Gauss Elimination Method:

To reduce the augmented matrix to row - echelon form you should follow the following steps:

Step 1: Locate the leftmost column that does not consist entirely of zeros.

Step 2: Interchange the top row with another row , if necessary , to bring
a nonzero entry to the top of the column found in Step 1.

Step 3: If the entry that is now at the top of the column found in Step 1 is b
, multiply the first row by 1/b in order to introduce a leading 1.

Step 4: Add suitable multiples of the top row to the rows below so that all
entries below the leading 1 become zeros.

Step 5: Now cover the top row in the matrix and begin again with Step 1
applied to the submatrix that remains. Continue in this way until the
entire matrix is in row - echelon form.

Example 1: **Solve the following system of linear equations by using the Gauss elimination method:**

$$5x_1 + 6x_2 = 7$$
$$3x_1 + 4x_2 = 5$$

Solution: The system of linear equations has the following augmented matrix

$$\left(\begin{array}{cc|c} 5 & 6 & 7 \\ 3 & 4 & 5 \end{array}\right) \xrightarrow{\frac{1}{5}R_1 \to R_1} \left(\begin{array}{cc|c} 1 & \frac{6}{5} & \frac{7}{5} \\ 3 & 4 & 5 \end{array}\right) \xrightarrow{R_2 - 3R_1 \to R_2}$$

$$\left(\begin{array}{cc|c} 1 & \frac{6}{5} & \frac{7}{5} \\ 0 & \frac{2}{5} & \frac{4}{5} \end{array}\right) \xrightarrow{\frac{5}{2}R_2 \to R_2} \left(\begin{array}{cc|c} 1 & \frac{6}{5} & \frac{7}{5} \\ 0 & 1 & 2 \end{array}\right)$$

In Row - Echelon Form

The last matrix is in row - echelon form . The corresponding reduced system is:

$x_1 + \frac{6}{5}x_2 = \frac{7}{5}$ *1*

$x_2 = 2$ *2*

Substitute the value of x_2 in equation (1) ,we get

$$x_1 + \frac{12}{5} = \frac{7}{5} \Rightarrow x_1 = \frac{7}{5} - \frac{12}{5} \Rightarrow x_1 = -\frac{5}{5} \Rightarrow x_1 = -1$$

Therefore the solution of the system is

$x_1 = -1$

$x_2 = 2$

Example 2: **Solve the following system of linear equations by using the Gauss elimination method:**

$4y + 2z = 1$
$2x + 3y + 5z = 0$
$3x + y + z = 11$

Solution: The system of linear equations has the following augmented matrix

$$\left(\begin{array}{ccc|c} 0 & 4 & 2 & 1 \\ 2 & 3 & 5 & 0 \\ 3 & 1 & 1 & 11 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} \textcircled{2} & 3 & 5 & 0 \\ 0 & 4 & 2 & 1 \\ 3 & 1 & 1 & 11 \end{array}\right) \xrightarrow{\frac{1}{2}R_1 \rightarrow R_1}$$

$$\left(\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{5}{2} & 0 \\ 0 & 4 & 2 & 1 \\ \textcircled{3} & 1 & 1 & 11 \end{array}\right) \xrightarrow{R_3 - 3R_1 \rightarrow R_3}$$

$$\left(\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{5}{2} & 0 \\ 0 & \textcircled{4} & 2 & 1 \\ 0 & -\frac{7}{2} & -\frac{13}{2} & 11 \end{array}\right) \xrightarrow{\frac{1}{4}R_2 \rightarrow R_2}$$

$$\left(\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{5}{2} & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} \\ 0 & -\frac{7}{2} & -\frac{13}{2} & 11 \end{array}\right) \xrightarrow{R_3 + \frac{7}{2}R_2 \to R_3}$$

$$\left(\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{5}{2} & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & -\frac{19}{4} & \frac{95}{8} \end{array}\right) \xrightarrow{-\frac{4}{19}R_3 \to R_3}$$

$$\left(\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{5}{2} & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 1 & -\frac{5}{2} \end{array}\right)$$

In Row - Echelon Form

The last matrix is in row - echelon form . The corresponding reduced system is:

$x + \frac{3}{2}y + \frac{5}{2}y = 0$ 1

$y + \frac{1}{2}Z = \frac{1}{4}$ 2

$Z = -\frac{5}{2}$ 3

Substitute the value of in equation (2) , we get Z

$$y - \frac{5}{4} = \frac{1}{4} \Rightarrow y = \frac{1}{4} + \frac{5}{4} \Rightarrow y = \frac{6}{4} \Rightarrow y = \frac{3}{2}$$

Substitute the values of *y* and *z* in equation (1) , we get

$$x + \frac{9}{4} - \frac{25}{4} = 0 \Rightarrow x = -\frac{9}{4} + \frac{25}{4} \Rightarrow x = \frac{16}{4} \Rightarrow x = 4$$

Therefore the solution of the system is

$x = 4$

$y = \frac{3}{2}$

$Z = -\frac{5}{2}$