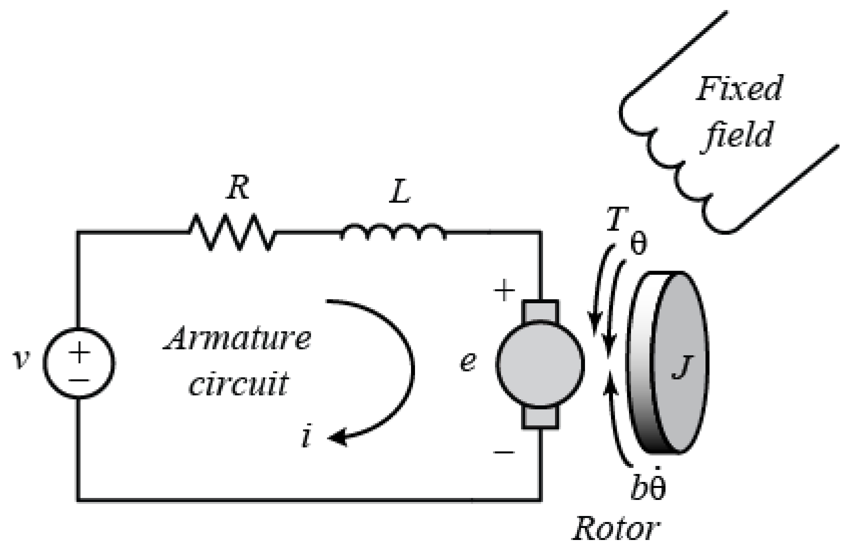
Modeling of permanent magnet DC motor (PMDC)

1. Modeling

A conventional actuator in control systems is the DC motor. It directly provides rotary motion and, coupled with wheels or drums and cables, can provide translational motion. The electric circuit of the armature and the free-body diagram of the rotor are shown in the following figure:



For this example, we will assume that the input of the system is the voltage source (*V*) applied to the motor's armature, while the output is the rotational speed of the shaft d(*theta*)/dt. The rotor and shaft are assumed to be rigid. We further consider a viscous friction model, that is, the friction torque is proportional to shaft angular velocity.

The physical parameters for our example are:

(J)	moment of inertia of the rotor	0.01 kg.m^2
(b)	motor viscous friction constant	0.1 N.m.s
(K_e)	electromotive force constant	0.01 V/rad/sec
(K_t)	motor torque constant	0.01 N.m/Amp
(R)	electric resistance	1 Ohm
(L)	electric inductance	0.5 H

In general, the torque generated by a DC motor is proportional to the armature current and the strength of the magnetic field. In this example, we will assume that the magnetic field is constant and, therefore, that the motor torque is proportional to only the armature current *i* by a constant factor K_t as shown in the equation below. This is referred to as an armature controlled motor.

$$T = k_t i$$

The back emf, *e*, is proportional to the angular velocity of the shaft by a constant factor K_e.

$$e = Ke\frac{d\theta}{dt}$$

Let that the motor torque and back emf constants are equal, that is,

$$K_t = K_e$$

therefore, we will use *K* to represent both the motor torque constant and the back emf constant.

This system will be modeled by summing the torques acting on the rotor inertia and integrating the acceleration to give velocity. Also, Kirchoff's laws will be applied to the armature circuit. First, we will model the integrals of the rotational acceleration and the rate of change of the armature current.

2. Building the model with Simulink

Next, we will apply **Newton's law** and **Kirchoff's law** to the motor system to generate the following equations:

$$J\frac{d^2\theta}{dt^2} = T - b\frac{d\theta}{dt} \quad \rightarrow \quad \frac{d^2\theta}{dt^2} = \frac{1}{J}\left(k_t i - b\frac{d\theta}{dt}\right)$$

$$L\frac{di}{dt} = -Ri + V - e \quad \rightarrow \quad \frac{di}{dt} = \frac{1}{L}\left(-Ri + V - Ke\frac{d\theta}{dt}\right)$$

The angular acceleration is equal to **1 / *J*** multiplied by the sum of two terms **(one positive, one negative).** Similarly,

The derivative of current is equal to **1 / *L*** multiplied by the sum of three terms **(one positive, two negative).**

Continuing to model these equations in Simulink, follow the steps given below.

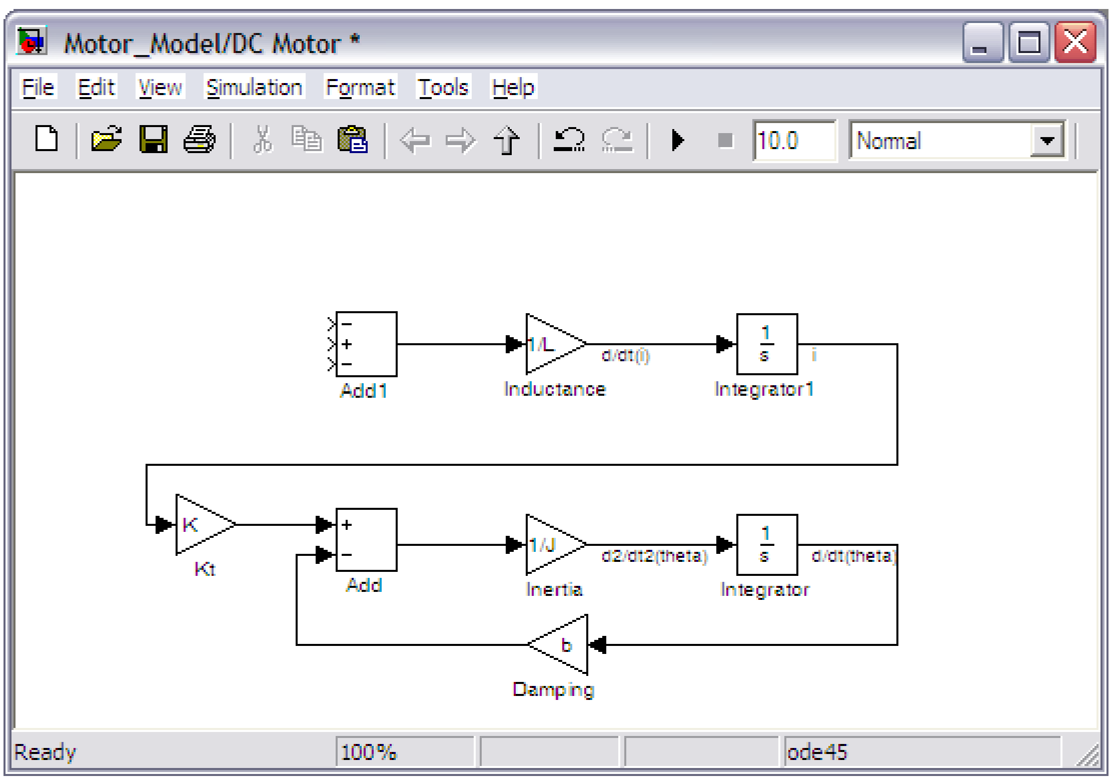
- Insert two Gain blocks from the Simulink/Math Operations library, one attached to each of the integrators.
- Edit the Gain block corresponding to angular acceleration by double-clicking it and changing its value to "1/J".
- Change the label of this Gain block to "Inertia" by clicking on the word "Gain" underneath the block.
- Similarly, edit the other Gain's value to "1/L" and its label to "Inductance".
- Insert two Add blocks from the Simulink/Math Operations library, one attached by a line to each of the Gain blocks.
- Edit the signs of the Add block corresponding to rotation to **"+ -"** since one term is positive and one is negative.
- Edit the signs of the other Add block to **"- + -"** to represent the signs of the terms in the electrical equation.

Now, we will add in the torques which are represented in the rotational equation. First, we will add in the damping torque.

- Insert a Gain block below the "Inertia" block. Next right-click on the block and select **Format > Flip Block** from the resulting menu to flip the block from left to right. You can also flip a selected block by holding down **Ctrl-I**.
- Set the Gain value to "b" and rename this block to "Damping".
- Tap a line (hold **Ctrl** while drawing or right-click on the line) off the rotational Integrator's output and connect it to the input of the "Damping" block.
- Draw a line from the "Damping" block output to the negative input of the rotational Add block.

Next, we will add in the torque from the armature.

- Insert a Gain block attached to the positive input of the rotational Add block with a line.
- Edit its value to "K" to represent the motor constant and Label it "Kt".
- Continue drawing the line leading from the current Integrator and connect it to the "Kt" block.



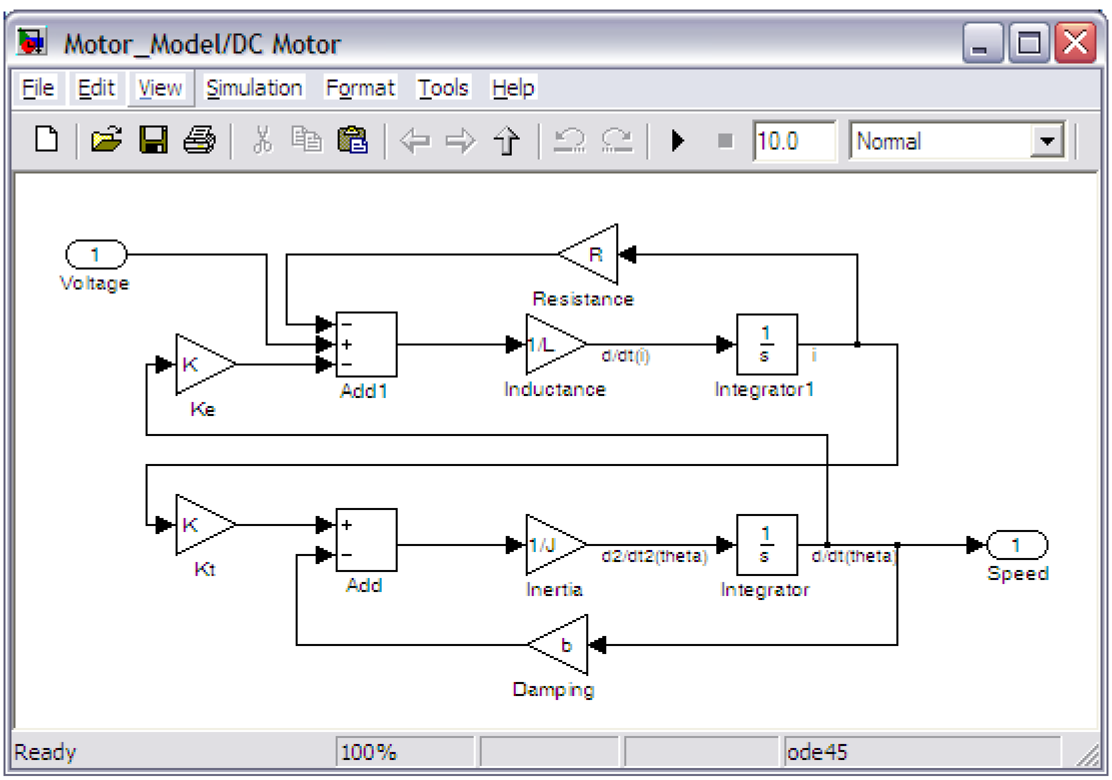
Now, we will add in the voltage terms which are represented in the electrical equation. First, we will add in the voltage drop across the armature resistance.

- Insert a Gain block above the "Inductance" block and flip it from left to right.
- Set the Gain value to "R" and rename this block to "Resistance".
- Tap a line off the current Integrator's output and connect it to the input of the "Resistance" block.
- Draw a line from the "Resistance" block's output to the upper negative input of the current equation Add block.

Next, we will add in the back emf from the motor.

- Insert a Gain block attached to the other negative input of the current Add block with a line.
- Edit it's value to "K" to represent the motor back emf constant and Label it "Ke".
- Tap a line off the rotational Integrator's output and connect it to the "Ke" block.
- Add In1 and Out1 blocks from the Simulink/Ports & Subsystems library and respectively label them "Voltage" and "Speed".

The final design should look like the example shown in the figure below.



To save all of these components as a single subsystem block, first select all of the blocks, then select **Create Subsystem** from the **Edit** menu. Name the subsystem "DC Motor" and then save the model. Your model should appear as follows.