Optics

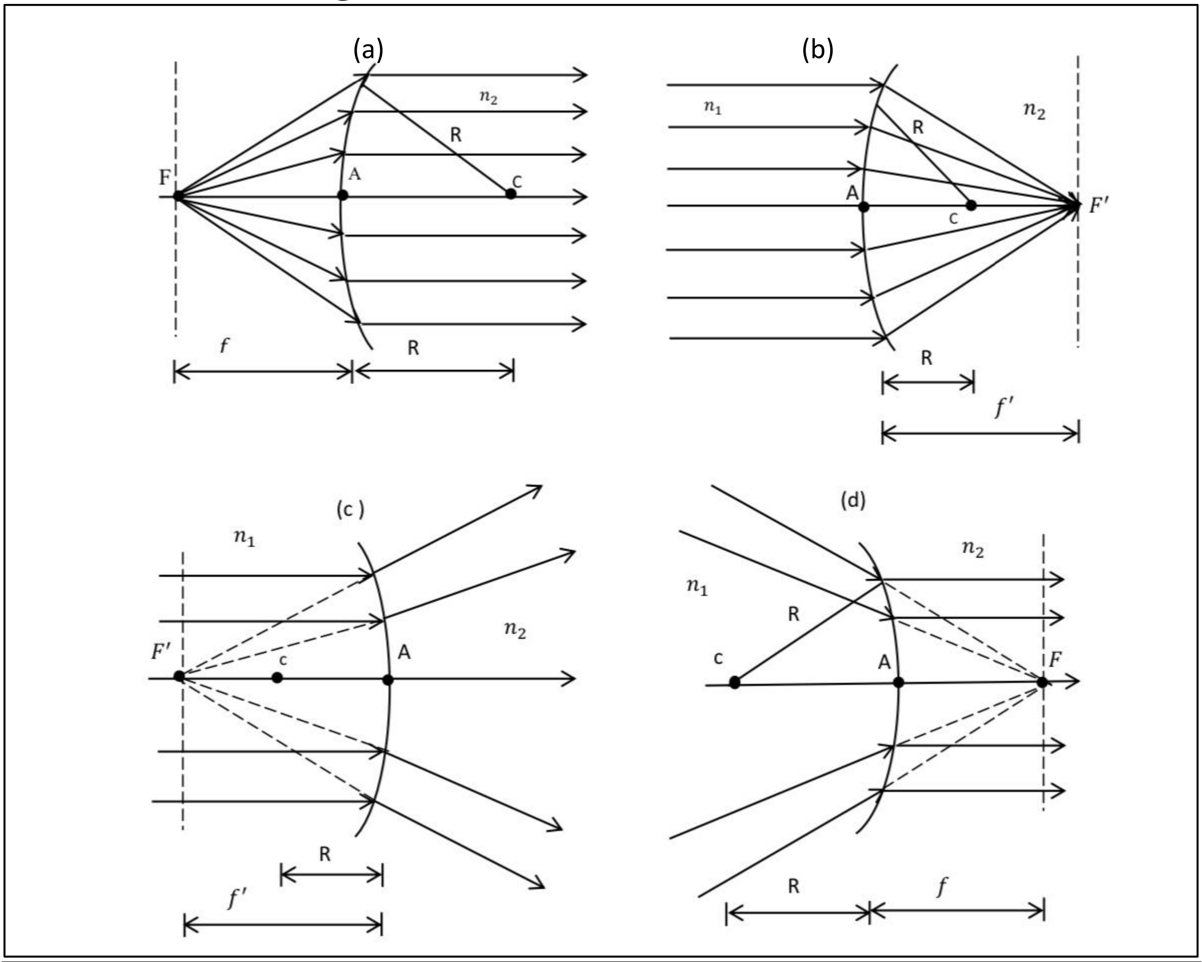
Second year / First semester
Lecture 4

Assist .lec. Mena Luay

Reflection and refraction at spherical surfaces

Spherical surfaces

In this chapter, we will treat refractive of light at a single spherical separating two media of different refractive indices, where a beam of light passes from one (n_1) medium into another (n_2). In all diagrams the incident light rays traveling from left to right. A convex surface therefore is one in which the center of curvature C lies to the right of the vertex, while a concave surface is one in which C lies to the left of the vertex as shown in figure.



- The principal axis in each diagram is straight line passes through the center of curvature C.
- The point A where the axis crosses the surface is called the vertex
- In diagram (a) rays are shown diverging from a point source F on the axis in the first medium and refracted into a beam everywhere parallel to the axis in the second medium.
- In diagram (b) a parallel incident beam is refracted and brought to a focus at the point F'.
- In diagram (C) a parallel incident beam is refracted to diverge as if it came from the point F'. F' in each case is called the secondary focal point and the distance f ' is called the secondary focal length.
- Diagram (d) shows a beam converging in the first medium toward the point F and then refracted into a parallel beam in the second medium. F in each of these two cases is called the primary focal point, and the distance f is called the primary focal length.

We now state that the primary focal point F is an axial point having the property that any ray coming from it or proceeding toward it travels parallel to the axis after refraction.
the secondary focal point F' is an axial point having the property that any incident ray traveling parallel to the axis will, after refraction, proceed toward, or appear to come from, F'.
A plane perpendicular to the axis and passing through either focal point is called a focal plane.

Gaussian formula

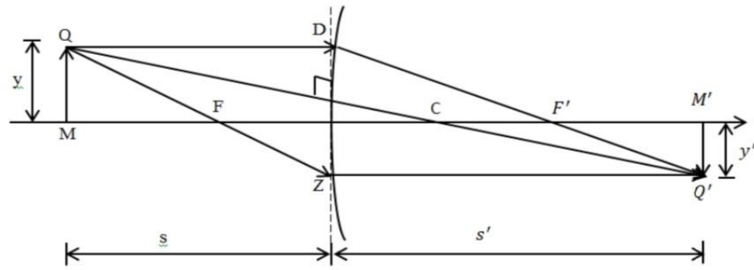


Figure 3.2: Rays leave the point object Q and passing through the spherical surface are brought to a focus at the point image Q'.

The refracting power (P) = $\frac{n}{F} = \frac{n'}{F'} = \frac{n}{s} + \frac{n'}{s'} = \frac{n'-n}{R}$

s ⟶ بعد الجسم

s´ ⟶ بعد الصورة

R⟶ نصف قطر التكور

P: refracting power when all distance are measured in meter. And the power is in units called diopters

Power → convex surface is positive
→ concave surface is negative

When an object at primary focal point F → s = f
→ **s′** = ∞

$$\frac{n}{f} + \frac{n'}{\infty} = \frac{n'-n}{R} \longrightarrow \frac{n}{f} = \frac{n'-n}{R}$$

When an object at primary focal point F' → $s' = f''$, → $s = \infty$

$$\frac{n}{\infty} + \frac{n'}{f'} = \frac{n'-n}{R} \longrightarrow \frac{n'}{f'} = \frac{n'-n}{R}$$

Magnification :-

In any optical system the ratio between the transverse dimension of the final image and corresponding dimension of the original object is called the lateral magnification.

Magnification $= m = \frac{y'}{y} = - \frac{ns'}{n's}$

m
- > 1 Magnification image (مكبرة)
- < 1 miniature image (مصغرة)
- $= 1$ object and image the same size (نفس حجم الجسم)

m
- → positive the image will be virtual and erect (معتدلة)
- → negative the image is real and invert (مقلوبة)

s′
- → (+) Real picture (حقيقية)
- → (-) Imaginary picture (خيالية)

R → (+) (محدبة)
R → (-) (مقعرة)

f , f′ → (+) (محدبة)
f , f′ → (-) (مقعرة)

s → (+) (الجسم على يسار السطح الكروي)
s → (-) (الجسم على يمين السطح الكروي)

Example-1- The end of a solid glass rod of index 1.50 is ground and polished to a hemispherical surface of radius 1 cm. A small object is placed in air on the axis 4 cm to the left of the vertex. Find the position of the image. Assume n = 1.00 for air.

Sol

n = 1, n' = 1.5, r = + 1.0 cm, and s = +4 cm. The unknown quantity is s'.

$$\frac{n}{s}+\frac{n'}{s'}=\frac{n'-n}{R}$$

$$\frac{1}{4}+\frac{1.5}{s'}=\frac{1.5-1}{1}$$

$$\frac{1.5}{s'}=\frac{1.5-1}{1}-\frac{1}{4}$$

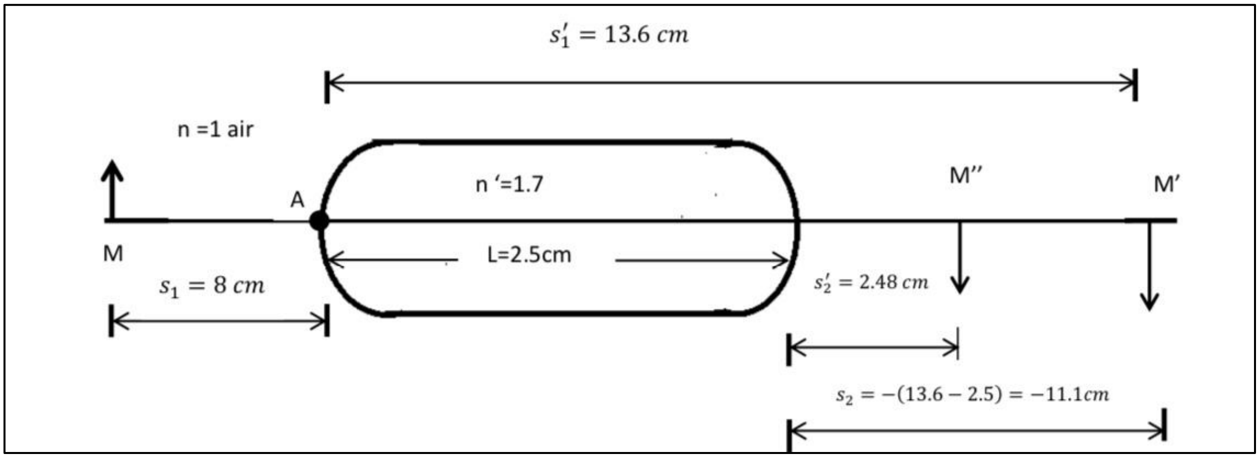
$$\frac{1.5}{s'}=\frac{0.5}{1}-\frac{1}{4}$$

$$\frac{1.5}{s'}=\frac{2-1}{4} \longrightarrow \frac{1.5}{s'}=\frac{1}{4}$$

$$s'=6\text{ cm}$$

That a real image is formed in the glass rod 6 cm to the right of the vertex

Example-2- The length of glass rod 2.5 cm and index 1.7 is polished of the two ends to a hemispherical surfaces of radius 2.8 cm , the left surface is convex and right surface is concave . A small object is placed in air on the axis 8 cm to the left surface. Find a- the final image characterize, b- the primary and secondary focal length for the left first surface.



Sol

1-To find the image from first surface on the left

$$\frac{n}{s_1} + \frac{n'}{s_1'} = \frac{n' - n}{R} \longrightarrow \frac{1}{8} + \frac{1.7}{s_1'} = \frac{1.7-1}{2.8}$$

$$\frac{1.7}{s_1'} = \frac{0.7}{2.8} - \frac{1}{8}$$

$$\frac{1.7}{s_1'} = \frac{5.6-2.8}{22.4} \longrightarrow \frac{1.7}{s_1'} = 0.125$$

$s_1' = 13.6$ cm (الصورة حقيقية)

$m' = -\frac{n s_1'}{n' s_1} = \frac{-1*13.6}{1.7*8} = -1$ (الصورة مقلوبة و نفس حجم الجسم)

2-The image position from first surface, to find s_2 is representing a virtual object for second surface

s_2 = - ($s_1{}' - L$)

s_2 = - (13.6 − 2.5) = -11.1 cm

1- to find the image it's formed from second surface

$n' = 1.7$, $n = 1$, $s_2 = -11.1$, R = -2.8

$$\frac{n'}{s_2} + \frac{n}{s_2{}'} = \frac{n - n'}{R} \longrightarrow \frac{1.7}{-11.1} + \frac{1}{s_2{}'} = \frac{1-1.7}{-2.8}$$

$$\frac{1}{s_2{}'} = \frac{1-1.7}{-2.8} + \frac{1.7}{11.1}$$

$$\frac{1.7}{s_2{}'} = \frac{-0.7}{-2.8} + \frac{1.7}{11.1} = \frac{7.77+4.76}{31.08}$$

$s_2{}' = 2.48$ cm (الصورة حقيقية)

$m'' = -\frac{n' s_2{}'}{n\, s_2}$ = -$\frac{(1.7)(2.48)}{(1)(-11.1)}$ = 0.37 (الصورة معتدلة ومصغرة)

To find the magnification for two surfaces

m= $m_1 * m_2$

m_1 / FOR FIRST SURFACE; m_2 /FOR SECOND SURFACE

m= (- 1)(0.37) = - 0.37

The final image is inverted miniature and real

b- to find the primary focal length when an object is placed at focal position foe first surface

$$\frac{n}{f_1} + \frac{n'}{\infty} = \frac{n' - n}{R_1} \longrightarrow \frac{1}{f_1} = \frac{1.7-1}{2.8}$$

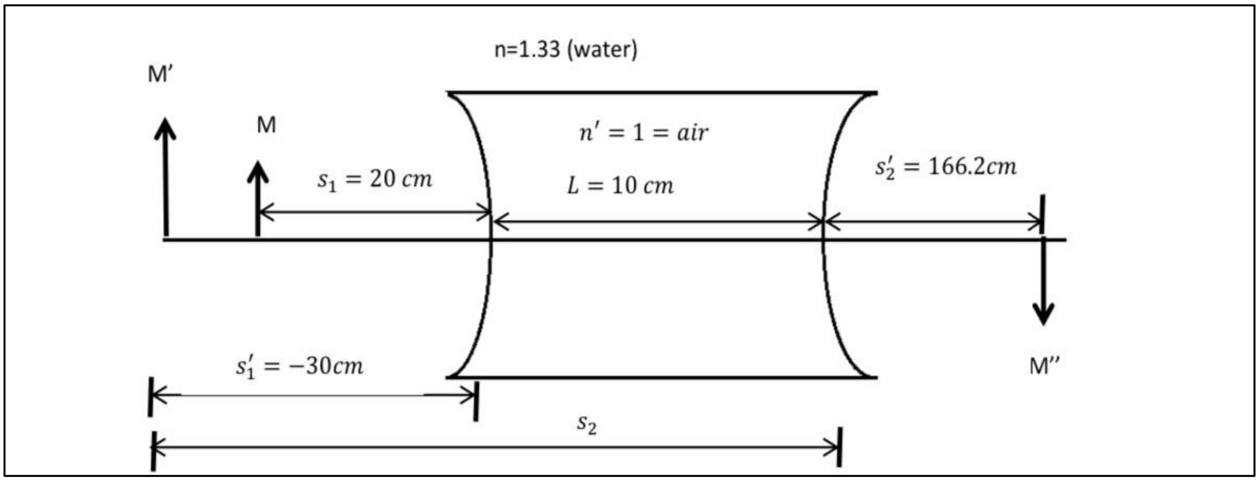
$f_1 = \frac{1}{0.25} = 4$ cm

To find the secondary focal length for first surface

$\frac{n}{\infty} + \frac{n'}{f_1'} = \frac{n'-n}{R_1} \longrightarrow \frac{1.7}{f_1'} = \frac{0.7}{2.8}$

$\frac{1.7}{f_1'} = 0.25 \longrightarrow f_1' = 6.8$ cm

Example-3- A hollow glass rod 10 cm long, both ends polished to semispherical surfaces as shown in figure below with radii 10 cm the rod is submerged in water of index 1.33 find the characteristic of the final image when an object is placed 20 cm from the vertex of the left surface.



sol

$\frac{n}{s_1} + \frac{n'}{s_1'} = \frac{n'-n}{R} \longrightarrow \frac{1.33}{20} + \frac{1}{s_1'} = \frac{1-1.33}{-10}$

$\frac{1}{s_1'} = \frac{-0.33}{-10} = 0.033 \longrightarrow s_1' = -30$ (الصورة خيالية)

$m' = -\frac{n s_1'}{n' s_1} = -\frac{(1.33)(-30)}{(1)(20)} = 2$ (الصورة معتدلة و مكبرة)

Now we find the second image

$s_2 = (s_1{}' + L) = 30+10 = 40$

$\frac{n'}{s_2} + \frac{n}{s_2{}'} = \frac{n-n'}{R} \longrightarrow \frac{1}{40} + \frac{1.33}{s_2{}'} = \frac{1.33-1}{10}$

$\frac{1.33}{s_2{}'} = \frac{0.33}{10} - \frac{1}{40} \longrightarrow s_2{}' = 166.25$ (الصورة حقيقية)

$m'' = -\frac{n' s_2{}'}{n\, s_2} = -\frac{(1)(166.25)}{(1.33)(40)} = -3$(الصورة مكبرة و مقلوبة)

To find the magnification for two surfaces

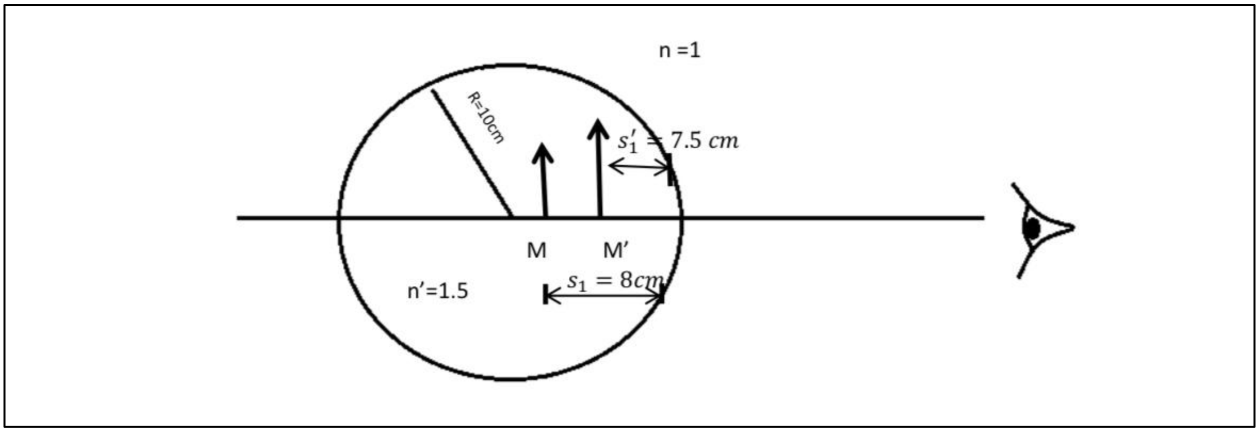
m= $m_1 * m_2$

m_1 / FOR FIRST SURFACE; m_2 /FOR SECOND SURFACE

m= (2)(-3.125) = -6

The final image is inverted and magnifying

Example-4- Find the final image characterize for object is placed 8 cm inside glass ball with radius 10 cm from human eye as shown in figure



sol

s_1 = 8 cm , R= 10cm , n= 1 , n'=1.5

$$\frac{n'}{s_1}+\frac{n}{s_1'}=\frac{n-n'}{R} \longrightarrow \frac{1.5}{8}+\frac{1}{s_1'}=\frac{1-1.5}{-10}=$$

$$\frac{1}{s_1'}=\frac{-0.5}{-10}-\frac{1.5}{8}$$

$s_1' = -7.3$ cm (صورة خيالية)

$m' = -\frac{n' s_1'}{n\, s_1} = -\frac{(1.5)(-7.3)}{(1)(8)} = 1.36$ (صورة معتدلة ومكبرة)