Al Mamoun University College
Medical Instrumentation
Engineering Technologies

Digital Signal Processing

(DSP)

For the Third Class

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Digital Signal Processing

Signal

A signal is defined as any ***physical quantity*** that varies with ***time***, ***space***, or any other ***independent variable*** or ***variables***.

Examples of signals that we encounter frequently are ***speech***, ***music***, ***picture***, and ***video signals***.

Mathematically, we describe a signal as a ***function*** of ***one*** or ***more independent variables***.

For example, the functions

$$s_1(t)=5t$$

$$s_2(t)=20t^2$$

Describes two signals, one that varies linearly with the independent variable t (time) and a second that varies quadratically with t.

Dimensional signals have two independent variables. For example, image is a 2 – D signal whose independent variables are the two spatial coordinates (x,y)

$s(x,y)=3x+2xy+10y^2$ (Two independent variables.)

Video is a 3 – dimensional signal whose independent variables are the two spatial coordinates, (x,y) and time (t).

Similarly, a 3 – D picture is also a 3 – D signal whose independent variables are the three spatial coordinates (x,y,z).

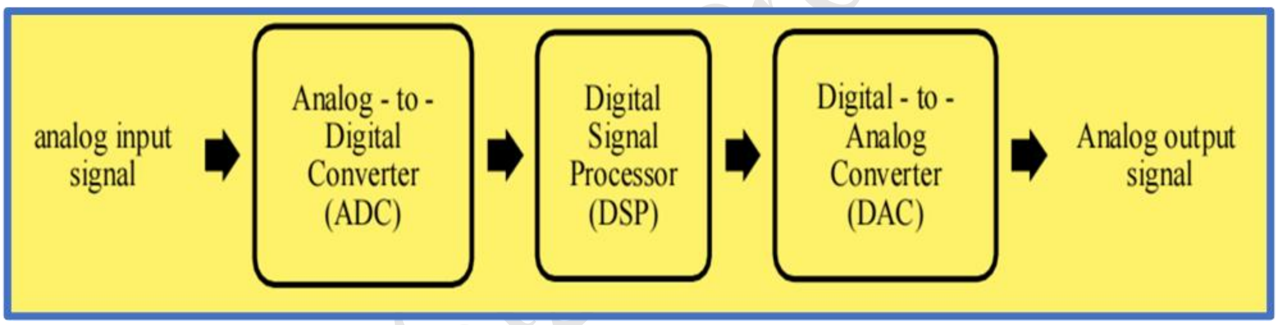
Processing

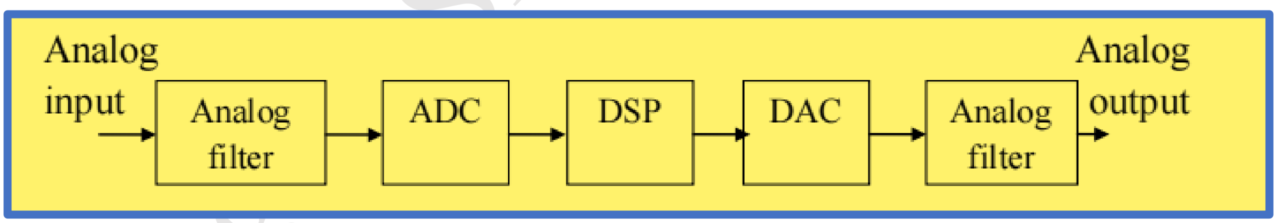
The operation performed on the signal by the system is called ***Signal Processing.***

Digital Signal Processing

Digital Signal Processing of signals may consist of a number of ***mathematical operations*** as specified by a ***software program***, in which case, the program represents an implementation of the system in software. Alternatively, digital processing of signals may also be performed by ***digital hardware*** (logic circuits). So, a digital system can be implemented as a combination of ***digital hardware and software***, each of which performs its own set of specified operations.

Block Diagram Representation of Digital Signal Processing





Advantages of Digital Signal Processing over Analog Signal Processing

1. A digital programmable system allows flexibility in reconfiguring the digital signal processing operations simply by changing the program.

Reconfiguration of an analog system usually implies a redesign of the hardware followed by testing and verification.

2. Tolerances in analog circuit components and power supply make it extremely difficult to control the accuracy of analog signal processor.

   A digital signal processor provides better control of accuracy requirements in terms of word length, floating – point versus fixed – point arithmetic, and similar factors.

3. Digital signals are easily stored on magnetic tapes and disks without deterioration or loss of signal fidelity beyond that introduced in A/D conversion. So the signals become transportable and can be processed offline.
4. Digital signal processing is cheaper than its analog counterpart.
5. Digital circuits are amenable for full integration. This is not possible for analog circuits because inductances of respectable value (μH or mH) require large space to generate flux.
6. The same digital signal processor can be used to perform two operations by time multiplexing, since digital signals are defined only at finite number of time instants.
7. Different parts of digital signal processor can work at different sampling rates.
8. It is very difficult to perform precise mathematical operations on signals in analog form but these operations can be routinely implemented on a digital computer using software.
9. Several filters need several boards in analog signal processing, whereas in digital signal processing, same DSP processor is used for many filters.

Disadvantages of Digital Signal Processing over Analog Signal Processing

1. Digital signal processors have increased complexity.
2. Signals having extremely wide bandwidths require fast – sampling – rate ADCs. Hence the frequency range of operation of DSPs is limited by the speed of ADC.
3. In analog signal processor, passive elements are used, which dissipate very less power.

In digital signal processor, active elements like transistors are used, which dissipate more power.

CLASSIFICATION OF SIGNALS

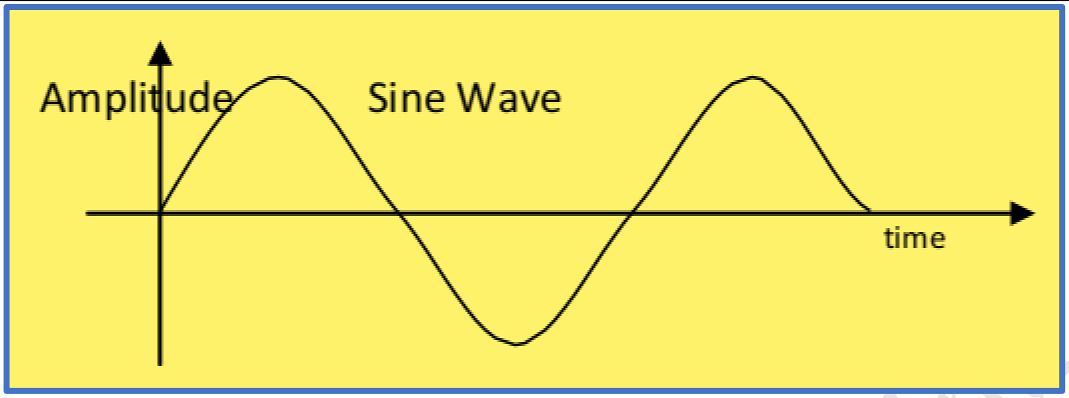
There are five methods of classifying signals based on different features :

(a) Based on independent variable.
(b) Depending upon the number of independent variable.
(c) Depending upon the certainity by which the signal can be uniquely described.
(d) Based on repetition nature.
(e) Based on reflection.

a) Based on independent variables

Analog signal:

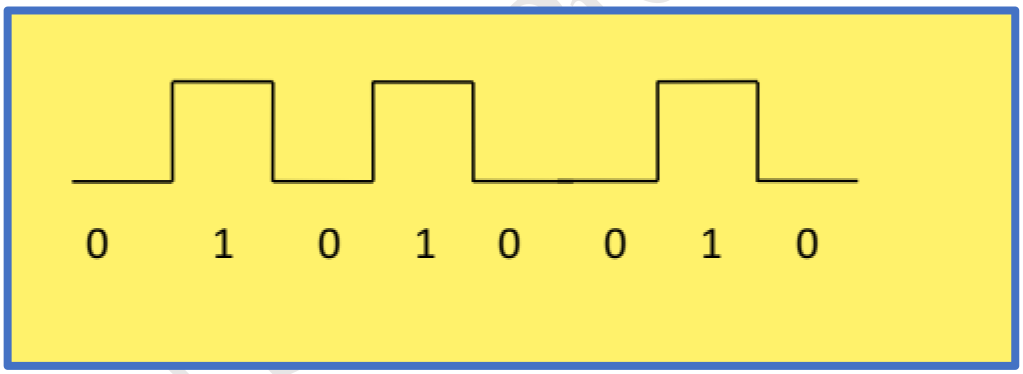
A signal that is defined for every instants of time is known as ***analog signal***. Analog signals are ***continuous*** in ***amplitude*** and ***continuous*** **in** ***time***. It is denoted by x(t). It is also called as ***continuous time signal***.



Continuous time signal

Digital signal

The signals that are ***discrete in time*** and ***quantized in amplitude*** is called digital signal.



Digital Signal

Continuous time signal:

A signal that is defined for every ***instants of time*** is known as ***continuous time signal***. Continuous time signals are ***continuous in amplitude and continuous in time.*** It is denoted by x(t).

e.g. Speech signal is an example of analog signal.

Discrete time signal:

A signal that is defined for ***discrete instants of time*** is known as ***discrete time signal***. Discrete time signals are ***continuous in amplitude and***

discrete in time. It is also obtained by sampling a continuous time signal. It is denoted by x(n).

e.g. Digitized music signal stored in CD-ROM disk.

(*b*) **Depending upon the number of independent variable.**

(*i*) *1-D Signals.* It is a function of a single independent variable.

e.g. (*a*) speech signal–independent variable is time.

(*b*) music signal.

(*ii*) *2-D Signal.* It is a function of two independent variables.

e.g. Photographic image signal–two independent variables are the two spatial variables.

Each frame of a black and white video signal is a 2D-image signal that is a function of two discrete spatial variable, with each frame occurring squentially at discrete instants of time.

(*iii*) *M-D Signal.* It is a function of 'M' independent variable in time.

e.g. Video signal.

The black and white video signal can be considered an example of a 3D signal where the three independent variables are two spatial variables and time.

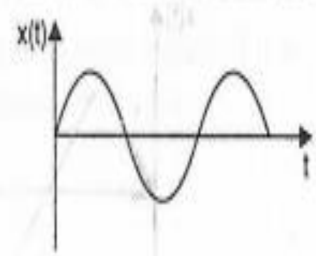
A colour video signal is a three-channel signal composed of three 3-D signals representing the three primary colours : red, green and blue (RGB).

c) Depending up on the certainly by which the signal can be uniquely described.

(*i*) *Deterministic Signal.* A signal that can be uniquely determined by a well-defined process such as a mathematical expression or rule, or table look-up is called a deterministic signal.

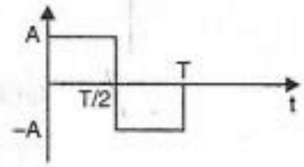
e.g. (*a*) A sinusoidal signal can be represented as,

$$v(t) = V_m \sin \omega t \text{ for } t \geq 0.$$



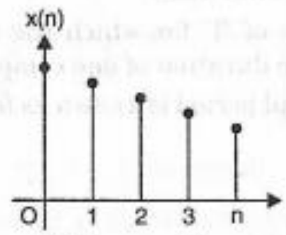
(*b*) A square signal can be defined as

$$x(t) = A \quad \text{for} \quad 0 < t < T/2$$
$$= -A \quad \text{for} \quad T/2 < t < T.$$

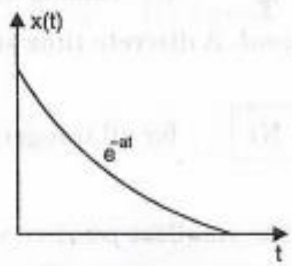


(*c*) An exponential,

discrete time signal $x(n) = e^{an}$ for $n \geq 0$



Continuous time signal $x(t) = e^{-at}$ for $n < 0$.



(*ii*) *Random Signal.* A signal that is generated in a random fashion and cannot be predicted ahead of time is called a "random signal".

e.g. Speech signal, ECG signal, EEG signals.

d)*Based on repetition nature.*

Periodic and Aperiodic signal

Periodic signal:

A signal is said to periodic if it ***repeats*** again and again over a certain period of time.

Aperiodic signal:

A signal that does ***not repeat*** at a definite interval of time is called aperiodic signal.

Continuous domain:

A Continuous time signal is said to periodic if it satisfies the condition

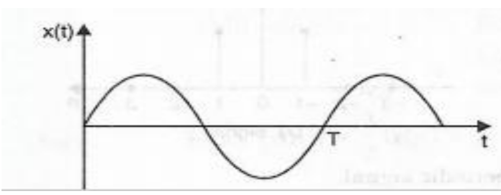
$$x(t) = x(t+T)$$

$where\ T\ is\ fundamental\ time\ period.$

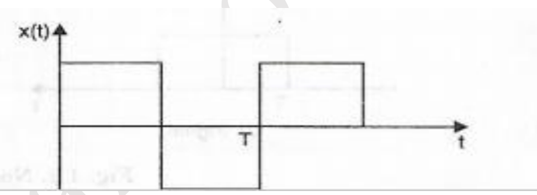
If the above condition is not satisfied then the signal is said to be Aperiodic.

If frequency $f = \frac{1}{T}$ Hz then the angular frequency $\boldsymbol{\Omega = 2\pi f = \frac{2\pi}{T}}$ rad/sec, fundamental time period $\boldsymbol{T = \frac{2\pi}{\Omega}}$,

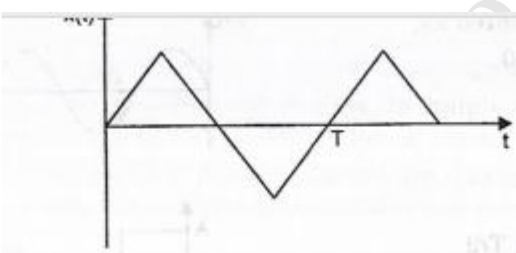
e.g. Sine wave, square wave, triangular wave etc.,



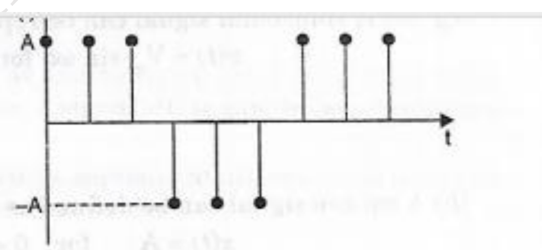
Sine wave



square wave



Triangular wave



discrete or impulse wave

Discrete domain:

A Discrete time signal is said to periodic if it satisfies the condition

$$x(n) = x(n+N)$$

$where\ N\ is\ fundamental\ time\ period$ (integer).

If the above condition is not satisfied then the signal is said to be

Aperiodic.

Fundamental time period N = $\frac{2\pi}{\omega}$, where ω is discrete fundamental angular frequency in rad/sec.

EXAMPLE: The signals

$$x_1(n) = a^n u(n) = \begin{cases} a^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$x_2(n) = \cos(n^2)$$

are not periodic, whereas the signal.

$$x_3(n) = e^{j\pi n/8}$$

is periodic and has a fundamental period of N = 16.

(sol: $x_3(n) = x_3(n+N) \Longrightarrow e^{j\left(\pi\frac{n}{8}\right)} = e^{j(2\pi f n)}$

$$\pi\frac{n}{8} = 2\pi f n$$

$$\Longrightarrow f = \frac{1}{16} \Longrightarrow \boldsymbol{N = \frac{1}{f}} \Longrightarrow \boldsymbol{N = 16}$$**)**

If $x_1(n)$ is a signal that is periodic with a period N_1, and $x_2(n)$ is another signal that is periodic with a period N_2, the sum

$$x(n) = x_1(n) + x_2(n)$$

will always be periodic and the fundamental period is

$$\boldsymbol{N} = \frac{\boldsymbol{N_1 N_2}}{\mathbf{gcd}\,(\boldsymbol{N_1}, \boldsymbol{N_2})}$$

where gcd (N_1, N_2) means the greatest common divisor of N_1 and N_2.

The same is true for the product.

$$x(n) = x_1(n)x_2(n)$$

will always be periodic and the fundamental period is

$$\boldsymbol{N} = \frac{\boldsymbol{N_1 N_2}}{\mathbf{gcd}\,(\boldsymbol{N_1}, \boldsymbol{N_2})}$$

Examples

1- Determine whether or not each of the following signals are periodic? If a signal is periodic, determine its fundamental period.

a- $x(t) = cos(t + \frac{\pi}{4})$ b- $x(t) = cos\left(\frac{\pi}{3}t\right) + sin(\frac{\pi}{4}t)$

$c - x(t) = cost + sin\sqrt{2}t$ t

sol:

a- $x(t) = x(t + T)$

$cos(t + \frac{\pi}{4}) = cos(t + \frac{\pi}{4} + T)$)

$cos(t + \frac{\pi}{4} + 2\pi) = cos(t + \frac{\pi}{4} + T)$)

$t + \frac{\pi}{4} + 2\pi = t + \frac{\pi}{4} + T) \Longrightarrow T = 2\pi$

b-sol: $x(t) = cos\left(\frac{\pi}{3}t\right) + sin(\frac{\pi}{4}t)$

$x_1(t) = cos\left(\frac{\pi}{3}t\right)$ is a periodic

$cos\left(\frac{\pi}{3}t\right) = cos(2\pi f t) \Rightarrow \left(\frac{\pi}{3}t\right) = (2\pi f t) \Rightarrow \frac{\pi}{3} = 2\pi f \Rightarrow f = \frac{1}{6}$

$$T_1 = \frac{1}{f} = 6$$

$x_1(t) = cos\left(\frac{\pi}{3}t\right)$ is a periodic

$sin\left(\frac{\pi}{4}t\right) = sin(2\pi f t) \Rightarrow \left(\frac{\pi}{4}t\right) = (2\pi f t) \Rightarrow \frac{\pi}{4} = 2\pi f \Rightarrow f = \frac{1}{8}$

$$T_2 = \frac{1}{f} = 8$$

So,

$\frac{T_1}{T_2} = \frac{6}{8} = \frac{3}{4}$ *is rational number* $\Rightarrow x(t)$ *is periodic with period*

$T = 3T_2 = 24\ sec$

Or $T = 4T_1 = 24\ sec$

c-$x(t) = cost + sin\sqrt{2}t$

sol: $x_1(t) = cost$ is a periodic

$(2\pi f t) = t \Rightarrow 2\pi f = 1 \Rightarrow f = \frac{1}{2\pi}$

$$T_1 = \frac{1}{f} = 2\pi$$

$x_1(t) = sin\sqrt{2}t$ is a periodic

$(2\pi f t) = \sqrt{2}t \Rightarrow 2\pi f = \sqrt{2} \Rightarrow f = \frac{\sqrt{2}}{2\pi} \Rightarrow T_2 = \frac{2\pi}{\sqrt{2}}$

So, $\frac{T_1}{T_2} = \frac{2\pi}{\frac{2\pi}{\sqrt{2}}} = \sqrt{2}$ *is not rational number* $\Rightarrow x(t)$ *is not periodic*

e)***Based on reflection.***

Even and Odd Signals

(i) *Even signal (symmetric).* A signal $x(t)$ or $x(n)$ is referred to as even signal, if it is identical to its time reversal counter part *i.e.*, with its about the origin.

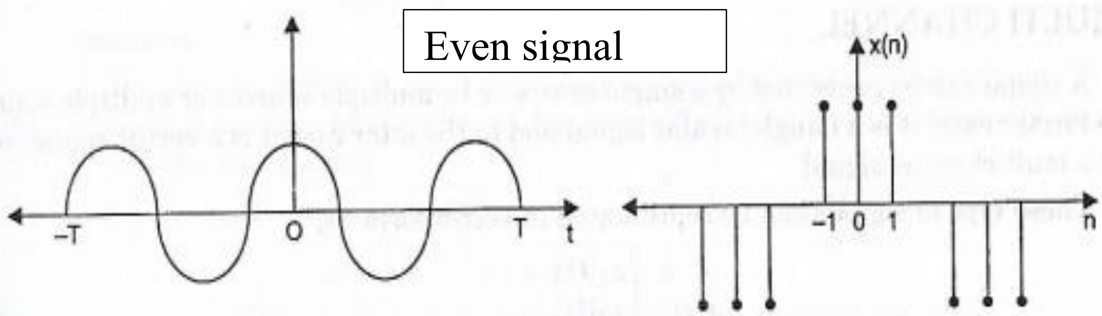
$$x(t) = x(-t) \quad \text{for all 't'} \quad \text{CT even signal.}$$

and

$$x(n) = x(-n) \quad \text{for all 'n'} \quad \text{DT even signal.}$$

*Even signals are symmetric with respect to vertical axis.

e.g.

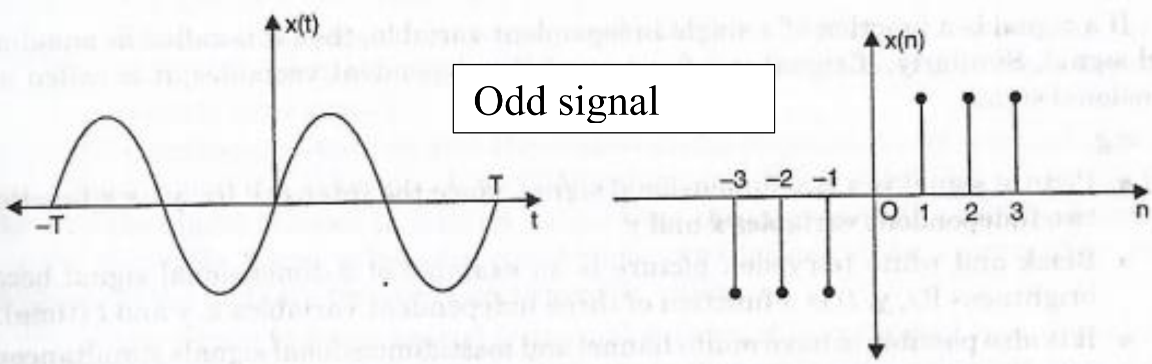


(ii) *Odd signal (antisymmetric).* A signal is said to be odd signal if,

$$x(t) = -x(-t) \quad \text{for} \quad \text{CT signal.}$$

$$x(n) = -x(-n) \quad \text{for} \quad \text{DT signal.}$$

e.g.



A real – valued discrete – time signal is called an ***Even Signal*** if it is identical with its reflection about the ***origin*** .i.e., it must be ***symmetrical*** about the vertical axis.

$$x(n) = x(-n) \quad \forall n$$

A real – valued discrete – time signal is called an ***Odd Signal*** if it is

antisymmetrical about the vertical axis.

$x(n) = -x(-n) \quad \forall n$

Every signal x[n] can be expressed as the sum of its even and odd components.

$$x[n] = x_e[n] + x_o[n]$$

Where

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

1. The sum of two even signals are even signal.
2. The sum of two odd signals are odd signal.
3. The sum of an even signal and an odd signal is neither even nor odd signal.
4. Product of even and odd sequences results is an odd sequence.
5. Product of two odd sequences results is an even sequence.
6. Product of two even sequences results is an even sequence.

System

A system is a ***physical device*** that performs an ***operation*** on a signal. For example, natural signals are generated by a system that responds to a stimulus or force. For eg., speech signals are generated by forcing air through the ***vocal cords***. Here, the vocal cord and the vocal tract constitute the ***system*** (also called the vocal cavity). The air is the stimulus.

The stimulus along with the system is called a signal source.

An electronic filter is also a ***system***. Here, the system performs an operation on the signal, which has the effect of ***reducing the noise*** and interference from the desired information – bearing signal.

When the signal is passed through a system, the signal is said to have been ***processed***.

Multiple Choice Questions

Circle the correct answer:-

1. **A signal is defined as any physical quantity that varies with**:
   a-time,
   b-space,
   c-any other independent variable or variables,
   d-**All of the above**

2. Discrete time signal is a function of:-
   a- Integer-valued variable n.
   b- Fractional variable, n.
   c- Time.
   d- Speed.

3. The signal $x(t) = e^{j\frac{\pi n}{8}}$ is periodic and has a fundamental period of N =:

a- 8 b-12 **c-16** d-4

4. If $x_1(n)$ is a signal that is periodic with a period N_1, and $x_2(n)$ is another signal that is periodic with a period N_2, the sum

$$x(n) = x_1(n) + x_2(n)$$

will always be periodic and the fundamental period is:

a-$N = \frac{N_1 N_2}{\gcd(N_1, N_2)}$ b-$N = \frac{N_1}{\gcd(N_1, N_2)}$

c-$N = \frac{N_2}{\gcd(N_1, N_2)}$ d- Any one of above.

4.A signal is even signal if it satisfies:

a-$x(t) = -x(t)$ b- $x(t) = -x(-t)$

c- $\boldsymbol{x(t) = x(-t)}$ d- $x(t) = x(2t)$

5.A signal is odd signal if it satisfies:

a-$x(t) = -x(t)$ **b- $\boldsymbol{x(t) = -x(-t)}$**

c- $x(t) = x(-t)$ d- $x(t) = x(2t)$

6. The odd part of a signal x(t) is:

a- $x(t) + x(-t)$ b- $x(t) - x(-t)$

c- $\frac{1}{2}(x(t) + x(-t))$ **d- $\boldsymbol{\frac{1}{2}(x(t) - x(-t))}$**

7. The even part of a signal x(t) is:

a- $x(t) + x(-t)$ b- $x(t) - x(-t)$

c- $\boldsymbol{\frac{1}{2}(x(t) + x(-t))}$ d- $\frac{1}{2}(x(t) - x(-t))$

8.The sum of two even signals are:

a- even signal. b-odd signal

c- neither even nor odd d- Any one of above

9.The sum of two odd signals are

a- even signal. **b-odd signal**

c- neither even nor odd signal d-Any one of above

10.The sum of an even signal and an odd signal is:

a- even signal. b-odd signal

c- neither even nor odd signal d- Any one of above

11.Product of even and odd signals results is:

a- an odd signal b- an even signal

c- neither even nor odd signal d- Any one of above

12.Product of two even signals is:

a- an odd signal **b- an even signal**

c- neither even nor odd signal d- Any one of above

13.Product of two odd signals is:

a- an odd signal **b- an even signal**

c- neither even nor odd signal d- Any one of above

14.The independent variable n in a signal x(n) represents:

a- Time b- Distance c- Spatial coordinate **d- Any one of above**

15. Discrete-time signals are often derived by:-

a- Sound. **b- Sampling a continuous-time signal**.

c- Digital signal. d- Random signal.

16. Discrete-time signal may be:-

a- Complex valued. b- Real valued.

c- Imaginary valued. **d- Any of above.**

17.If x(n) is a discrete-time signal, then the value of x(n) at non integer value of 'n' is:

a- Zero b-Positive c- Negative **d- Not defined**

Explanation: For a discrete time signal, the value of x(n) exists only at integral values of n. So, for a non- integer value of 'n' the value of x(n) does not exist.

18. **Which of the following should be done in order to convert a continuous-time signal to a discrete-time signal?**

a- Sampling b- Differentiating

c- Integrating d- None of the mentioned

Explanation: The process of converting a continuous-time signal into a discrete-time signal by taking samples of continuous time signal at discrete time instants is known as 'sampling'.

9. The relation between analog frequency 'F' and digital frequency 'f' is:

a- F=f*T (where T is sampling period)

b- f=F*T

c- No relation

d- None of the mentioned

*Explanation: Consider an analog signal of frequency 'F', which when sampled periodically at a rate $F_s=1/T$ samples per second yields a frequency of f=F/Fs => f=F*T.*

Fill in the following blanks with the correct answers?

1.Analog signals are .... in amplitude and ……….. in time.

Ans. (**continuous**)

2. The signals that are discrete in ……… and quantized in…….. is called digital signal.

Ans. (**Time**, **amplitude**)

3. A digital system can be implemented as a combination of digital …….. and ………, each of which performs its own set of specified operations.

Ans. (hardware, software)

4.Examples of random signal are ………., ……….., and ………

Ans.(Speech signal, ECG signals, EEG signals)

5.Examples of 1-D signal are ……….,and …………

Ans. (Speech signal, music)

6. Examples of 2-D signal are ……….,and …………
Ans. (photographic image, black and white video)

7.Examples of M-D signal is…………
Ans. (video)

8. A signal is said to ……… if it repeats again and again over a certain period of time.
Ans. (periodic)

9. A Discrete time signal is said to periodic if it satisfies the condition………….. $where\ N\ is\ fundamental\ time\ period$ **(integer).**
Ans.($x(n) = x(n + N)$ **)**

10. A continuous time signal is said to be periodic if it satisfies the condition………….. $where\ T\ is\ fundamental\ time\ period.$
Ans.($x(t) = x(t + T)$ **)**

11. A real – valued discrete – time signal is called an Even Signal if it must be symmetrical about the …………...
Ans. (vertical axis)

12. A real – valued discrete – time signal is called an *Odd Signal* if it is *antisymmetrical* about the …………..

Ans. (vertical axis)

H.W

Show that whether the following signals are odd or even?

a)x(t)=t b)x(t)=5cos(3t) c) x(t)=sin(t)

Ans. **a-odd, b-even, c-odd.**