H.W

Show that whether the following signals are odd or even?

a)x(t)=t b)x(t)=5cos(3t) c) x(t)=sin(t)

Ans. a-odd, b-even, c-odd.

Sol:

a)x(t)=t $\Longrightarrow \mathrm{x}(-\mathrm{t}) = -\mathrm{t} = -\mathrm{x}(\mathrm{t}) \Longrightarrow x(t) = -x(t)$

so $x(t) = -x(-t) \quad \Longrightarrow \quad x(t)$ is odd.

b)x(t)=5cos(3t) $\Longrightarrow \mathrm{x}(-\mathrm{t}) = 5\cos(-3\mathrm{t}) = 5\cos(3\mathrm{t}) = \mathrm{x}(\mathrm{t})$

so $x(t) = x(-t) \quad \Longrightarrow \quad x(t)$ is even.

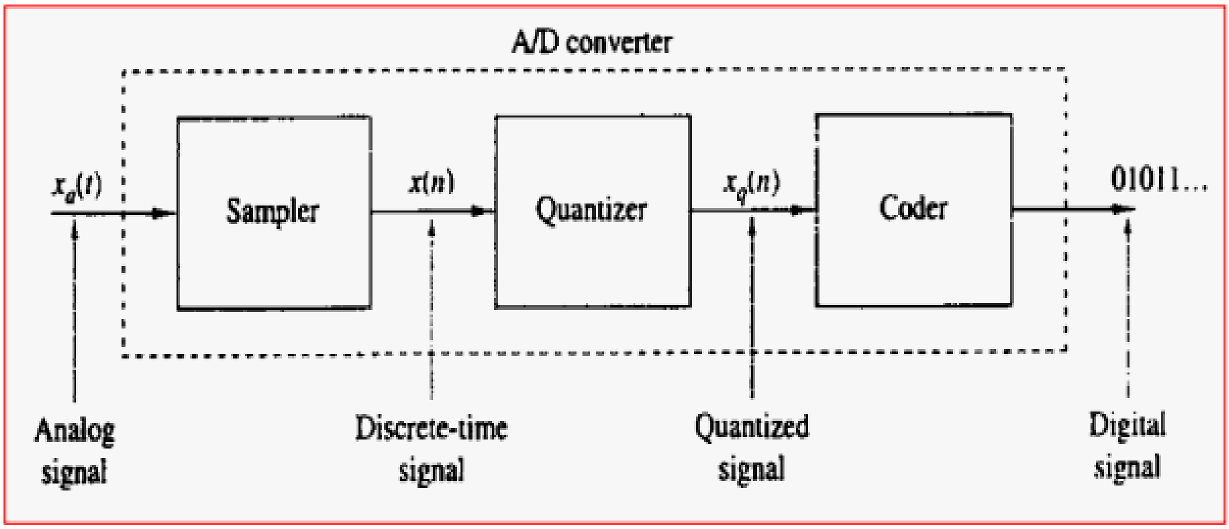
c) x(t)=sin(t) $\Longrightarrow x(-t) = \sin(-\mathrm{t}) = -\sin(\mathrm{t}) = -\mathrm{x}(\mathrm{t})$

so $x(t) = -x(-t) \quad \Longrightarrow \quad x(t)$ is odd.

Analog – to – digital and Digital – to Analog Conversion:

Most signals of practical interest, such as biological signals are analog. To process analog signals by digital means, it is first necessary to convert them in digital form. That is, to convert them to a sequence of numbers having finite precision. This procedure is called analog – to – digital (A/D) conversion.

Conceptually, we view A/D conversion as a three – step process. This process is illustrated in the figure below



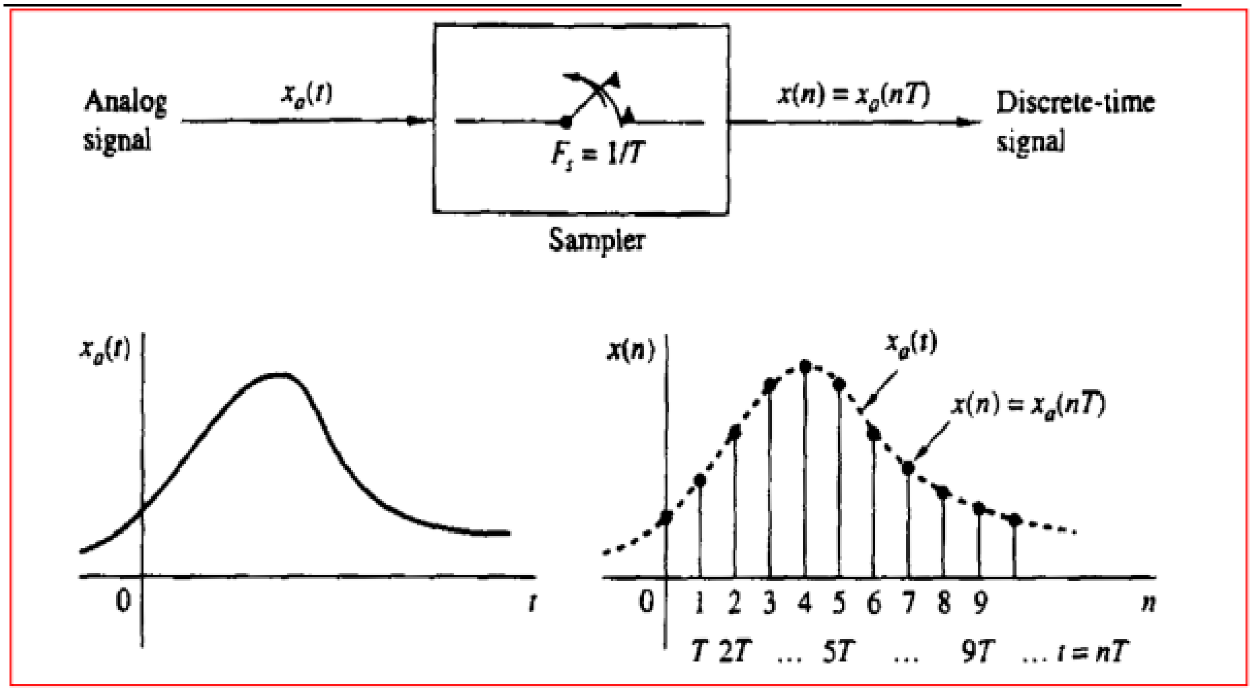
1. Sampling. This is the conversion of continuous – time signal into a discrete –time signal obtained by taking "samples" of the continuous – time signal at discrete – time instants. Thus, if $x_a(t)$ is the input to the sampler, the output is $x_a(nT) \equiv x(n)$, where T is called the sampling interval.
2. Quantization. This is the conversion of a discrete – time continuous – valued signal into a discrete – time, discrete – valued (digital) signal. The value of each signal sample is represented by a value selected from a finite set of possible values. The difference between the unquantized sample x(n) and the quantized output $x_q(n)$ is called the quantization error.

3. Coding. In the coding process, each discrete value $x_q(n)$ is represented by a b –bit binary sequence.

Sampling of Analog Signals.

There are many ways to sample an analog signal. We limit our discussion to periodic or uniform sampling, which is the type of sampling used most often in practice. This is described by the relation:

$$x(n) = x_a(nT), \qquad -\infty < n < \infty$$



The time interval T between successive samples is called the sampling period or sample interval and its reciprocal $1/T = F_s$ is called the sampling rate or sampling frequency.

The variables t and n are linearly related through the sampling period T or, equivalently, through the sampling rate $F_s = 1/T$, as

$$t = nT = \frac{n}{F_s}$$

As a consequence of above equation, there exists a relationship between the frequency variable F (or Ω) for analog signals and the frequency variable f (or ω) for discrete – time signals. To establish this relationship, consider an analog sinusoidal signal of the form:

$$x_a(t) = A\cos(2\pi Ft + \theta)$$

$$x_a(nT) \equiv x(n) = A\cos(2\pi FnT + \theta)$$

$$= A\cos\left(\frac{2\pi nF}{F_s} + \theta\right)$$

The frequency variables F and f are linearly related as:

$$f = \frac{F}{F_s}$$

Or, equivalently, as $\omega = \Omega T$

The frequency variable f is relative or normalized frequency. We can use f to determine the frequency F in hertz only if the sampling frequency F_s is known.

The relations are summarized in following table

Continuous – time signals	Discrete – time signals
$\Omega = 2\pi F$	$\omega = 2\pi f$
$\frac{radians}{sec}$ Hz	$\frac{radians}{sample}$ $\frac{cycles}{sample}$

→ $\omega = \Omega T,\ f = F/F_s$

← $\Omega = \omega/T,\ F = f.F_s$

Continuous – time signals	Discrete – time signals
$-\infty < \Omega < \infty$	$-\pi/T \leq \omega \leq \pi/T$
$-\infty < F < \infty$	$-F_s/2 \leq f \leq F_s/2$

From these relations we observe that

$$f_{max} = \frac{F_s}{2} = \frac{1}{2T}$$

$$\omega_{max} = \pi F_s = \frac{\pi}{T}$$

The sampling theorem

If the highest frequency contained in an analog signal $\mathbf{x_a(t)}$ is $\mathbf{F_{max} = B}$ and the signal is sampled at a rate $\mathbf{F_s > 2F_{max} \equiv 2B}$, then $\mathbf{x_a(t)}$ can be exactly recovered from its sample values using the interpolation function:

$$g(t) = \frac{\sin 2\pi Bt}{2\pi Bt}$$

Thus $\mathbf{x_a(t)}$ may be expressed as

$$x_a(t) = \sum_{n=-\infty}^{\infty} x_a\left(\frac{n}{F_s}\right) g\left(t - \frac{n}{F_s}\right)$$

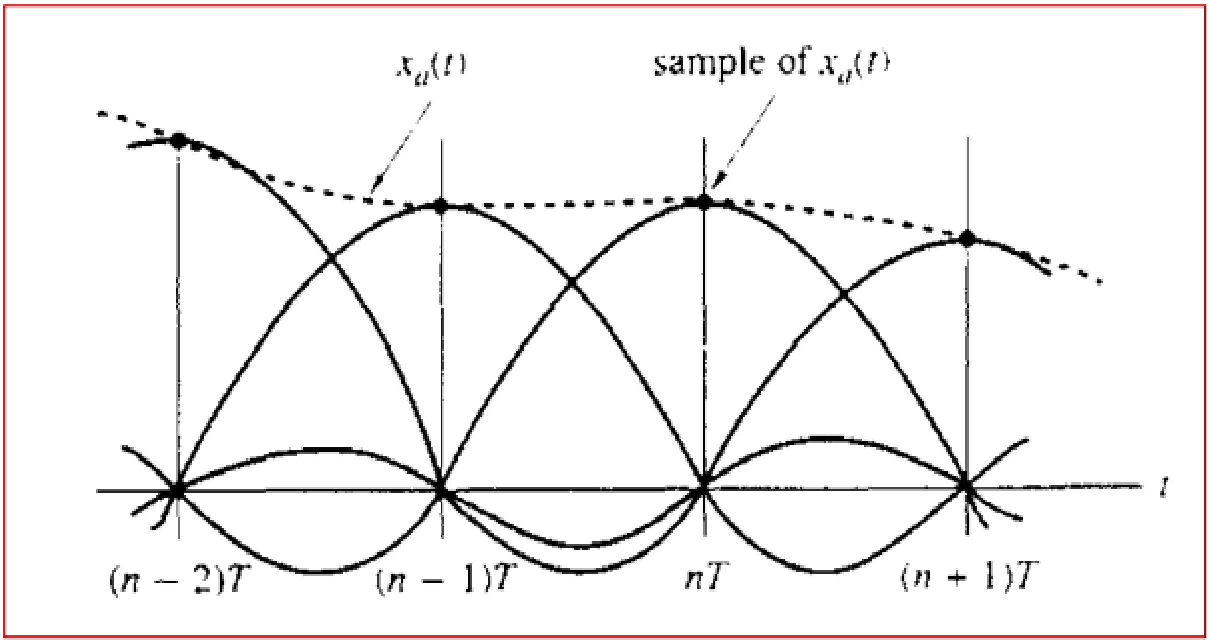
Where $x_a(\frac{n}{F_s}) = x_a(nT) \equiv x(n)$ are the samples of $x_a(t)$.

When the sampling of $x_a(t)$ is performed at the minimum sampling rate $F_s = 2B$, the reconstruction formula becomes

$$x_a(t) = \sum_{n=-\infty}^{\infty} x_a\left(\frac{n}{2B}\right) \frac{\sin 2\pi B(t - \frac{n}{2B})}{2\pi B(t - \frac{n}{2B})}$$

And the sampling rate $F_N = 2B$ is called ***Nyquist rate***.

Figure below illustrate the ideal **D/A** conversion process using the interpolation function.



Example
Consider the analog signal

$$x_a(t) = 3\cos 50\pi + 10\sin 300\pi - \cos 100\pi$$

What is the Nyquist rate for this signal?

Solution

The frequencies present in the signal above are

$F_1 = 25$ Hz $F_2 = 150$ Hz $F_3 = 50$ Hz

Thus $F_{max} = 150$ Hz and $F_s > 2F_{max} = 300$ Hz $= F_N$ is the Nyquist rate.

Example

Consider the analog signal

$x_a(t) = 3cos2000\pi + 5sin6000\pi + 10cos12000\pi$

a) What is the Nyquist rate for this signal?
b) Assume now that we sample this signal using a sampling rate $F_s =$ 5000 samples/sec. what is the discrete – time signal obtained after sampling?
c) What is the analog signal ya (t) we can reconstruct from the samples if we use ideal interpolation?

Solution

a) The frequencies existing in the analog signal are

$F_1 = 1$KHz, $F_2 = 3$ KHz, $F_3 = 6$ KHz

Thus $F_{max} = 6$ KHz, and according to the sampling theorem
$F_s > 2\,F_{max} = 12$ KHz and the Nyquist rate is $F_N = 12$ KHz

b)

Since we have chosen $F_s = 5$ KHz, the folding frequency is

$$\frac{F_s}{2} = 2.5\ KHz$$

And this is the maximum frequency that can be represented uniquely by the sampled signal. We obtain

$$x(n) = x_a(nT) = x_a\left(\frac{n}{F_s}\right)$$
$$= 3cos2\pi\left(\frac{1}{5}\right)n + 5sin2\pi\left(\frac{3}{5}\right)n + 10cos2\pi\left(\frac{6}{5}\right)n$$
$$= 3cos2\pi\left(\frac{1}{5}\right)n + 5sin2\pi\left(1-\frac{2}{5}\right)n + 10cos2\pi\left(1+\frac{1}{5}\right)n$$
$$= 3cos2\pi\left(\frac{1}{5}\right)n + 5sin2\pi\left(-\frac{2}{5}\right)n + 10cos2\pi\left(\frac{1}{5}\right)n$$

Finally, we obtain

$$x(n) = 13cos2\pi\left(\frac{1}{5}\right)n - 5sin2\pi\left(\frac{2}{5}\right)n$$

Finally, we obtain

$$x(n) = 13cos2\pi\left(\frac{1}{5}\right)n - 5sin2\pi\left(\frac{2}{5}\right)n$$

c) Since only the frequency components at 1 KHz and 2 KHz are present in the sampled signal, the analog signal we can recover is

$$y_a(t) = 13\cos 2000\pi t - 5\sin 4000\pi t$$

Which is obviously different from the original signal $x_a(t)$. This distortion of the original analog signal was caused by the aliasing effect, due to the low sampling rate used.

Quantization of Continuous – Amplitude signal

The process of converting a discrete – time continuous – amplitude signal into a digital signal by expressing each sample value as a finite (instead of an infinite) number of digits, is called quantization. The error introduced in representing the continuous – valued signal by a finite set of discrete value levels is called quantization error or quantization noise.

We denote the quantizer operation on the samples x(n) as Q[x(n)] and let $x_q(n)$ denote the sequence of quantized samples at the output of the quantizer. Hence

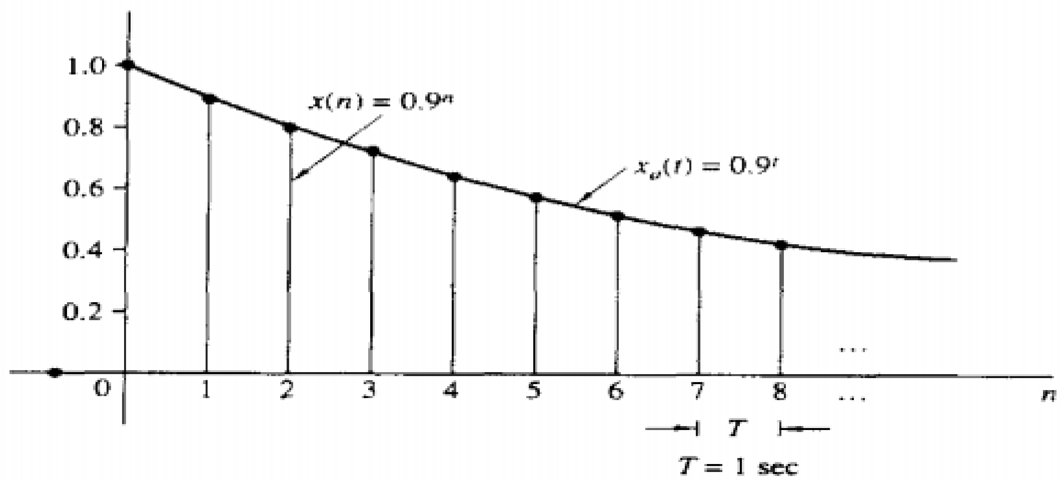
$$x_q(n) = Q\,[x\,(n)]$$

Then the quantization error is a sequence $e_q(n)$ defined as the difference between the quantized value and the actual sample value. Thus

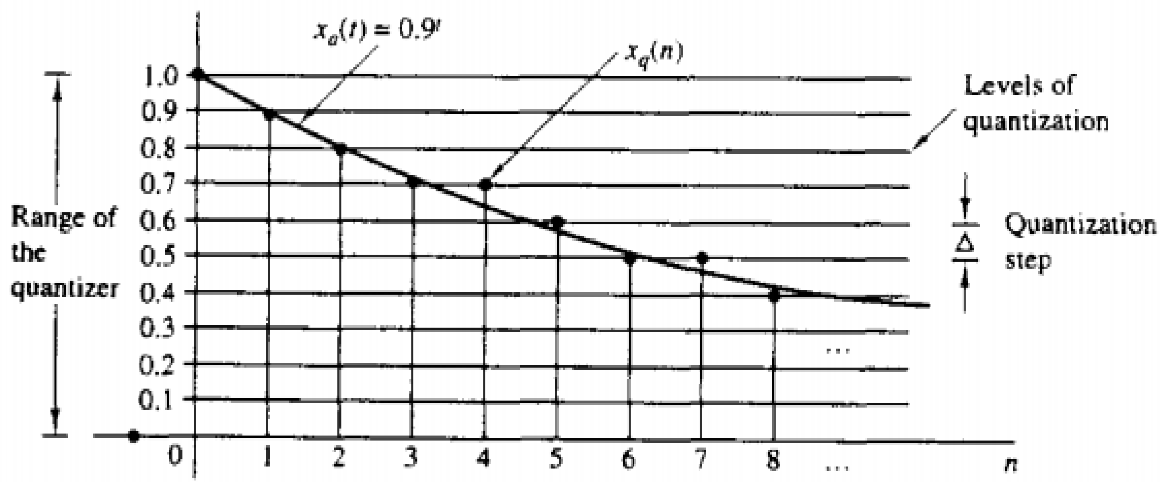
$$\boldsymbol{e_q(n) = x_q(n) - x\,(n)}$$

Let us consider the discrete – time signal

$$x(n) = \begin{cases} 0.9^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$



(a)



(b)

TABLE 1.2 NUMERICAL ILLUSTRATION OF QUANTIZATION WITH ONE SIGNIFICANT DIGIT USING TRUNCATION OR ROUNDING

n	$x(n)$ Discrete-time signal	$x_q(n)$ (Truncation)	$x_q(n)$ (Rounding)	$e_q(n) = x_q(n) - x(n)$ (Rounding)
0	1	1.0	1.0	0.0
1	0.9	0.9	0.9	0.0
2	0.81	0.8	0.8	−0.01
3	0.729	0.7	0.7	−0.029
4	0.6561	0.6	0.7	0.0439
5	0.59049	0.5	0.6	0.00951
6	0.531441	0.5	0.5	−0.031441
7	0.4782969	0.4	0.5	0.0217031
8	0.43046721	0.4	0.4	−0.03046721
9	0.387420489	0.3	0.4	0.012579511

1- Sampling Process is:

a- Is the process of converting continuous time signal to discrete time signal.

b- Is the process of converting continuous time signal to digital signal.

c- Is the process of converting discrete time signal to digital signal.

d- Is the process of converting digital signal to analog signal.

2- Quantization Process is:

a- Conversion of discrete time discrete valued signal to digital signal.

b- Conversion of discrete time continuous valued signal to discrete time discrete valued signal.

c- Conversion of discrete time continuous valued signal to continuous time discrete valued signal.

d- Conversion of digital signal to continuous time discrete valued signal.

3- Coding Process is:

a- Representing of continuous valued signal by binary numbers.

b- Representing of continuous valued signal by real numbers.
c- **Representing of discrete valued signal by binary numbers.**

4. Most signals of practical interest, such as biological signals are:

a- Continuous in nature.

b- Discrete in nature.

c- Random in nature.

5. ADC is the abbreviation of:

a- Digital to Analog Converter.

b- Analog to Digital Converter.

c- Analog to Analog Converter.

d- Digital to Digital Converter.

6. DAC is the abbreviation of

a- Frequency Sampling.

b- Time sampling.

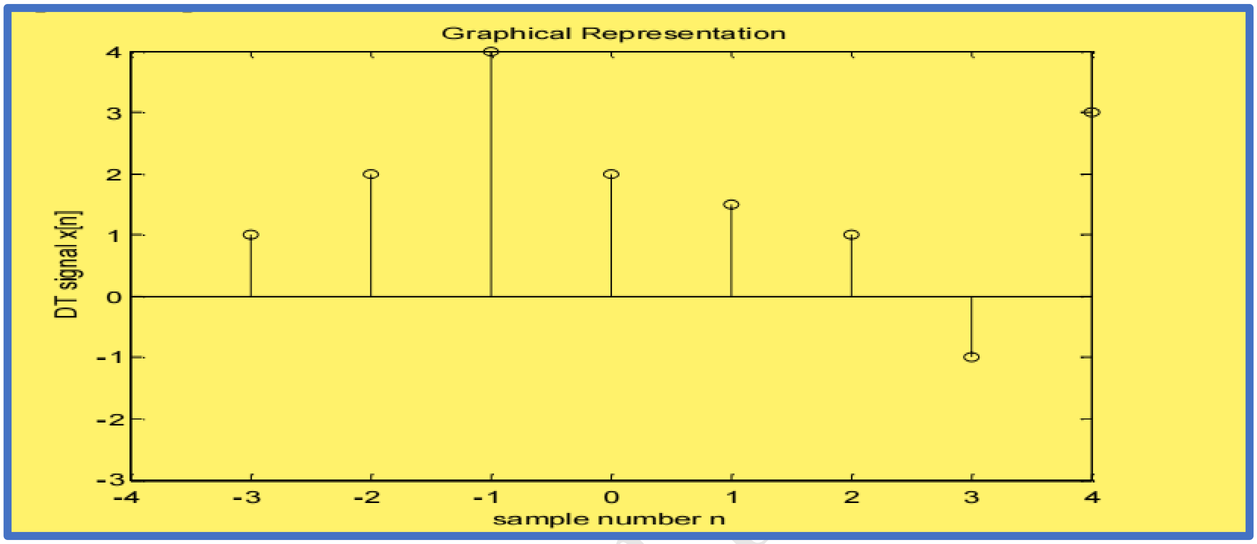
c- Digital to Analog Converter.

d- Impulse sampling

Discrete time signal representation

The different representations of a discrete time signal are

1. Graphical Representation



2. Functional representation

$$x[n] = \begin{cases} 1 & \text{for } n=1,2,3 \\ 2 & \text{for } n=4 \\ 0 & \text{otherwise} \end{cases}$$

3. Tabular representation

N	-	- - - - - -	-2	-1	0	1	2	3	4	5	- - - - - -
x [n]	-	- - - - - -	0	0	1	1	4	1	0	0	- - - - - -

4. Sequence representation

x [n] = { - , -, -. -, - , 0, 0, 1, 4, 1, 0, 0, - , - , - , - }
↑

the above is a representation of a two – sided infinite duration sequence, and the symbol indicates the time origin (n = 0).

If the sequence is zero for n < 0, it can be represented as

x [n] = { 1, 4, 1, 2, - , - , - , - }

A finite duration sequence can be represented as

x[n] = { 3, -1, -2, 5, 0, 4, -1}
↑

This is referred to as a 7 – point sequence.

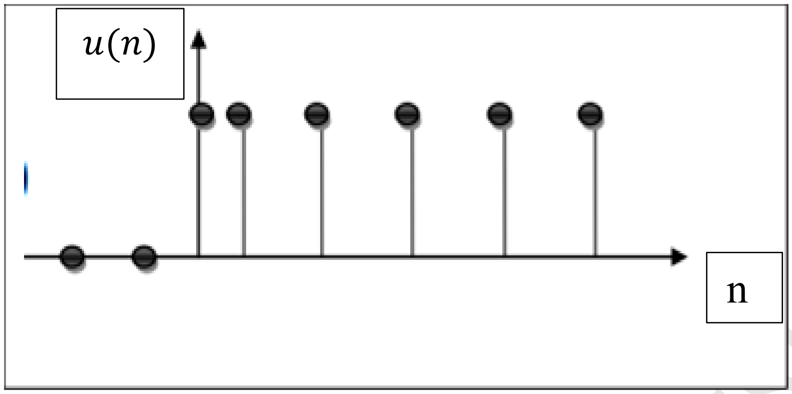
Basic type of digital signal:

Many DSP algorithms are linear. The response of a linear algorithm, or processor, to a number of signals applied simultaneously equals the summation of its responses to each signal applied separately. Thus if we can define the responses to each signal applied separately. Thus we can define the response of a linear processor to basic signals, we can predicate its response of a linear processor to basic signals; we can predict its response to more complication environment (summation of a number of simpler basic signals).

1. ***Unit step function***

   The unit step function u[n] is defined as

$$u(n) = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

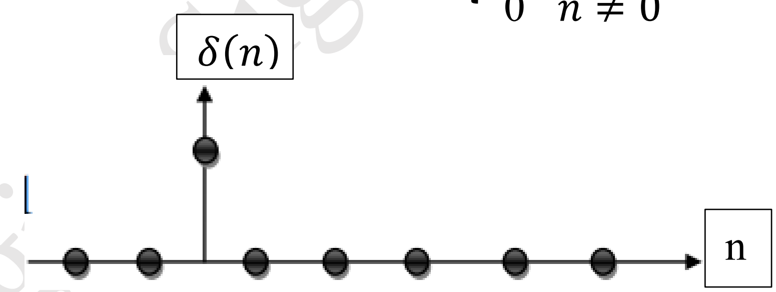


unit step function

2- *Unite impulse function*

The unit impulse function δ[n] defined as

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



unit impulse function

The relationship between unite step and unite impulse is given by

$$u(n) = \sum_{m=-\infty}^{n} \delta(n)$$

Conversely, we can easily generate δ[n] from u[n]. The recurrence formula:

$$\boldsymbol{\delta(n) = u(n) - u(n-1)}$$

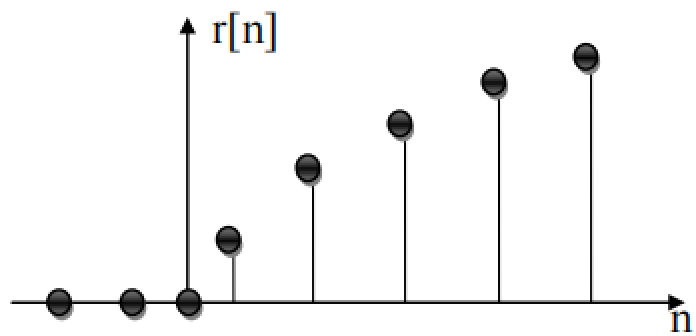
holds good for all integer values of n.

3- Unit ramp function

The unit ramp function is defined as

$$r(n) = nu(n)$$

since u(n) is zero for n< 0 so also ramp function is zero for n<0



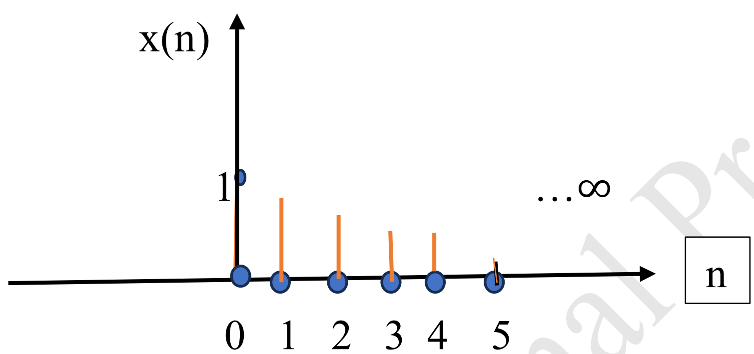
n	**…**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**…**
r(n)	**…**	**0**	**0**	**0**	**0**	**1**	**2**	**3**	**…**

4- Exponential signal

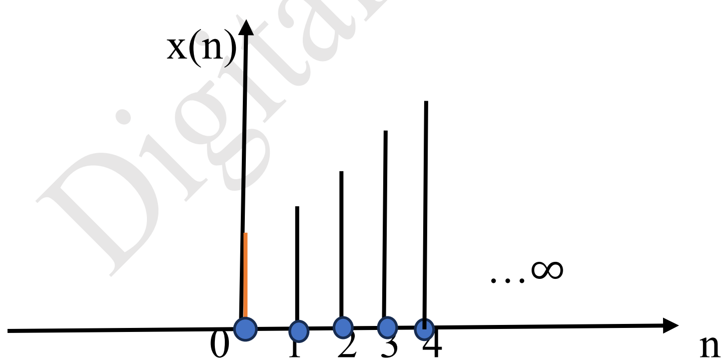
Finally, an exponential signal is defined by

$$x(n) = \begin{cases} a^n & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

When $0 < a < 1$



When $a > 1$



Where a may be a real or complex number. Of particular interest is the exponential sequence that is formed when $a = e^{j\omega_o}$, where ω_o, is a real number. In this case, x(n) is a complex exponential

$$x(n) = a^{jn\omega_o} = \cos(n\omega_o) + j\sin(n\omega_o)$$

5- Sinusoidal signal

Cosinusoidal signal is defined as $x(n) = A\cos(\omega n)$	**Sinusoidal signal** is defined as $x(n) = A\sin(\omega n)$

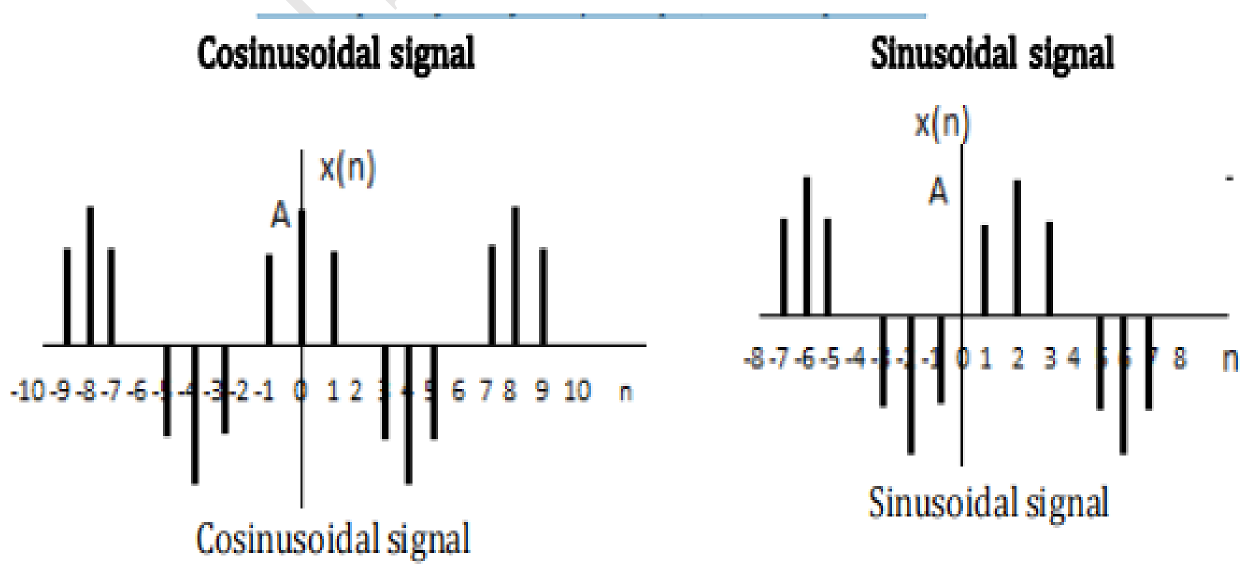
where $\omega = 2\pi f = \frac{2\pi}{N}\, m,$ and

ω is frequency in radians/sample

m is smallest integer

f is frequency in cycles/sample,

A is amplitude



6- **Complex Exponential signal**
is defined as

$$\boldsymbol{x\ (n) = a^n e^{j(\omega_o n)}}$$

$$= a^n[\cos\ (\ a\omega_o n) + j sin(\omega_o n)\ ]$$

$where\ x_r\ (n) = a^n \cos\ (\ a\omega_o n)\ and \quad x_i\ (n) = a^n \cos\ (\ a\omega_o n)$