# Average (Mean) value

The average value of any current or voltage is the value indicated on a (d-c) meter

$$\boxed{G \text{ (average value)} = \frac{\text{algebraic sum of areas}}{\text{length of curve}}}$$

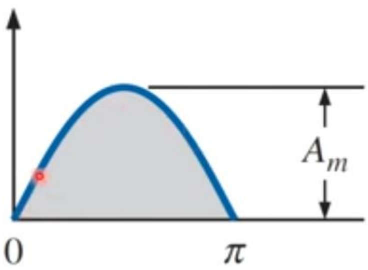
Where **G** (Represents the average value of current or voltage)

OR $$G = \frac{1}{T}\int_0^T i\,dt$$

For the account algebraic sum of areas

$$Area = \int_0^{\pi} A_m \sin\theta \, d\theta$$

$$\boxed{\text{Area} = 2A_m}$$




Note:

- Areas above the axis assigned a positive sign, and those below, a negative sign

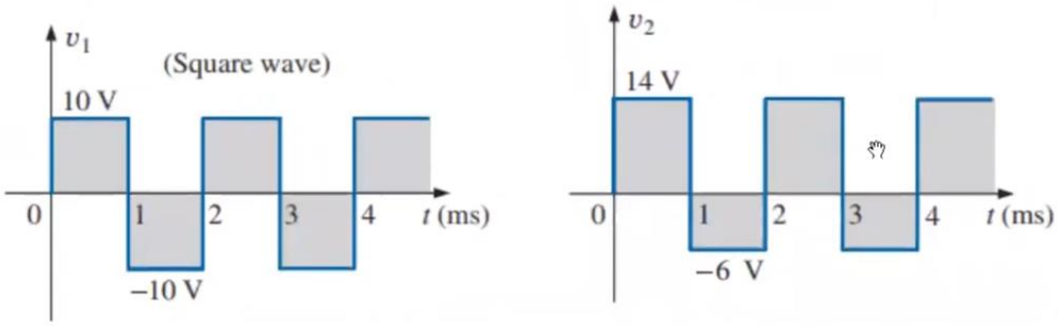
- A positive average value is then above the axis, and negative value, below

- The average value of any current and voltage is the value indicated a Dc meter

- The average value of any symmetrical waveform (positive prate equal to a negative prate) is zero

- The integration is useful mathematical tool to find areas under the curves

Ex: Determine the average value of the waveforms in Figure below:




(a) (b)

Sol: (a)

$$G = \frac{(10\text{ V})(1\text{ ms}) - (10\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{0}{2\text{ ms}} = \mathbf{0\ V}$$

(b)

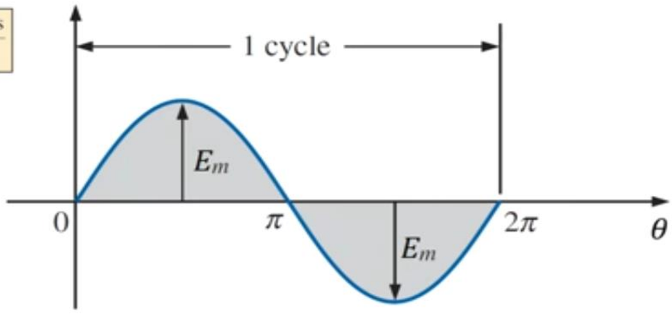
$$G = \frac{(14\text{ V})(1\text{ ms}) - (6\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{14\text{ V} - 6\text{ V}}{2} = \frac{8\text{ V}}{2} = \mathbf{4\ V}$$

**EXAMPLE 2:** Determine the average value of the sinusoidal waveform in Figure shown.

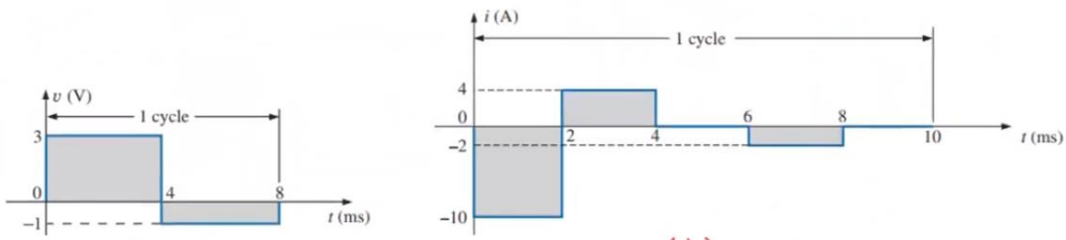
***Solution:*** By inspection it is fairly obvious that
***the average value of a pure sinusoidal waveform over one full cycle is zero.***

$$G\text{ (average value)} = \frac{\text{algebraic sum of areas}}{\text{length of curve}}$$

$$G = \frac{+2E_m - 2E_m}{2\pi} = 0\,V$$



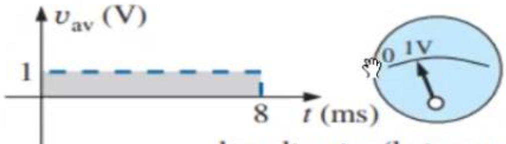

Ex: Determine the average value of the waveforms over one full cycle in Figure below:




(a) (b)

Sol:

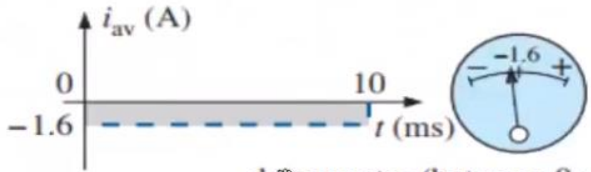
$$\text{a.}\quad G = \frac{+(3\text{ V})(4\text{ ms}) - (1\text{ V})(4\text{ ms})}{8\text{ ms}} = \frac{12\text{ V} - 4\text{ V}}{8} = \mathbf{1\ V}$$




dc voltmeter (between 0 and 8 ms)

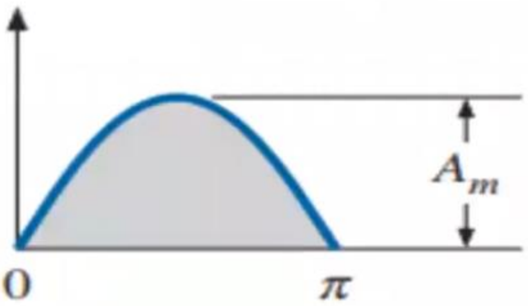
$$\text{b.}\quad G = \frac{-(10\text{ V})(2\text{ ms}) + (4\text{ V})(2\text{ ms}) - (2\text{ V})(2\text{ ms})}{10\text{ ms}}$$

$$= \frac{-20\text{ V} + 8\text{ V} - 4\text{ V}}{10} = -\frac{16\text{ V}}{10} = \mathbf{-1.6\ V}$$




dc ammeter (between 0 and 10 ms)

Ex: Determine the average value of the waveforms Figure below:

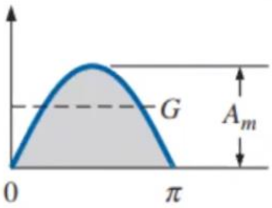



$$\text{Area} = \int_0^{\pi} A_m \sin\alpha\, d\alpha$$

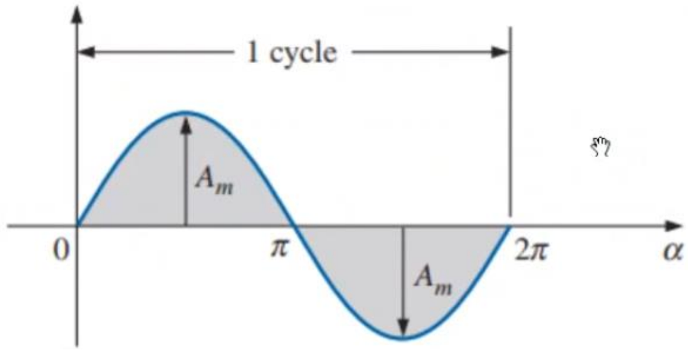
$$\begin{aligned}\text{Area} &= A_m[-\cos\alpha]_0^{\pi}\\ &= -A_m(\cos\pi - \cos 0^\circ)\\ &= -A_m[-1 - (+1)] = -A_m(-2)\end{aligned}$$

$$= 2A_m$$

$$\boxed{G = \frac{2A_m}{\pi} = 0.637A_m}$$



**Ex:** Determine the average value of the sinusoidal waveform in Figure below:

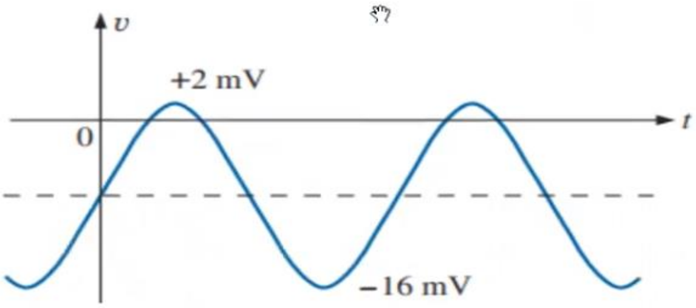



Sol:

***the average value of a pure sinusoidal waveform over one full cycle is zero.***

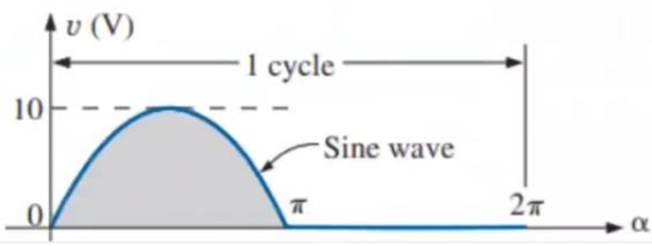
$$G = \frac{+2A_m - 2A_m}{2\pi} = \mathbf{0\ V}$$

Ex: Determine the average value of the waveform in Figure below:




***Solution:*** The peak-to-peak value of the sinusoidal function is 16 mV + 2 mV = 18 mV. The peak amplitude of the sinusoidal waveform is, therefore, 18 mV/2 = 9 mV. Counting down 9 mV from 2 mV (or 9 mV up from −16 mV) results in an average or dc level of **−7 mV,** as noted by the dashed line in Fig.

Ex: Determine the average value of the waveform in Figure below




***Solution:***

$$G = \frac{2A_m + 0}{2\pi} = \frac{2(10\ \text{V})}{2\pi} \cong \mathbf{3.18\ V}$$

# Effective value (r.m.s.)

## Root-Mean-Squar

The r.m.s. value of an alternating current is given by steady (dc) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.

It is also known as the effective or virtual value of the alternating current; the former being used more extensively.

That generated by current (I) for time (t) in a resistance (R)

Heat generated = $I^2.R.t$ (Joules)

The standard form of a sinusoidal alternating current is $i = I_m \sin \omega t = I_m \sin \theta$

The power delivered by the ac supply at any instant of time is

$$P_{ac} = (i_{ac})^2 R = (I_m \sin \omega t)^2 R = (I_m^2 \sin^2 \omega t) R$$

However

$$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t)$$

Therefore,

$$P_{ac} = I_m^2 \left[ \frac{1}{2}(1 - \cos 2\omega t) \right] R$$

$$P_{ac} = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2} \cos 2\omega t$$

The *average power* delivered by the ac source is just the first term $\left(\frac{I_m^2 R}{2}\right)$, since the average value of a cosine wave is zero even though the wave may have twice the frequency of the original input current waveform.

Equating the average power delivered by the ac generator to that delivered by the dc source,

$$P_{av(ac)} = P_{dc}$$

$$\frac{I_m^2 \cancel{R}}{2} = I_{dc}^2 \cancel{R}$$

$$\frac{I_m^2}{2} = I_{dc}^2$$

which, in words, states that,

*the equivalent dc of a sinusoidal current or voltage is $\frac{1}{\sqrt{2}}$ or 0.707 of its peak value.*

The equivalent dc value is called the **rms** or **effective value** of the sinusoidal quantity.

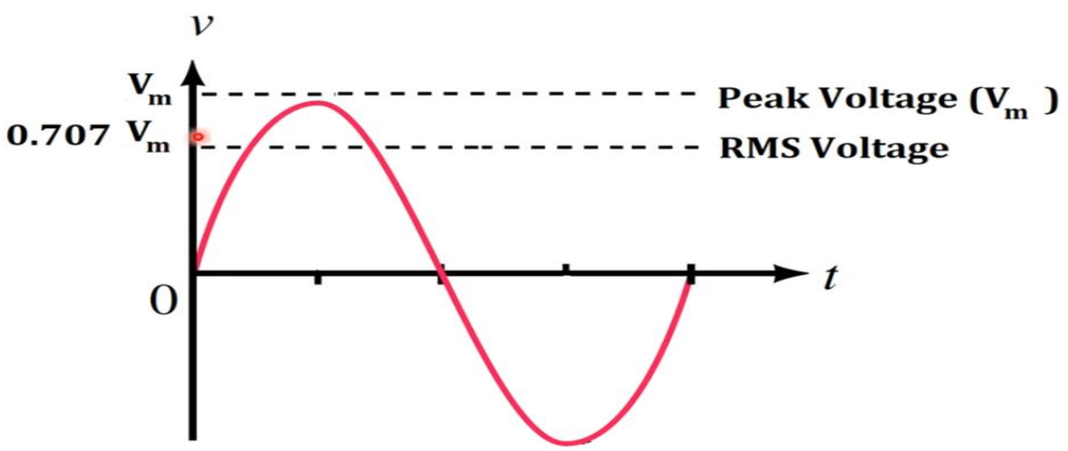
$$I_{rms} = \frac{1}{\sqrt{2}} I_m = 0.707 I_m$$

$$E_{rms} = \frac{1}{\sqrt{2}} E_m = 0.707 E_m$$

and

$$I_m = \sqrt{2} I_{rms} = 1.414 I_{rms}$$

$$E_m = \sqrt{2} E_{rms} = 1.414 E_{rms}$$



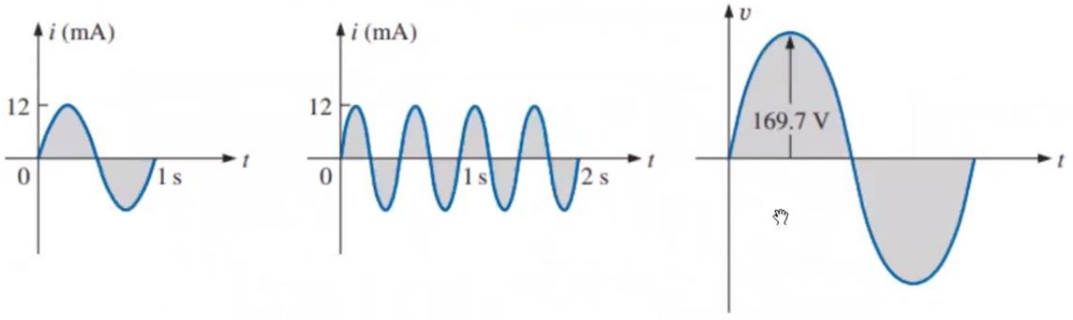

In general, The root mean square value (effective value) of any quantity plotted as a function of time can be found by using the following equation

$$I_{rms} = \sqrt{\frac{\int_0^T i^2(t)\,dt}{T}}$$

which means

$$I_{rms} = \sqrt{\frac{\text{area } (i^2(t))}{T}}$$

**EX:** Find the rms values of the sinusoidal waveform in each part in Figure below:



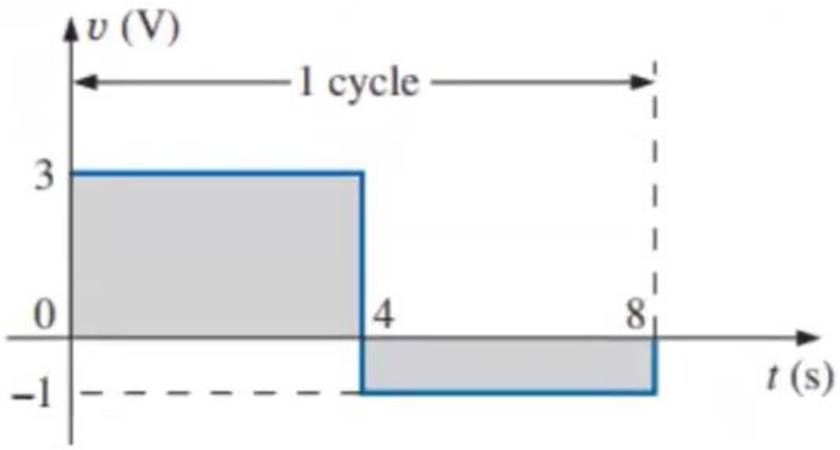

(a)  (b)  (c)

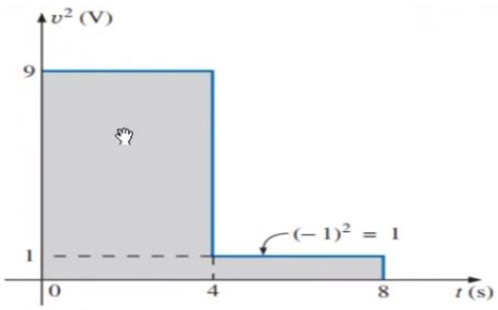
(a), $I_{rms} = 0.707(12 \times 10^{-3}\ \text{A}) = \mathbf{8.48\ mA.}$

(b) $I_{rms} = 0.707(12 \times 10^{-3}\ \text{A}) = \mathbf{8.48\ mA.}$ note that frequency did not change on effective value.

(c), $V_{rms} = 0.707(169.73\ \text{V}) \cong 120\text{V}$

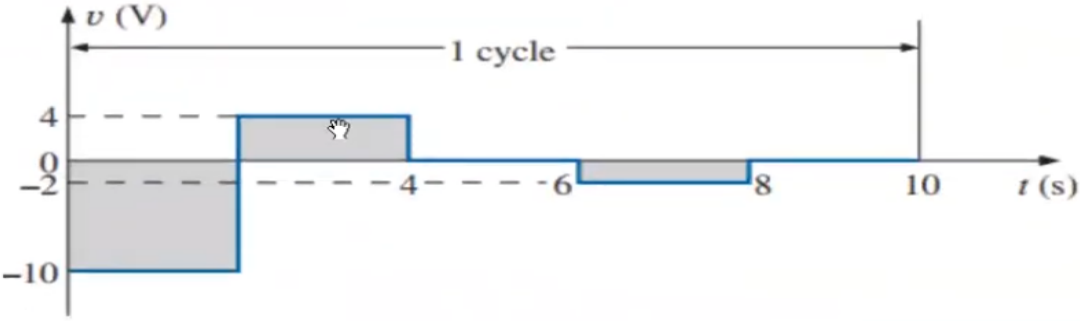
**EX:** Find the rms value of the waveform in Figure below:



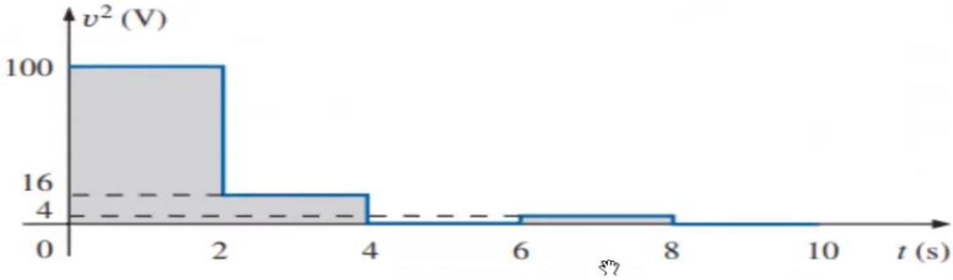




$$V_{rms} = \sqrt{\frac{(9)(4) + (1)(4)}{8}} = \sqrt{\frac{40}{8}} = \mathbf{2.24\ V}$$

**Ex:** Calculate the rms value of the voltage in Figure




Sol:




$$v_{rms} = \sqrt{\frac{100X2 + 16X2 + 4X2}{10}} = 4.899V$$

# Form Factor

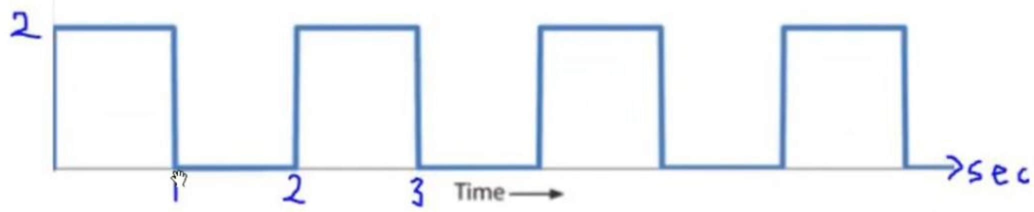
$$Form\ Factor = \frac{rms\ value}{average\ value}$$

For sinusoidal waveform $= \frac{0.707I_m}{0.637I_m} = 1.1$

# Peak Factor

$$Peak\ Factor = \frac{maximum\ value}{rms\ value}$$

**Ex:** Determine the form factor and peak factor of the square wave in Figure below:




Sol:

$$G = \frac{2X1}{2} = 1V$$

$$rms = \sqrt{\frac{4x1}{2}} = 1.414$$

$$\text{Form factor} = \frac{1.414}{1} = 1.414$$

$$\text{peak factor} = \frac{2}{1.414} = 1.414$$