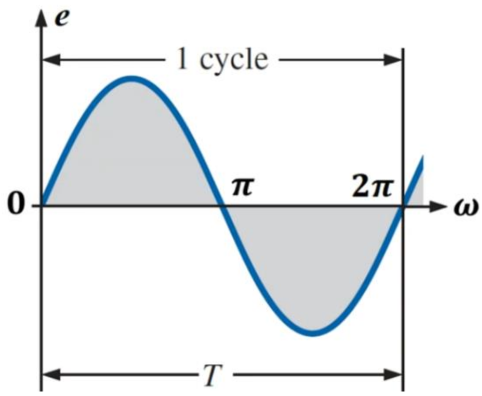
$$360^o = 2\pi \; radians$$

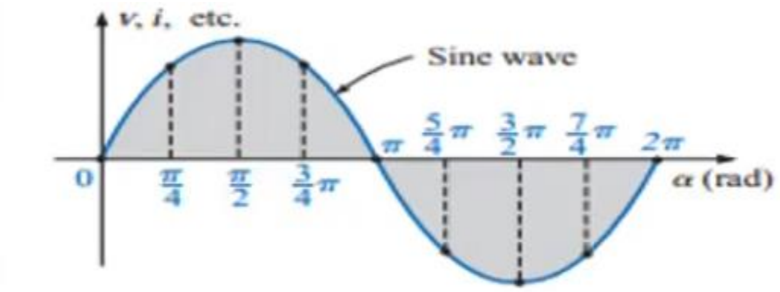
$$1\; rad. = \frac{360^o}{2\pi} = \frac{180^o}{\pi} = 57.3^o$$

The conversions equations between the two are the following:

$$\text{Radians} = \left(\frac{\pi}{180^\circ}\right) \times (\text{degrees})$$

$$\text{Degrees} = \left(\frac{180^\circ}{\pi}\right) \times (\text{radians})$$





The velocity with which the radius vector rotates about the center, called the **angular velocity,** can be determined from the following equation:

$$\text{Angular velocity} = \frac{\text{distance (degrees or radians)}}{\text{time (seconds)}}$$

$$\omega = \frac{\alpha}{t}$$

$$\alpha = \omega t$$

Where: $\omega(omega)$ is typically provided in radians per second (rad/sec)

the angle α *(alpha)* is the distance in (radians).

t is time in sec.

the time required to complete one revolution is equal to the period (T) of the sinusoidal waveform and the distance (α) , will equal to (2π), then

$$\omega = \frac{\alpha}{t} = \frac{2\pi}{T}$$

$$T = \frac{1}{f}$$

$\omega = 2\pi f$ (rad/sec)

Applying these equations, we find

$$\mathbf{90°:}\ \text{Radians} = \frac{\pi}{180°}(90°) = \frac{\boldsymbol{\pi}}{\mathbf{2}}\ \textbf{rad}$$

$$\mathbf{30°:}\ \text{Radians} = \frac{\pi}{180°}(30°) = \frac{\boldsymbol{\pi}}{\mathbf{6}}\ \textbf{rad}$$

$$\frac{\boldsymbol{\pi}}{\mathbf{3}}\ \textbf{rad}:\ \text{Degrees} = \frac{180°}{\pi}\left(\frac{\pi}{3}\right) = \mathbf{60°}$$

$$\frac{\mathbf{3}\boldsymbol{\pi}}{\mathbf{2}}\ \textbf{rad:}\ \text{Degrees} = \frac{180°}{\pi}\left(\frac{3\pi}{2}\right) = \mathbf{270°}$$

12. Find the angular velocity of a waveform with a period of
a. 2 s. **b.** 0.3 ms. **c.** 4 μs. **d.** $\frac{1}{26}$ s.

Sol:

a. $\omega = \frac{2\pi}{T} = \frac{2\pi}{2\text{ s}} = \mathbf{3.14\ rad/s}$

b. $\omega = \frac{2\pi}{0.3 \times 10^{-3}\text{ s}} = \mathbf{20.94 \times 10^{3}\ rad/s}$

c. $\omega = \frac{2\pi}{4 \times 10^{-6}\text{ s}} = \mathbf{1.57 \times 10^{6}\ rad/s}$

d. $\omega = \frac{2\pi}{1/25\text{ s}} = \mathbf{157.1\ rad/s}$

13. Find the angular velocity of a waveform with a frequency of

a. 50 Hz. **b.** 600 Hz.
c. 2 kHz. **d.** 0.004 MHz.

Sol:

a. $\omega = 2\pi f = 2\pi(50\text{ Hz}) = \mathbf{314.16\ rad/s}$

b. $\omega = 2\pi f = 2\pi(600\text{ Hz}) = \mathbf{3769.91\ rad/s}$

c. $\omega = 2\pi f = 2\pi(2\text{ kHz}) = \mathbf{12.56 \times 10^3\ rad/s}$

d. $\omega = 2\pi f = 2\pi(0.004\text{ MHz}) = \mathbf{25.13 \times 10^3\ rad/s}$

14. Find the frequency and period of sine waves having an angular velocity of

a. 754 rad/s. **b.** 8.4 rad/s.
c. 6000 rad/s. **d.** $\frac{1}{16}$ rad/s.

Sol:

a. $\omega = 2\pi f = \frac{2\pi}{T} \Rightarrow f = \frac{\omega}{2\pi}$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$f = \frac{\omega}{2\pi} = \frac{754\text{ rad/s}}{2\pi} = \mathbf{120\ Hz},\ \mathbf{T = 8.33\ ms}$

b. $f = \frac{\omega}{2\pi} = \frac{8.4\text{ rad/s}}{2\pi} = \mathbf{1.34\ Hz},\ \mathbf{T = 746.27\ ms}$

c. $f = \frac{\omega}{2\pi} = \frac{6000\text{ rad/s}}{2\pi} = \mathbf{954.93\ Hz},\ \mathbf{T = 1.05\ ms}$

d. $f = \frac{\omega}{2\pi} = \frac{1/16\ \text{r}}{}$ **00.5 ms**

***15.** Given $f = 60$ Hz, determine how long it will take the sinusoidal waveform to pass through an angle of 45°.

***16.** If a sinusoidal waveform passes through an angle of 30° in 5 ms, determine the angular velocity of the waveform.

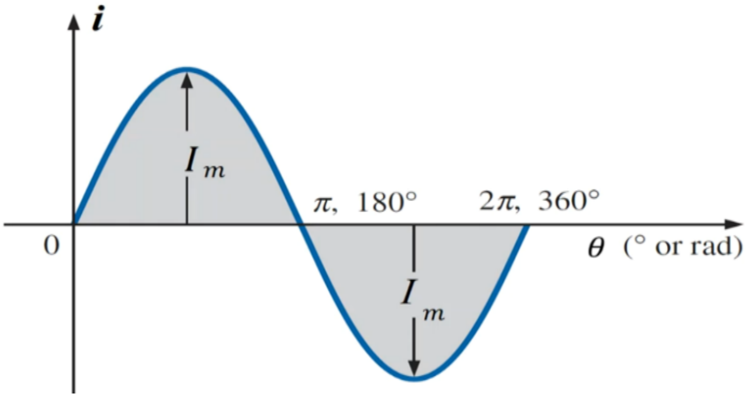
GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE

$$i = I_m \sin(\theta)$$

$$i = I_m \sin(\omega t)$$

$$i = I_m \sin(2\pi f t)$$

$$i = I_m \sin(\frac{2\pi}{T} t)$$

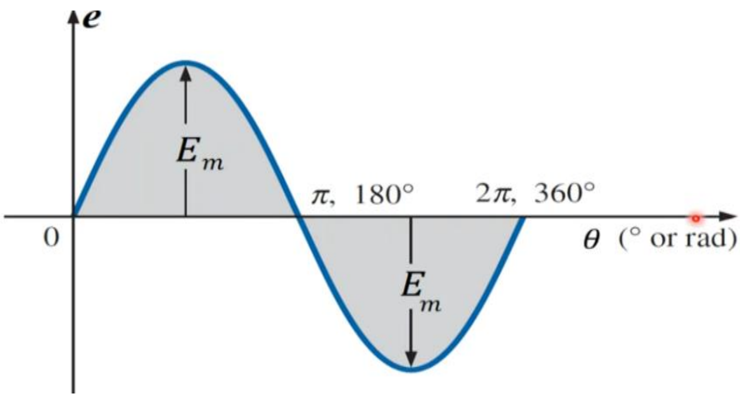


$$e = E_m \sin(\theta)$$

$$e = E_m \sin(\omega t)$$

$$e = E_m \sin(2\pi f t)$$

$$e = E_m \sin(\frac{2\pi}{T} t)$$



EXAMPLE

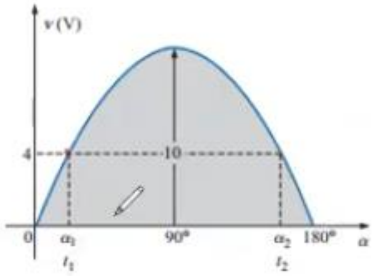
a. Determine the angle at which the magnitude of the sinusoidal function $v = 10 \sin 377t$ is 4 V.

b. Determine the time at which the magnitude is attained.

Solutions:

a.

$$\alpha_1 = \sin^{-1}\frac{v}{E_m} = \sin^{-1}\frac{4\text{ V}}{10\text{ V}} = \sin^{-1} 0.4 = \mathbf{23.578^\circ}$$



However, Figure ... reveals that the magnitude of 4 V (positive) will be attained at two points between 0° and 180°. The second intersection is determined by

$$\alpha_2 = 180^\circ - 23.578^\circ = \mathbf{156.422^\circ}$$

In general, therefore, keep in mind that Equations ( ) will provide an angle with a magnitude between 0° and 90°.

b. Eq. ( ): $\alpha = \omega t$, and so $t = \alpha/\omega$. However, α must be in radians. Thus,

$$\alpha\text{ (rad)} = \frac{\pi}{180^\circ}(23.578^\circ) = 0.411\text{ rad}$$

and

$$t_1 = \frac{\alpha}{\omega} = \frac{0.411\text{ rad}}{377\text{ rad/s}} = \mathbf{1.09\text{ ms}}$$

For the second intersection,

$$\alpha\text{ (rad)} = \frac{\pi}{180^\circ}(156.422^\circ) = 2.73\text{ rad}$$

$$t_2 = \frac{\alpha}{\omega} = \frac{2.73\text{ rad}}{377\text{ rad/s}} = \mathbf{7.24\text{ ms}}$$