Text Book:

Mechanics of Materials (James M. Gere and Barry J. Goodno)

Additional References:

1- ***Mechanics of Materials (R. C. Hibbeler)***
2- ***Mechanics of Materials (Ferdinand P. Beer et. al.)***

1.1 - Introduction to Mechanics of Materials

Mechanics of materials is a branch of applied mechanics that deals with the behavior of solid bodies subjected to various types of loading. Other names for this field of study are *strength of materials* and *mechanics of deformable bodies*.

The principal objective of mechanics of materials is to determine the stresses, strains, and displacements in structures and their components due to the loads acting on them.

An understanding of mechanical behavior is essential for the safe design of all types of structures, whether artificial limbs, orthotics, medical equipment as well airplanes and antennas, buildings and bridges, machines and motors, or ships and spacecraft.

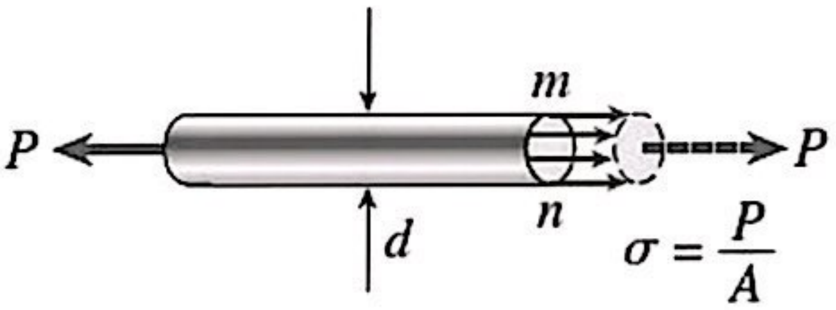
Theoretical analyses and experimental results have equally important roles in mechanics of materials. Theories are used to derive formulas and equations for predicting mechanical behavior but these expressions cannot be used in practical design unless the physical properties of the materials are known

1.2 - Normal Stress and Strain

The most fundamental concepts in mechanics of materials are stress and strain

Stress has units of force per unit area and is denoted by the Greek letter σ (sigma).

$$\sigma = \frac{P}{A}$$



units of force per unit of area.

British units

Pounds per square inch (psi) or kips per square inch (ksi).

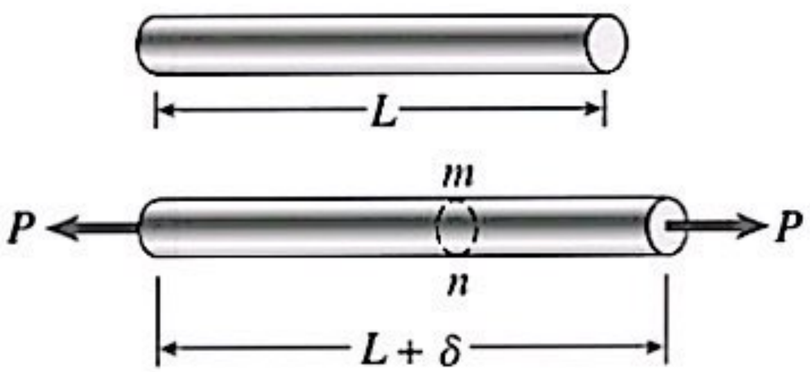
SI units

Newton's per square meter (N/m^2), that is, pascals (Pa).

If the bar is in tension, the strain is called a **tensile strain**, representing an elongation or stretching of the material. If the bar is in compression, the strain is a **compressive strain** and the bar shortens. Tensile strain is usually taken as positive and compressive strain as negative.

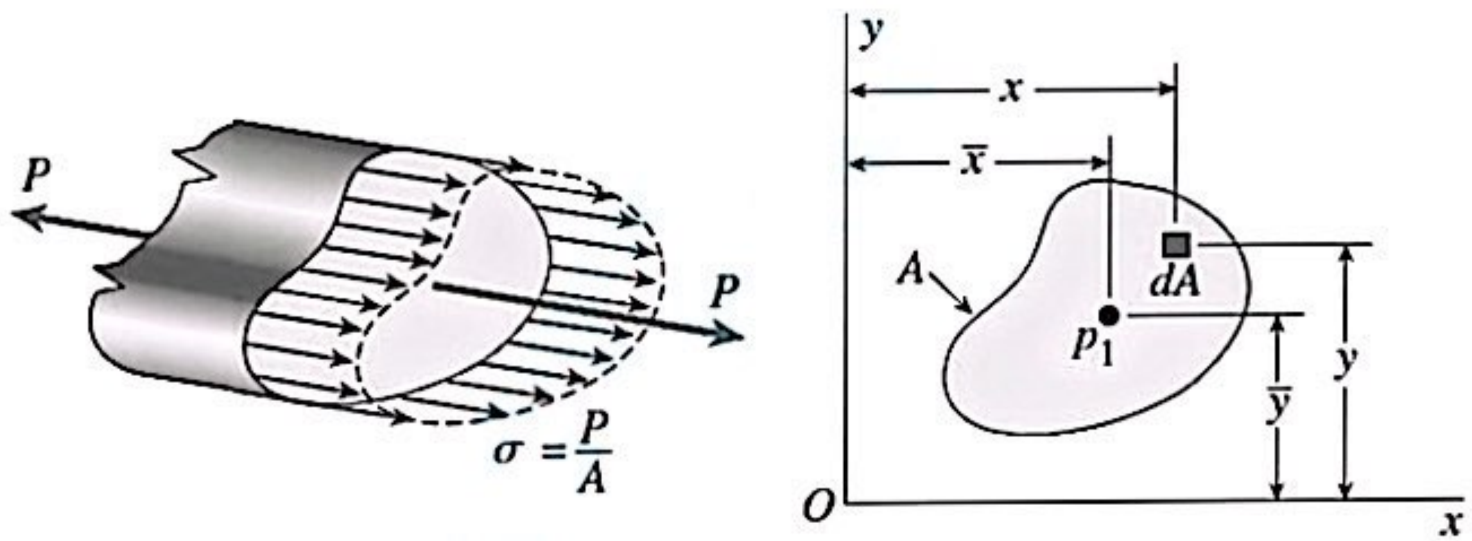
Normal Strain is the *elongation per unit length*, and is denoted by the Gr letter ε (epsilon).

$$\epsilon = \frac{\delta}{L}$$



The uniaxial stress, including stresses in directions other than the longitudinal direction of the bar

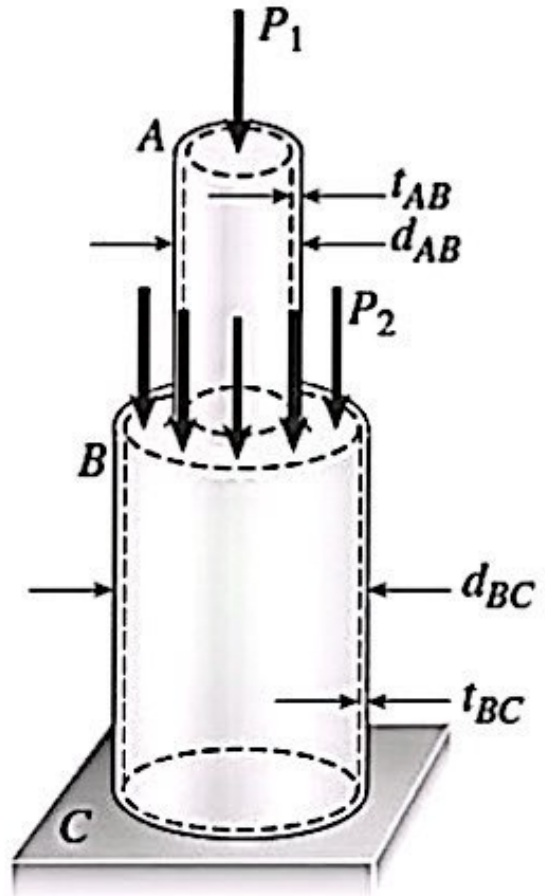
Line of Action of the Axial Forces for a Uniform Stress Distribution



Mechanics of Materials.

A hollow circular post *ABC* (see figure) supports a load $P_1 = 1700$ lb acting at the top. A second load P_2 is uniformly distributed around the cap plate at *B*. The diameters and thicknesses of the upper and lower parts of the post are $d_{AB} = 1.25$ in., $t_{AB} = 0.5$ in., $d_{BC} = 2.25$ in., and $t_{BC} = 0.375$ in., respectively.

(a) Calculate the normal stress σ_{AB} in the upper part of the post.
(b) If it is desired that the lower part of the post have the same compressive stress as the upper part, what should be the magnitude of the load P_2?
(c) If P_1 remains at 1700 lb and P_2 is now set at 2260 lb, what new thickness of *BC* will result in the same compressive stress in both parts?



Solution

PART (a)

$P_1 = 1700 \qquad d_{AB} = 1.25 \qquad t_{AB} = 0.5$

$d_{BC} = 2.25 \qquad t_{BC} = 0.375$

$$A_{AB} = \frac{\pi[\,d_{AB}^2 - (d_{AB} - 2t_{AB})^2]}{4}$$

$$A_{AB} = 1.178 \qquad \sigma_{AB} = \frac{P_1}{A_{AB}}$$

$$\sigma_{AB} = 1443 \text{ psi} \quad \leftarrow$$

PART (b)

$$A_{BC} = \frac{\pi[\,d_{BC}^2 - (d_{BC} - 2t_{BC})^2]}{4}$$

$$A_{BC} = 2.209 \qquad P_2 = \sigma_{AB}A_{BC} - P_1$$

$$P_2 = 1488 \text{ lbs} \quad \leftarrow$$

CHECK: $\dfrac{P_1 + P_2}{A_{BC}} = 1443$ psi

1.4 - Elasticity, Plasticity, and Creep

Elasticity property of a material is that returns to its original dimensions during unloading.
Plasticity is the characteristic of a material by which it undergoes inelastic strains beyond the strain at the elastic limit.
Creep is the elongation when loaded for long periods of time.

1.5 - Linear Elasticity, Hooke's Law, and Poisson's Ratio

Hooke's Law

The linear relationship between stress and strain $\sigma = E\epsilon$
E is a constant of proportionality known as the **modulus of elasticity** (**Young's modulus**) for the material.

Poisson's Ratio

When a prismatic bar is loaded in tension, the axial elongation is accompanied by **lateral contraction**

Poisson's ratio. usually denoted by the Greek letter ν (nu), expressed by the equation

$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\epsilon'}{\epsilon}$$

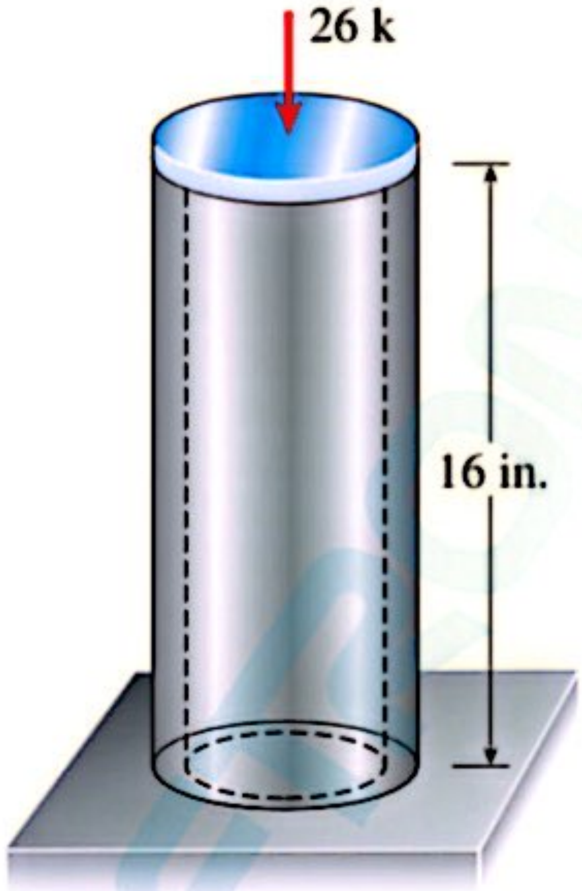
Poisson's ratio remains constant throughout the linearly elastic range.

The **homogeneous** material have the same composition (and hence the sar elastic properties) at every point. However, having a homogeneous mate does not mean that the elastic properties at a particular point are the sam all *directions*. ***Materials*** having the same properties in all directions (wheth axial, lateral, or any other direction) are said to be **isotropic**. If the properties differ in various directions, the material is **anisotropic.**

In this course, all examples and problems are solved with the assumption that the material is linearly elastic, homogeneous, and isotropic.

A short post constructed from a hollow circular tube of aluminum supports a compressive load of 26 kips (Fig. 1-5). The inner and outer diameters of the tube are $d_1 = 4.0$ in. and $d_2 = 4.5$ in., respectively, and its length is 16 in. The shortening of the post due to the load is measured as 0.012 in.

Determine the compressive stress and strain in the post. (Disregard the weight of the post itself, and assume that the post does not buckle under the load.)



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Solution

Assuming that the compressive load acts at the center of the hollow tube, we can use the equation $\sigma = P/A$ (Eq. 1-1) to calculate the normal stress. The force P equals 26 k (or 26,000 lb), and the cross-sectional area A is

$$A = \frac{\pi}{4}\left(d_2^2 - d_1^2\right) = \frac{\pi}{4}\left[(4.5\text{ in.})^2 - (4.0\text{ in.})^2\right] = 3.338\text{ in.}^2$$

Therefore, the compressive stress in the post is

$$\sigma = \frac{P}{A} = \frac{26{,}000\text{ lb}}{3.338\text{ in.}^2} = 7790\text{ psi}$$

The compressive strain (from Eq. 1-2) is

$$\epsilon = \frac{\delta}{L} = \frac{0.012\text{ in.}}{16\text{ in.}} = 750 \times 10^{-6}$$

Thus, the stress and strain in the post have been calculated.

Note: As explained earlier, strain is a dimensionless quantity and no units are needed. For clarity, however, units are often given. In this example, ϵ could be written as 750×10^{-6} in./in. or 750 μin./in.

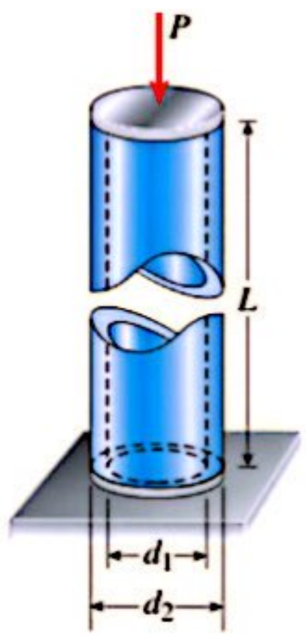


FIG. 1-23 Example 1-3. Steel pipe in compression

A steel pipe of length $L =$ 4.0 ft, outside diameter $d_2 = 6.0$ in., and inside diameter $d_1 = 4.5$ in. is compressed by an axial force $P = 140$ k (Fig. 1-23). The material has modulus of elasticity $E = 30{,}000$ ksi and Poisson's ratio $\nu = 0.30$.

Determine the following quantities for the pipe: (a) the shortening δ, (b) the lateral strain ϵ', (c) the increase Δd_2 in the outer diameter and the increase Δd_1 in the inner diameter, and (d) the increase Δt in the wall thickness.

Solution

The cross-sectional area A and longitudinal stress σ are determined as follows:

$$A = \frac{\pi}{4}\left(d_2^2 - d_1^2\right) = \frac{\pi}{4}\left[(6.0 \text{ in.})^2 - (4.5 \text{ in.})^2\right] = 12.37 \text{ in.}^2$$

$$\sigma = -\frac{P}{A} = -\frac{140 \text{ k}}{12.37 \text{ in.}^2} = -11.32 \text{ ksi (compression)}$$

Because the stress is well below the yield stress (see Table H-3, Appendix H), the material behaves linearly elastically and the axial strain may be found from Hooke's law:

$$\epsilon = \frac{\sigma}{E} = \frac{-11.32 \text{ ksi}}{30{,}000 \text{ ksi}} = -377.3 \times 10^{-6}$$

The minus sign for the strain indicates that the pipe shortens.

(a) Knowing the axial strain, we can now find the change in length of the pipe (see Eq. 1-2):

$$\delta = \epsilon L = (-377.3 \times 10^{-6})(4.0 \text{ ft})(12 \text{ in./ft}) = -0.018 \text{ in.}$$

The negative sign again indicates a shortening of the pipe.

(b) The lateral strain is obtained from Poisson's ratio (see Eq. 1-10):

$$\epsilon' = -\nu\epsilon = -(0.30)(-377.3 \times 10^{-6}) = 113.2 \times 10^{-6}$$

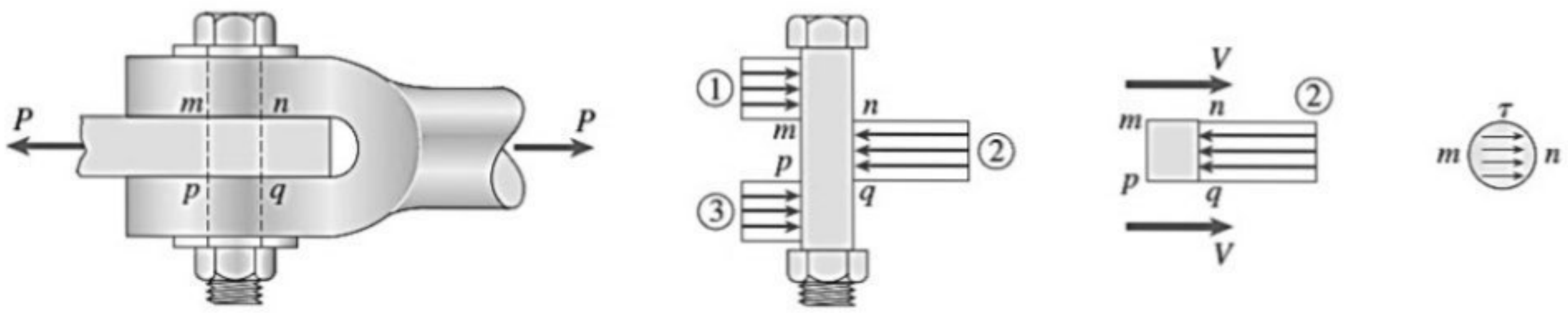
The positive sign for ϵ' indicates an increase in the lateral dimensions, as expected for compression.

1.6 - Shear Stress and Strain

Shear stress, that acts *tangential* to the surface of the material. Shear stresses are customarily denoted by the Greek letter τ (tau).

Figure show connection consists of a flat bar A, a clevis C, and a bolt B that passes through holes in the bar and clevis. Under the action of the tensile loads P, the bar and clevis will press against the bolt in **bearing**, and contact stresses, called **bearing stresses**.

(a)



average bearing stress σ_b by dividing the total bearing force F_b by the bearing area A_b:

$$\sigma_b = \frac{F_b}{A_b}$$

The **bearing area** is defined as the projected area of the curved bearing surface.

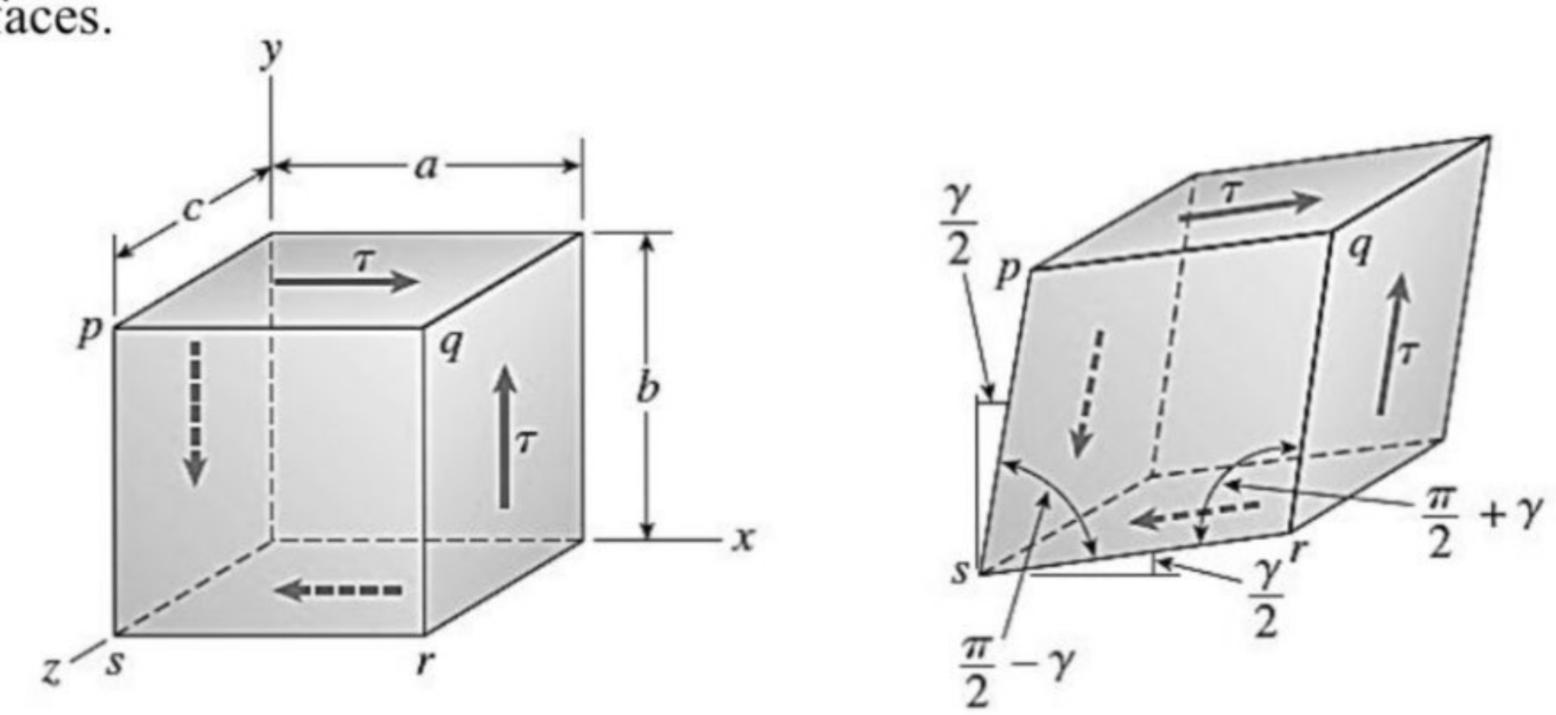
The **average shear stress** on the cross section of a bolt is obtained by dividing the total shear force V by the area A of the cross section on which it acts, as follows:

$$\tau_{aver} = \frac{V}{A}$$

The general observations regarding shear stresses acting on a rectangular element:

1. Shear stresses on opposite (and parallel) faces of an element are equal in magnitude and opposite in direction.

2. Shear stresses on adjacent (and perpendicular) faces of an element are equal in magnitude and have directions such that both stresses point toward, and both point away from, the line of intersection of the faces.



The angle γ is a measure of the **distortion**, or change in shape, of the element and is called the **shear strain**. Because shear strain is an angle, it is usually measured in degrees or radians.

Hooke's law in shear: $\tau = G\gamma$

in which G is the **shear modulus of elasticity** (also called the *modulus of rigidity*).

The shear modulus G has the same **units** as the tension modulus E, The moduli of elasticity in tension and shear are related by the following equation:

$$G = \frac{E}{2(1+\nu)}$$

Example 1-4

A punch for making holes in steel plates is shown in Fig. 1-29a. Assume that a punch having diameter $d = 20$ mm is used to punch a hole in an 8-mm plate, as shown in the cross-sectional view (Fig. 1-29b).

If a force $P = 110$ kN is required to create the hole, what is the average shear stress in the plate and the average compressive stress in the punch?

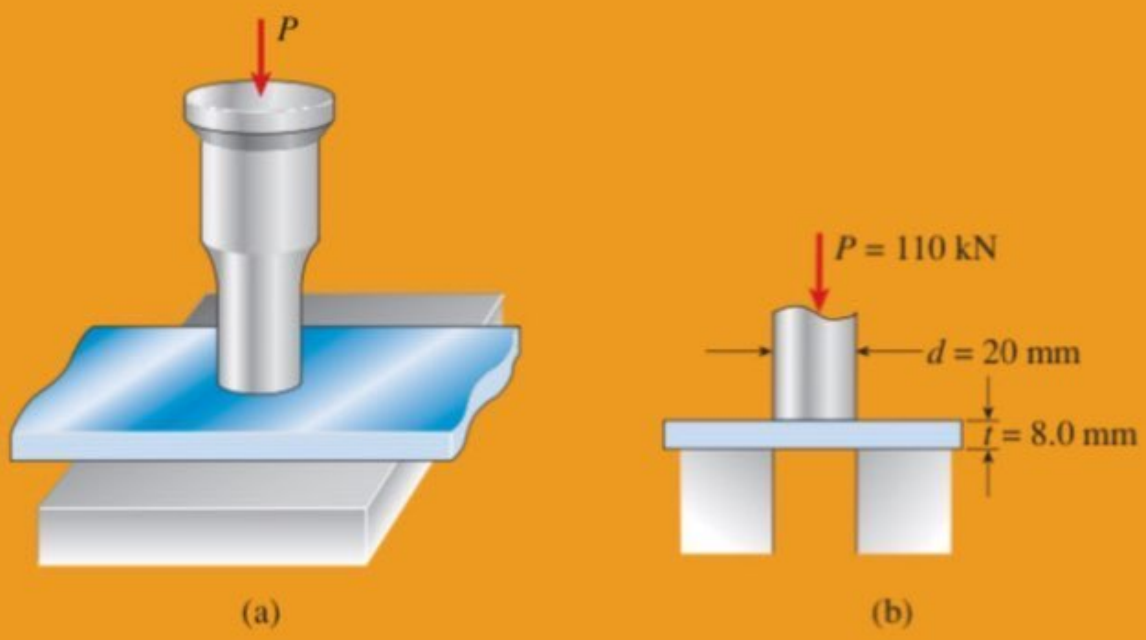


FIG. 1-29 Example 1-4. Punching a hole in a steel plate

Solution

The average shear stress in the plate is obtained by dividing the force P by the shear area of the plate. The shear area A_s is equal to the circumference of the hole times the thickness of the plate, or

$$A_s = \pi dt = \pi(20 \text{ mm})(8.0 \text{ mm}) = 502.7 \text{ mm}^2$$

in which d is the diameter of the punch and t is the thickness of the plate. Therefore, the average shear stress in the plate is

$$\tau_{aver} = \frac{P}{A_s} = \frac{110 \text{ kN}}{502.7 \text{ mm}^2} = 219 \text{ MPa}$$

The average compressive stress in the punch is

$$\sigma_c = \frac{P}{A_{punch}} = \frac{P}{\pi d^2/4} = \frac{110 \text{ kN}}{\pi(20 \text{ mm})^2/4} = 350 \text{ MPa}$$

in which A_{punch} is the cross-sectional area of the punch.

Note: This analysis is highly idealized because we are disregarding impact effects that occur when a punch is rammed through a plate. (The inclusion of such effects requires advanced methods of analysis that are beyond the scope of mechanics of materials.)

Example 1-5

A steel strut S serving as a brace for a boat hoist transmits a compressive force $P = 12$ k to the deck of a pier (Fig. 1-30a). The strut has a hollow square cross section with wall thickness $t = 0.375$ in. (Fig. 1-30b), and the angle θ between the strut and the horizontal is 40°. A pin through the strut transmits the compressive force from the strut to two gussets G that are welded to the base plate B. Four anchor bolts fasten the base plate to the deck.

The diameter of the pin is $d_{pin} = 0.75$ in., the thickness of the gussets is $t_G = 0.625$ in., the thickness of the base plate is $t_B = 0.375$ in., and the diameter of the anchor bolts is $d_{bolt} = 0.50$ in.

Determine the following stresses: (a) the bearing stress between the strut and the pin, (b) the shear stress in the pin, (c) the bearing stress between the pin and the gussets, (d) the bearing stress between the anchor bolts and the base plate, and (e) the shear stress in the anchor bolts. (Disregard any friction between the base plate and the deck.)

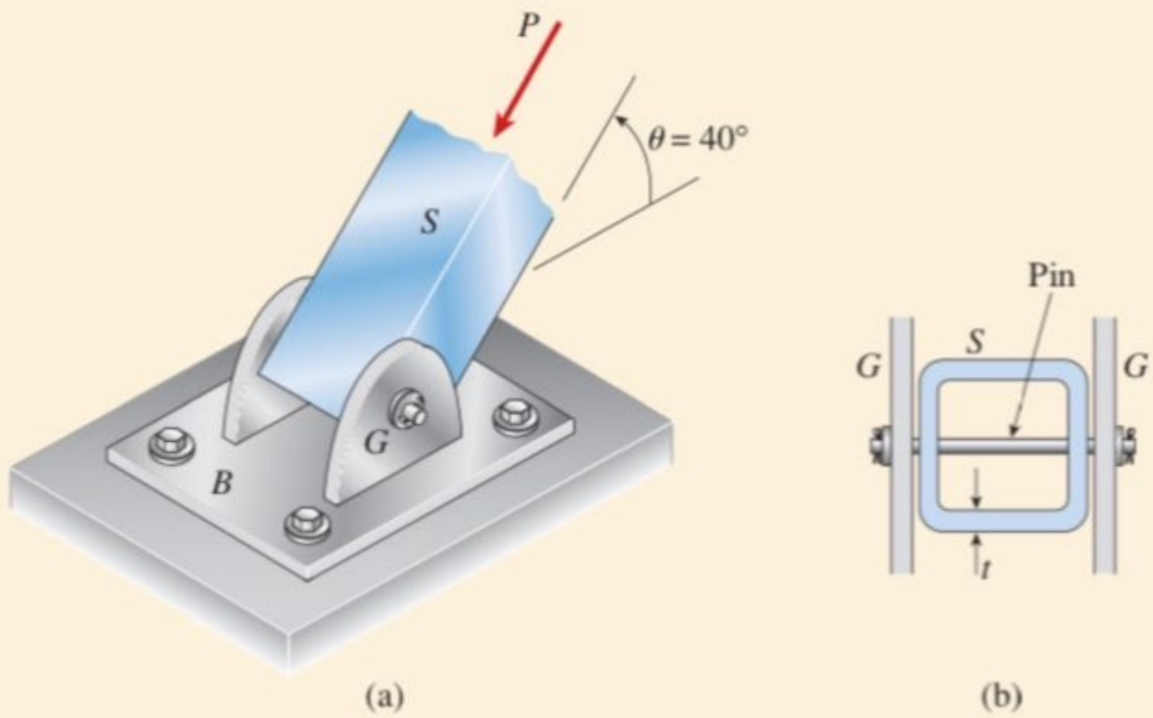


FIG. 1-30 Example 1-5. (a) Pin connection between strut S and base plate B (b) Cross section through the strut S

Solution

(a) *Bearing stress between strut and pin.* The average value of the bearing stress between the strut and the pin is found by dividing the force in the strut by the total bearing area of the strut against the pin. The latter is equal to twice the thickness of the strut (because bearing occurs at two locations) times the diameter of the pin (see Fig. 1-30b). Thus, the bearing stress is

$$\sigma_{b1} = \frac{P}{2td_{pin}} = \frac{12 \text{ k}}{2(0.375 \text{ in.})(0.75 \text{ in.})} = 21.3 \text{ ksi}$$

This bearing stress is not excessive for a strut made of structural steel.

(b) *Shear stress in pin.* As can be seen from Fig. 1-30b, the pin tends to shear on two planes, namely, the planes between the strut and the gussets. Therefore, the average shear stress in the pin (which is in double shear) is equal to the total load applied to the pin divided by twice its cross-sectional area:

$$\tau_{pin} = \frac{P}{2\pi d_{pin}^2/4} = \frac{12 \text{ k}}{2\pi(0.75 \text{ in.})^2/4} = 13.6 \text{ ksi}$$

The pin would normally be made of high-strength steel (tensile yield stress greater than 50 ksi) and could easily withstand this shear stress (the yield stress in shear is usually at least 50% of the yield stress in tension).

(c) *Bearing stress between pin and gussets.* The pin bears against the gussets at two locations, so the bearing area is twice the thickness of the gussets times the pin diameter; thus,

$$\sigma_{b2} = \frac{P}{2t_G d_{pin}} = \frac{12 \text{ k}}{2(0.625 \text{ in.})(0.75 \text{ in.})} = 12.8 \text{ ksi}$$

which is less than the bearing stress between the strut and the pin (21.3 ksi).

(d) *Bearing stress between anchor bolts and base plate.* The vertical component of the force P (see Fig. 1-30a) is transmitted to the pier by direct bearing between the base plate and the pier. The horizontal component, however, is transmitted through the anchor bolts. The average bearing stress between the base plate and the anchor bolts is equal to the horizontal component of the force P divided by the bearing area of four bolts. The bearing area for one bolt is equal to the thickness of the base plate times the bolt diameter. Consequently, the bearing stress is

$$\sigma_{b3} = \frac{P \cos 40°}{4t_B d_{bolt}} = \frac{(12 \text{ k})(\cos 40°)}{4(0.375 \text{ in.})(0.50 \text{ in.})} = 12.3 \text{ ksi}$$

(e) *Shear stress in anchor bolts.* The average shear stress in the anchor bolts is equal to the horizontal component of the force P divided by the total cross-sectional area of four bolts (note that each bolt is in single shear). Therefore,

$$\tau_{bolt} = \frac{P \cos 40°}{4\pi d_{bolt}^2/4} = \frac{(12 \text{ k})(\cos 40°)}{4\pi(0.50 \text{ in.})^2/4} = 11.7 \text{ ksi}$$

Any friction between the base plate and the pier would reduce the load on the anchor bolts.

Chapter two: - Axially Loaded Members

2.1. Introduction: - Structural components subjected only to tension or compressions are known as **axially loaded members**. Examples of axially loaded bars are truss members, long bone etc.

2.2 Changes in Lengths of Axially Loaded Members

When determining the changes in lengths of axially loaded members, it is convenient to begin with a **coil spring** in its **natural length** L (also called its *unstressed length*, *relaxed length*, or *free length*),

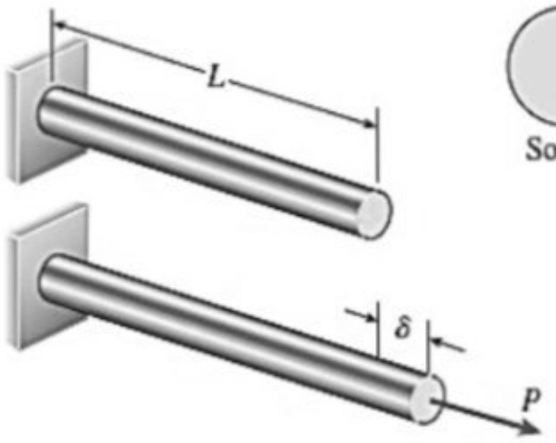
$$P = k\delta \qquad \delta = fP$$

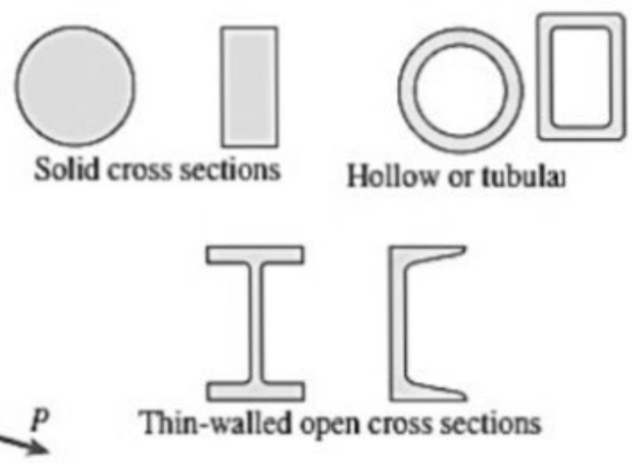
In which k and f are constants of proportionality. The constant k is called the **stiffness** of the spring and is defined as the force required to produce a unit elongation, that is, $k = P/\delta$. Similarly, the constant f is known as the **flexibility** and is defined as the elongation produced by a load of unit value, that is, $f = \delta/P$.

Let us also assume that the material is **linearly elastic**, which means that it follows Hooke's law

Prismatic Bars

Axially loaded bars elongate under tensile loads and shorten under compressive loads,





$$\delta = \frac{PL}{EA}$$

stiffness $k = \dfrac{EA}{L}$ and flexibility $f = \dfrac{L}{EA}$

The assembly shown in Fig. 4–6*a* consists of an aluminum tube AB having a cross-sectional area of 400 mm^2. A steel rod having a diameter of 10 mm is attached to a rigid collar and passes through the tube. If a tensile load of 80 kN is applied to the rod, determine the displacement of the end C of the rod. Take $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.

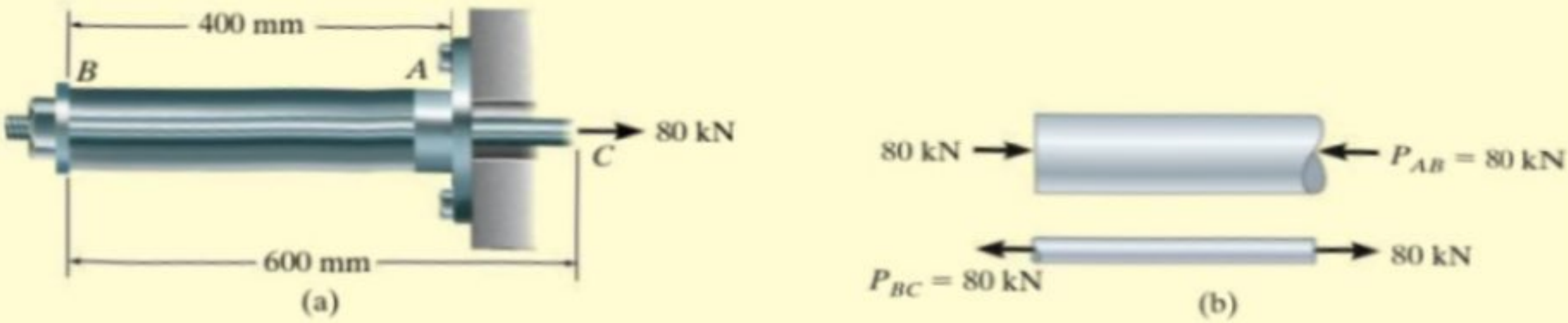


Fig. 4–6

SOLUTION

Internal Force. The free-body diagram of the tube and rod segments in Fig. 4–6*b*, shows that the rod is subjected to a tension of 80 kN and the tube is subjected to a compression of 80 kN.

Displacement. We will first determine the displacement of end C with respect to end B. Working in units of newtons and meters, we have

$$\delta_{C/B} = \frac{PL}{AE} = \frac{[+80(10^3)\text{ N}]\,(0.6\text{ m})}{\pi\,(0.005\text{ m})^2[200\,(10^9)\text{ N/m}^2]} = +0.003056\text{ m} \rightarrow$$

The positive sign indicates that end C moves *to the right* relative to end B, since the bar elongates.

The displacement of end B with respect to the *fixed* end A is

$$\delta_B = \frac{PL}{AE} = \frac{[-80(10^3)\text{ N}](0.4\text{ m})}{[400\text{ mm}^2(10^{-6})\text{ m}^2/\text{mm}^2][70(10^9)\text{ N/m}^2]}$$
$$= -0.001143\text{ m} = 0.001143\text{ m} \rightarrow$$

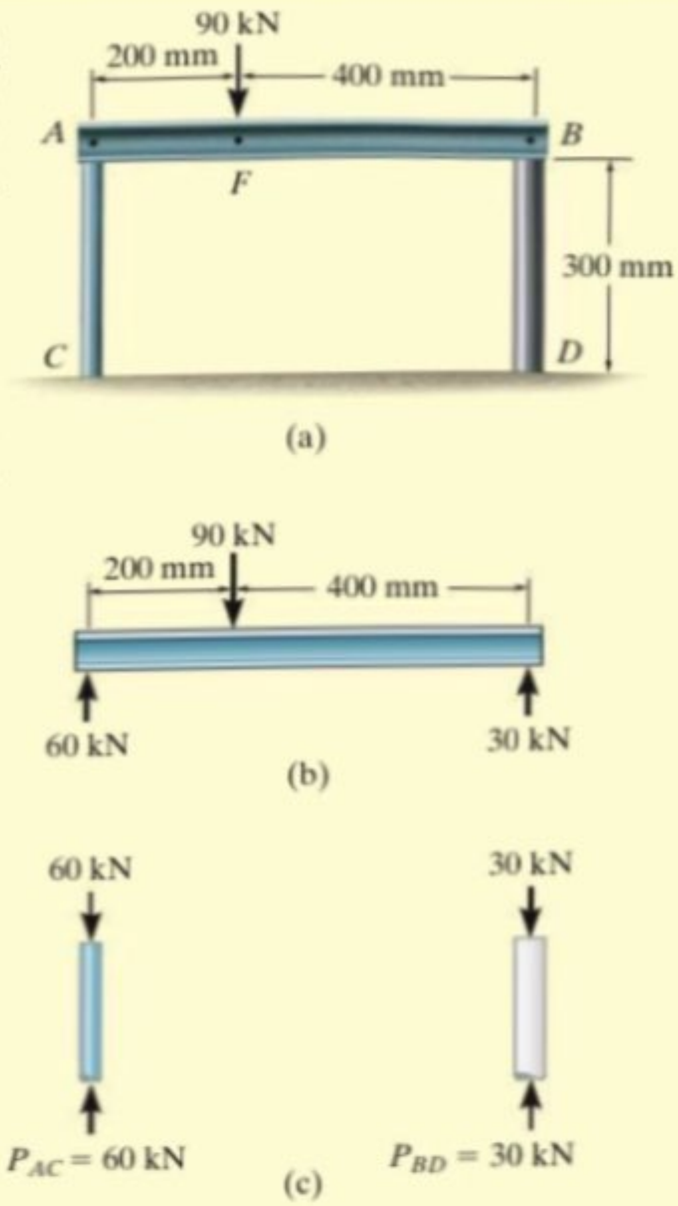
Here the negative sign indicates that the tube shortens, and so B moves to the *right* relative to A.

Since both displacements are to the right, the displacement of C relative to the fixed end A is therefore

$$(\overset{+}{\rightarrow}) \qquad \delta_C = \delta_B + \delta_{C/B} = 0.001143\text{ m} + 0.003056\text{ m}$$
$$= 0.00420\text{ m} = 4.20\text{ mm} \rightarrow \qquad \textit{Ans.}$$

EXAMPLE 4.2

Rigid beam AB rests on the two short posts shown in Fig. 4–7*a*. AC is made of steel and has a diameter of 20 mm, and BD is made of aluminum and has a diameter of 40 mm. Determine the displacement of point F on AB if a vertical load of 90 kN is applied over this point. Take $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.



SOLUTION

Internal Force. The compressive forces acting at the top of each post are determined from the equilibrium of member AB, Fig. 4–7*b*. These forces are equal to the internal forces in each post, Fig. 4–7*c*.

Displacement. The displacement of the top of each post is

Post AC:

$$\delta_A = \frac{P_{AC}L_{AC}}{A_{AC}E_{st}} = \frac{[-60(10^3)\text{ N}](0.300\text{ m})}{\pi(0.010\text{ m})^2[200(10^9)\text{ N/m}^2]} = -286(10^{-6})\text{ m}$$
$$= 0.286\text{ mm}\downarrow$$

Post BD:

$$\delta_B = \frac{P_{BD}L_{BD}}{A_{BD}E_{al}} = \frac{[-30(10^3)\text{N}](0.300\text{ m})}{\pi(0.020\text{ m})^2[70(10^9)\text{ N/m}^2]} = -102(10^{-6})\text{ m}$$
$$= 0.102\text{ mm}\downarrow$$

A diagram showing the centerline displacements at A, B, and F on the beam is shown in Fig. 4–7*d*. By proportion of the blue shaded triangle, the displacement of point F is therefore

$$\delta_F = 0.102\text{ mm} + (0.184\text{ mm})\left(\frac{400\text{ mm}}{600\text{ mm}}\right) = 0.225\text{ mm}\downarrow \qquad \textit{Ans.}$$

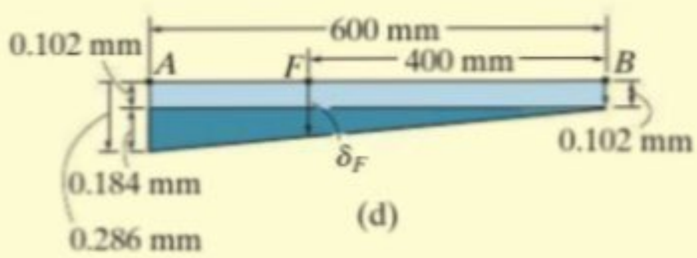


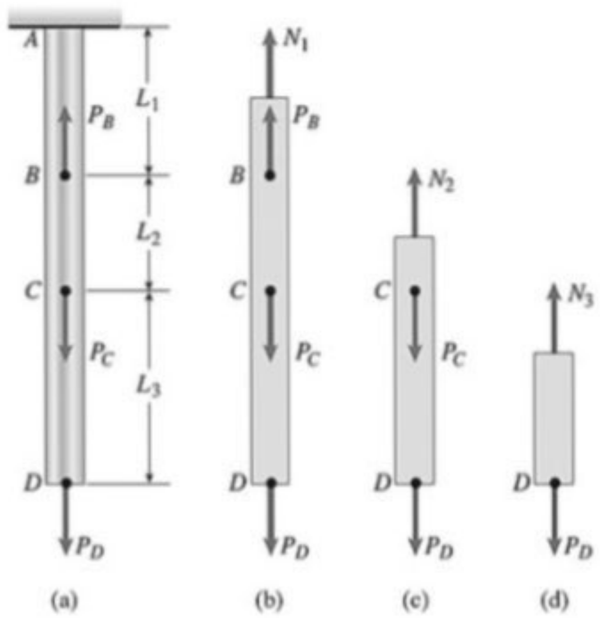
Fig. 4–7

2.3 Changes in Lengths Under Nonuniform Conditions

Bars with Intermediate Axial Loads

1. Identify the segments of the bar (segments AB, BC, and CD) as segments 1, 2, and 3, respectively.
2. Determine the internal axial forces N_1, N_2, and N_3 in segments 1, 2, and 3, respectively, from the free-body diagrams

$$N_1 = -P_B + P_C + P_D \qquad N_2 = P_C + P_D \qquad N_3 = P_D$$



3. Determine the changes in the lengths of the segments

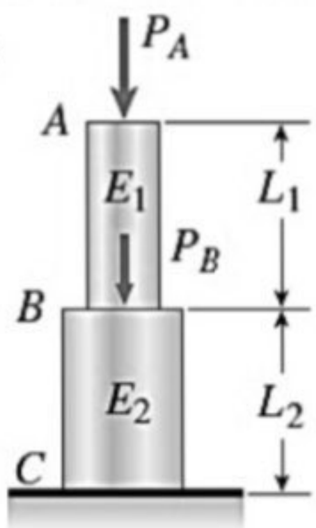
$$\delta_1 = \frac{N_1 L_1}{EA} \qquad \delta_2 = \frac{N_2 L_2}{EA} \qquad \delta_3 = \frac{N_3 L_3}{EA}$$

4. Add δ_1, δ_2, and δ_3 to obtain δ, the change in length of the entire bar:

$$\delta = \sum_{i=1}^{3} \delta_i = \delta_1 + \delta_2 + \delta_3$$

Bars Consisting of Prismatic Segments

This same general approach can be used when the bar consists of several prismatic segments, each having different axial forces, different dimensions, and different materials



$$\delta = \sum_{i=1}^{n} \frac{N_i L_i}{E_i A_i}$$