Chapter one: The nature and propagation of light

1-1The nature of light:

Until the time of Isaac Newton (1642-1727), most scientists thought that light consisted of streams of particles (called corpuscles) emitted by light sources .Galileo and others tried (unsuccessfully) to measure the speed of light. Around ،1665 evidence of wave properties of light began to be discovered. By the early 19.th century, evidence that light is a wave had grown very persuasive .In 1873, James Clerk Maxwell predicted the existence of electromagnetic waves and calculated their speed of propagation. This development, along with the experimental work of Heinrich Hem starting in 1887, showed conclusively that light is indeed an electromagnetic wave.

1-2 Sources of light:

Several effects associated with emission and absorption of light reveal a particle aspect, in that the energy carried by light waves is packaged in discrete bundles called **photons** or **quanta**.

These apparently contradictory **wave** and **particle** properties.

The fundamental sources of all electromagnetic radiation are electric charges in accelerated motion.

All bodies emit electromagnetic radiation as a result of thermal motion of their molecules; this radiation, called **thermal radiation**, is a mixture of different wavelengths.

A light source that has attained prominence in the last forty years is the laser. In laser, by atoms are induced to emit light in a cooperative, coherent fashion. The result is a very narrow beam of radiation that can be enormously intense and that is much more nearly **monochromatic**, or **single-frequency**, than light from any other source. The speed of light in vacuum is defined to be :

$$c = 2.99792458 * 10^8 \text{ m/s} \qquad \text{or} \qquad 3*10^8 \text{ m/ s}$$

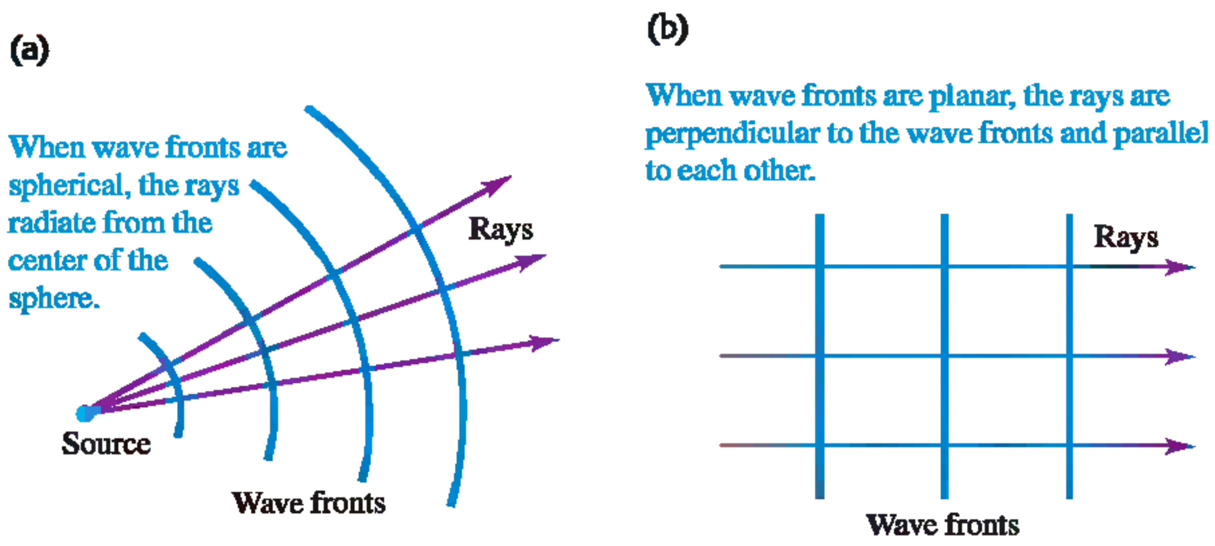
1-3 Waves, Wave Fronts, and Rays:

We define a wave front as the locus of all adjacent points at which the phase of vibration of a physical quantity associated with the wave is the same. That is, at any instant, all points on a wave front are at the same part of the cycle of their variation.

When electromagnetic waves are radiated by a small light source, we can represent the wave fronts as spherical surfaces concentric with the source or, as in Fig.a. Far away from the source, where the radii of the spheres have become very large.

In a **particle theory** of light, rays **are the paths of the particles**.

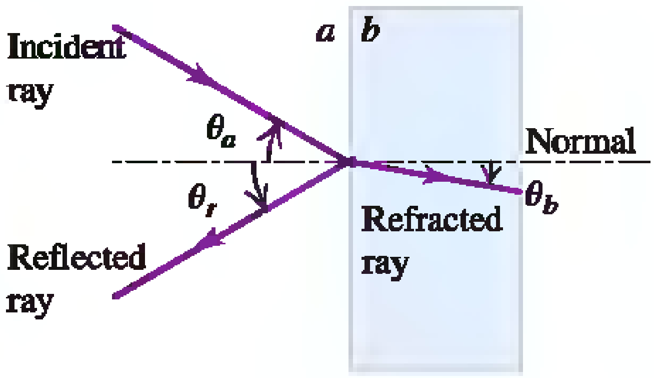
From the **wave** viewpoint a ray **is an imaginary line along the direction of travel of the wave.** In Fig.a the rays are the radii of the spherical wave fronts, and in Fig.b they are straight lines perpendicular to the wave fronts.



1-4 Reflection and Refraction:

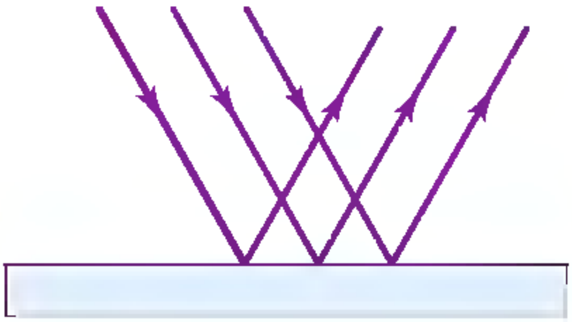
When a light wave strikes a smooth interface separating two transparent materials (such as air and glass or water and glass), the wave is in general partly reflected and partly refracted (transmitted) into the second material. We describe the directions of the incident, reflected, and refracted (transmitted) rays at a smooth interface between two optical materials in terms of the angles they make with the normal (perpendicular) to the surface at the point of incidence, as shown in Fig.c If the interface is rough, both the transmitted light and the reflected light are scattered in various directions, and there is no single angle of transmission or reflection.

(c) The representation simplified to show just one set of rays

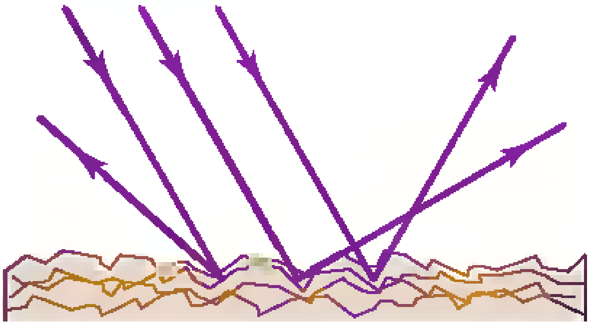


Reflection at a definite angle from a very smooth surface is called **specular reflection** (from the Latin word for "mirror"); scattered reflection from a rough surface is called **diffuse reflection**.

(a) Specular reflection



(b) Diffuse reflection



The index of refraction of an optical material (also called the refractive index), denoted by **n**. It is the ratio of the speed of light c in vacuum to the speed in the material:

$$n = \frac{c}{v} \qquad \text{(index of refraction)}$$

Light always travels more slowly in a material than in vacuum, so the value of n in anything other than vacuum is always greater than unity. For vacuum, **n = 1**. Since n is a ratio of two speeds, it is a pure number without units.

1-5 The Laws of Reflection and Refraction:

1. The incident, reflected, and refracted rays and the normal to the surface all lie in the same plane, The plane of the three rays is perpendicular to the plane of the boundary surface between the two materials. We always draw ray diagrams so that the incident, reflected, and refracted rays are in the plane of the diagram.

2. The angle of reflection **θr** is equal to the angle of incidence **θa** for all wavelengths and for any pair of materials.

$$\theta_a = \theta_r \qquad \text{(law of reflection)} \ldots\ldots\ldots\ldots 1$$

This relationship, together with the observation that the incident and reflected rays and the normal all lie in the same plane, is called the **law of reflection.**

3. For monochromatic light and for a given pair of materials, **a** and **b**, on opposite sides of the interface, the ratio of the sines of the angles **θa** and **θb** , where both angles are measured from the normal to the surface, is equal to the inverse ratio of the two indexes of refraction:

$$\frac{\sin\theta_a}{\sin\theta_b} = \frac{n_b}{n_a} \qquad \ldots\ldots\ldots\ldots 2$$

or

$$n_a \sin\theta_a = n_b \sin\theta_b \qquad \text{(law of refraction)} \ldots\ldots\ldots 3$$

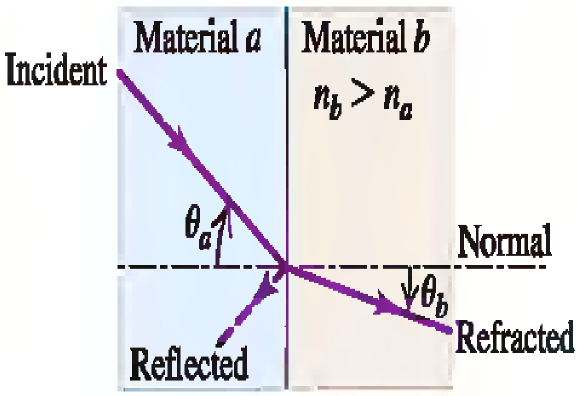
Is called the **law of refraction or Snell's law**.

When a ray passes from one material **a** into another material **b** having a larger index of refraction (**nb > na**) and hence a slower wave speed, the angle **θb** with the normal is smaller in the second material than the angle **θa** in the first hence the ray is bent toward the normal Fig.a.

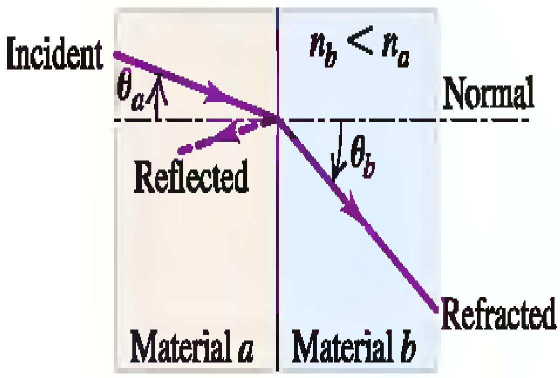
When the second material has a smaller index of refraction than the first material (**nb < na**) and hence a faster wave speed, the ray is bent away from the normal Fig.b.

No matter what the materials on either side of the interface, in the case of normal incidence the transmitted ray is not bent at all Fig.c. In this case **θa = 0** and **sinθa = 0**, so from Eq. (3) **θb** is also equal to zero, so the transmitted ray is also normal to the interface.

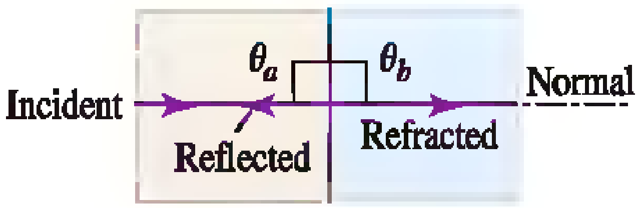
(a) A ray entering a material of *larger* index of refraction bends *toward* the normal.



(b) A ray entering a material of *smaller* index of refraction bends *away from* the normal.



(c) A ray oriented along the normal does not bend, regardless of the materials.



1-6 Index of Refraction and the Wave Aspects of Light:

The frequency **f** of the wave does not change when passing from one material to another. That is, the number of wave cycles arriving per unit time must equal the number leaving per unit time; this is a statement that the boundary surface cannot create or destroy waves.

The wavelength **λ** of the wave is different in general in different materials. This is because in any material, **υ=λf** , since **f** is the same in any material as in vacuum and **υ** is always less than the wave speed **c** in vacuum, **λ** is also correspondingly reduced. Thus the wavelength **λ** of light in a material is less than the wavelength λ_0 of the same light in vacuum. From the above discussion, $f = c/\lambda_0 = \upsilon/\lambda$

$$n = c/\upsilon$$

$$\lambda = \lambda_0/n \qquad \text{(wavelength of light in a material)}$$

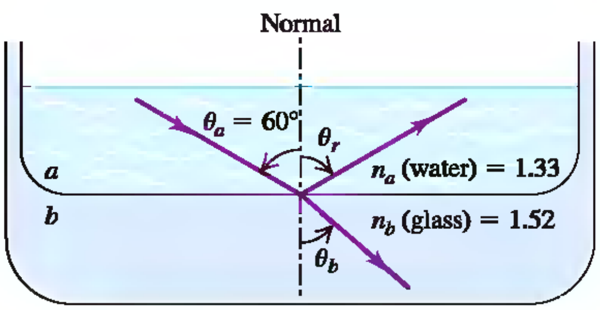
Example.1:

In Fig., material a is water and material b is a glass with index of refraction 1.52. If the incident ray makes an angle of 60° with the normal, find the directions of the reflected and refracted rays.

Solution :

This is a problem in geometric optics. We are given the incident angle and the index of refraction of each material, and we need to find the reflected and refracted angles.

Figure shows the rays and angles for this situation .The target variables are the reflected angle **θ**r and the refracted angle **θb**. Since **nb** is greater than **na** the refracted angle must be smaller than the incident angle **θa.**



The angle the reflected ray makes with the normal is the same as that of the incident ray, so To find the direction of the refracted ray, so

$\theta r = \theta a = 60^o$

To find the direction of the refracted ray, we use Snell's law, with **$n_a = 1.33$** **$n_b = 1.52$** and **$\theta a = 60.0^o$** We find::

$$n_a \sin\theta_a = n_b \sin\theta_b$$

$$\sin\theta_b = \frac{n_a}{n_b}\sin\theta_a = \frac{1.33}{1.52}\sin 60.0^\circ = 0.758$$

$$\theta_b = 49.3^\circ$$

Example. 2 :

The wavelength of the red light from a helium-neon laser is 633 nm in air but 474 nm in the aqueous humor inside your eyeball of human being. Calculate the index of refraction of the aqueous humor and the speed and frequency of the light in this substance.

Solution:

The key ideas here are the relationship between index of refraction n and wave speed **υ** and the relationsillp between index of refraction and wavelength **λ**.

$n = c/\upsilon$, as well as **$\lambda = \lambda_o/n$**. It will also be helpful to use the relationship **$\upsilon = \lambda f$** among wave speed, wavelength, and frequency.

$\lambda_o = 633$ nm

$\lambda = \lambda_o /n$ $\quad$ $n = \lambda_o / \lambda = 633/474 = 1.34$

This is about the same index of refraction as for water. Then **$n = c/\upsilon$**

$\upsilon = c /n = 3.00 * 10^8 /1.34 = 2.25 * 10^8$ m/s

Finally, from **$\upsilon = \lambda f$** $\quad$ **$f = \upsilon / \lambda =$**

$f_o = c/ \lambda_o = 3.00 * 10^8 /633 = 4.74 * 10^{14}$Hz

Chapter two: Reflation and Refraction at Plane Surfaces

2-1 Huygens Principle:

Huygens assumed that **every point of a wave front may be considered the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the wave**. The new wave front at a later time is then found by constructing a surface tangent to the secondary wavelets or, as it is called, the envelope of the wavelets .

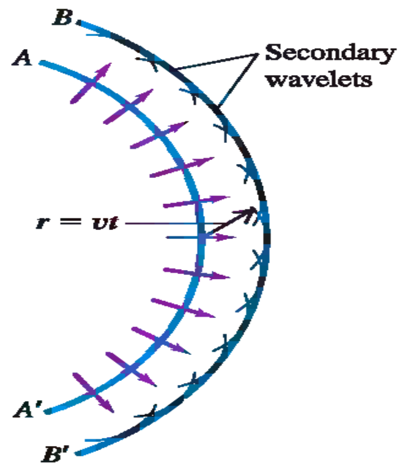


Fig.1

2-2 Derivation of the Law Refraction from Huygens Principle:

In Fig.2 we consider a wave front, represented by line **AA'**, arrived at the boundary surface **SS'** between two transparent materials **a** and **b**, with indexes of refraction $\mathbf{n_a}$ and $\mathbf{n_b}$ and wave speeds $\boldsymbol{\upsilon_a}$ and $\boldsymbol{\upsilon_b}$.

Those originating near the upper end of **AA'** travel with speed $\boldsymbol{\upsilon_a}$ and, after a time interval **t**, are spherical surfaces of radius $\boldsymbol{\upsilon_a t}$. The wavelet originating at point **A** however, is traveling in the second material **b** with speed $\boldsymbol{\upsilon_b}$ and at time **t** is a spherical surface of radius $\boldsymbol{\upsilon_b t}$.

respectively.

We draw $\mathbf{OQ} = \boldsymbol{\upsilon_a t}$ perpendicular to **AQ**, and we draw $\mathbf{AB} = \boldsymbol{\upsilon_b t}$, perpendicular to **BO**. From the right triangle **AOQ**,

$$\sin\theta_a = \frac{v_a t}{AO} \qquad \text{……….1}$$

and from the right triangle *AOB*,

$$\sin\theta_b = \frac{v_b t}{AO} \qquad \text{……….2}$$

Combining these, we find

$$\frac{\sin\theta_a}{\sin\theta_b} = \frac{v_a}{v_b} \qquad \text{………3}$$

We have defined the index of refraction n of a material as the ratio of the speed of light c in vacuum to its speed **υ** in the material:

$\mathbf{n_a = c/υ_a}$ **and** $\mathbf{n_b = c/υ_b}$ Thus:

$$\frac{n_b}{n_a} = \frac{c/v_b}{c/v_a} = \frac{v_a}{v_b} \qquad \text{…………4}$$

$$\frac{\sin\theta_a}{\sin\theta_b} = \frac{n_b}{n_a} \quad \text{or} \qquad \text{…………..5}$$

$$n_a \sin\theta_a = n_b \sin\theta_b \qquad \text{……………6}$$

Which we recognize as Snell's law, Eq. 6. So we have derived **Snell's law** from a wave theory.

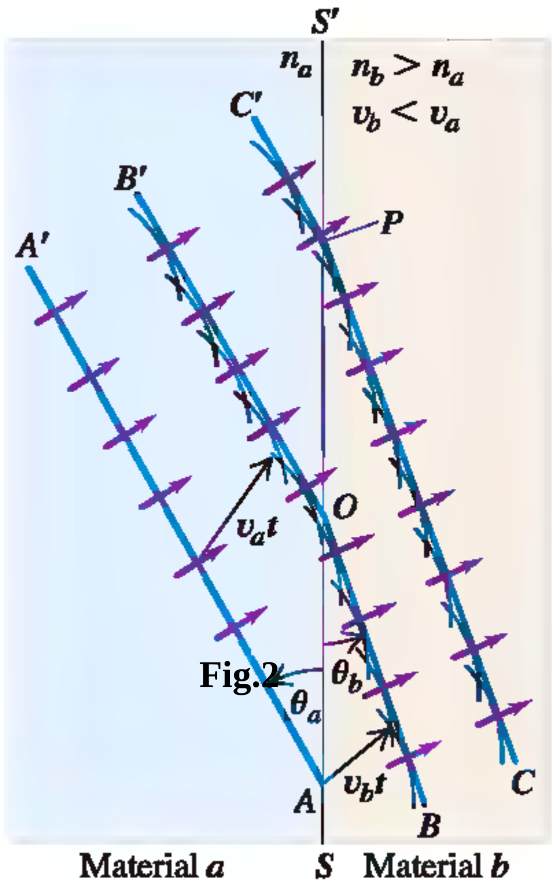


Fig.2

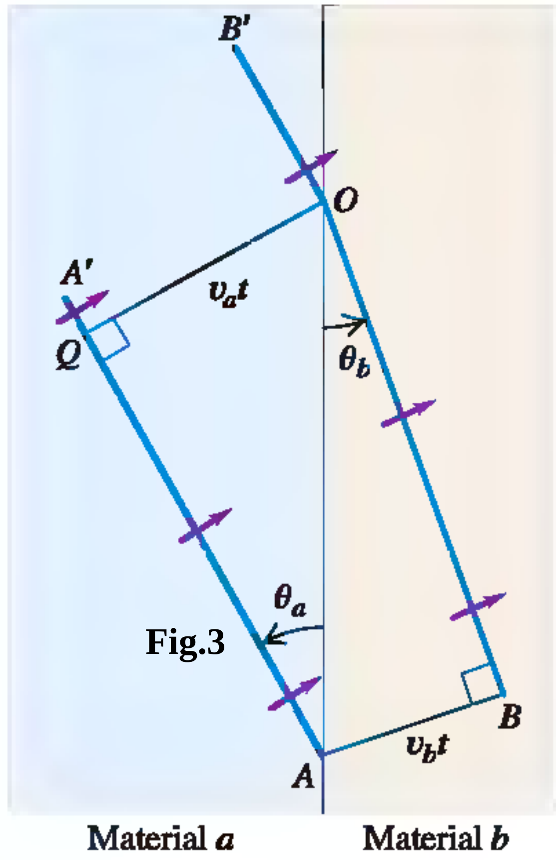


Fig.3

2-3 Total Internal Reflection:

All of the light can be reflected back from the interface, with none of it being transmitted, even though the second material is transparent. Fig.4 shows how this can occur.

Several rays are shown radiating from a point source in material **a** with index of refraction $\mathbf{n_a}$ the rays strike the surface of a second material **b** with index $\mathbf{n_b}$ where $\mathbf{n_a > n_b}$.

For instance, materials **a** and b could be water and air, respectively. From Snell's law of refraction:

$$\sin\theta_b = \frac{n_a}{n_b}\sin\theta_a$$

Because $\mathbf{n_a/n_b}$ is greater than unity $\mathbf{\sin\theta_b}$ is larger than $\mathbf{\sin\theta_a}$ the ray is bent away from the normal.

In ray 3 in the diagram, which emerges just grazing the surface at an angle of refraction of **90°**.

The angle of incidence for which the refracted ray emerges tangent to the surface is called the critical angle, denoted by θ_{crit}.

If the angle of incidence is larger than the critical angle, the sine of the angle of refraction, as computed by Snell's law, would have to be greater than unity, which is impossible. Beyond the critical angle, the ray **cannot** pass into the upper material; it is trapped in the lower material and is completely reflected at the boundary surface.

This situation **called total internal reflection**

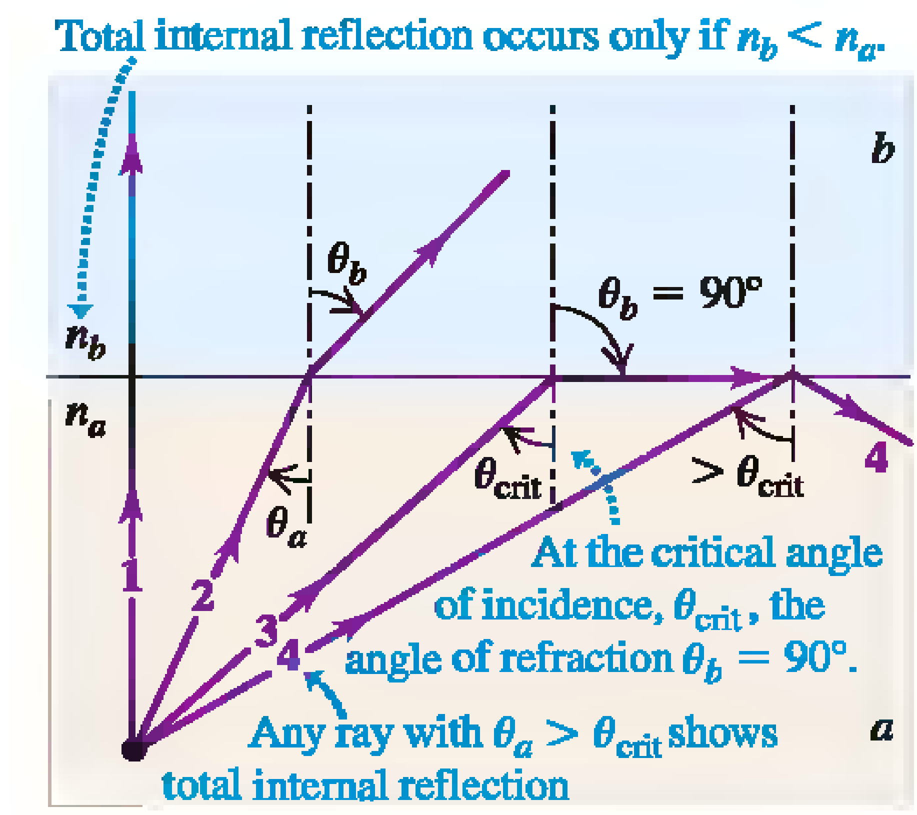
$$\sin\theta_{crit} = \frac{n_b}{n_a}$$ **(Critical angle for total internal reflection)**

Total internal reflection can occur only if two conditions are met:

1- $\mathbf{n_b}$ must be less than $\mathbf{n_a}$,

2-The critical angle must be smaller than the angle of incidence.

Fig.4



2-4 Refraction by Prims:

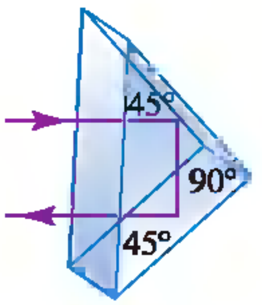
The light will be totally reflected if it strikes the glass-air surface at an angle of 41.1° or larger. Because the critical angle is slightly smaller than 45°, it is possible to use a prism with angles of 45° -45° - 90° as a totally reflecting surface.

As reflectors, totally reflecting prisms have some advantages over metallic surfaces such as ordinary coated-glass mirrors. While no metallic surface reflects 100% of the light incident on it, light can be totally reflected by a prism. The reflecting properties of a prism have the additional advantages of being permanent and unaffected by tarnishing .

A 45°-45°-90° prism, used as in Fig5a, is called a **Porro prism**.

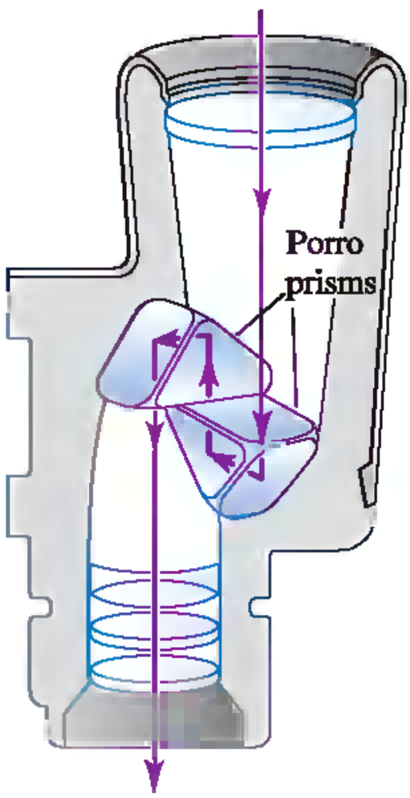
The total change of direction of the rays is 180°. Binoculars often use combinations of two Porro prisms, as in Fig5b.

(a) Total internal reflection in a Porro prism



If the incident beam is oriented as shown, total internal reflection occurs on the 45° faces (because, for a glass–air interface, $\theta_{crit} = 41.1°$).

(b) Binoculars use Porro prisms to reflect the light to each eyepiece.



2-5 Dispersion:

The speed of light in vacuum is the same for all wavelengths, but the speed in a material substance is different for different wavelengths. Therefore the index of refraction of a material depends on wavelength.

The dependence of wave speed and index of refraction on wavelength is called dispersion.

In most materials the value of **n** decreases with increasing wavelength and decreasing frequency, and thus **n** increases with decreasing wavelength and increasing frequency. In such a material, light of longer wavelength has greater speed than light of shorter wavelength .

The deviation (change of direction) produced by the prism increases with increasing index of refraction and frequency and decreasing wavelength. Violet light is deviated most, and red is deviated least; other colors are in intermediate positions. When it comes out of the prism, the light is spread out into a fan-shaped beam, the light is said to be **dispersed** into a spectrum.

The amount of dispersion depends on the difference between the refractive indexes for violet light and for red light.

We can see that for a substance such as fluorite, the difference between the indexes for red and violet is small, and the dispersion will also be small.

A better choice of material for a prism whose purpose is to produce a spectrum would be silicate flint glass, for which there is a larger difference in the value of **n** between red and violet.

2-6 Absorption:

No material is perfectly transparent, as light passes through any optical medium (except vacuum) its energy is partially absorbed, increasing the internal energy in the material and the intensity (power per unit area) is corresponding attenuated.

When a beam of light pass through a thin sheet of material of thickness **dx**, the decrease **dI** in its intensity **I** is found to be proportional to the initial intensity **I** and to the thickness **dx**. Thus

$$dI = -\alpha\, I\, dx \qquad \text{..............(1)}$$

The proportional constant **α** which depend on the material is called the **absorption coefficient**. The intensity after passage through a slab of finite thickness **x** can be obtained by integrating equ.1.

$$I = I_o e^{-\alpha x} \qquad \text{..............(2)}$$

Where I_o is the intensity at **x=0**. Equ.2 called its **Lambert's law**.

Example: A certain glass has $\alpha = 10\,m^{-1}$. Light passing through a flat slab at normal incidence is decreased in intensity by **5%**. What is the thickness of the slab?

Solution:

$$I = I_o e^{-\alpha x}$$

$$\ln I = \ln I_o - \alpha x$$

$$\ln I/I_o = -\alpha x$$

$$x = (-1/\alpha) \ln (I/I_o)$$

$$I/I_o = 0.95$$

$$x = -(1/10) \ln 0.95$$

$$x = 0.00513\ m$$

$$\text{or} \quad x = 0.513\ cm$$

Chapter Three: Geometric Optics

3-1 Reflection and Refraction at a Plane Surface:

1-We first need the concept of object as it is used in optics.

2-Point object that has no physical extent .

3- Extended object that real object with length, width, and height.

4- After the rays are reflected, their directions are the same as though they had come from point **P'**. We call point **P** an object point and point **P'** the corresponding image point, and we say that the reflecting surface forms an image of point **P** as shown in fig. 1.

Fig. 1

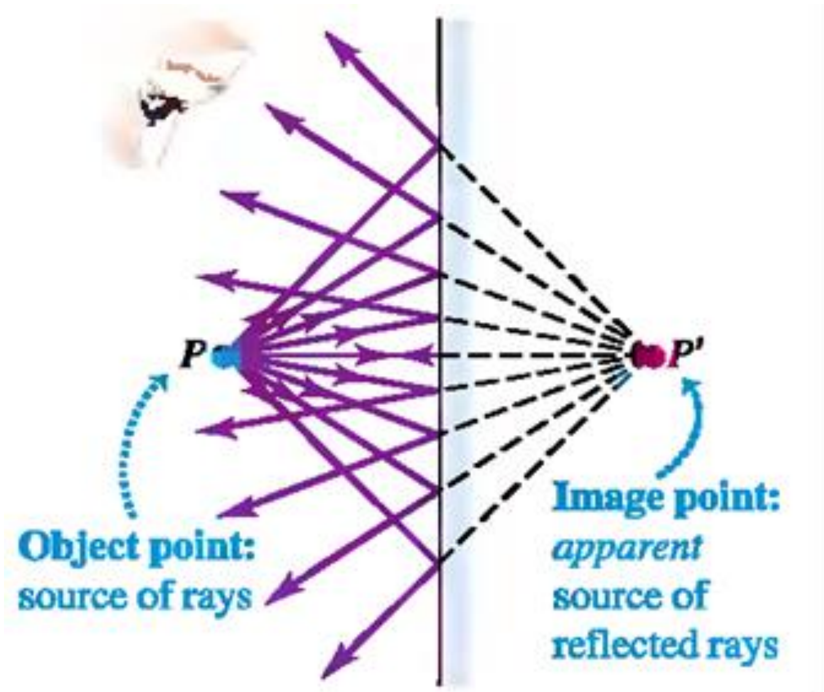
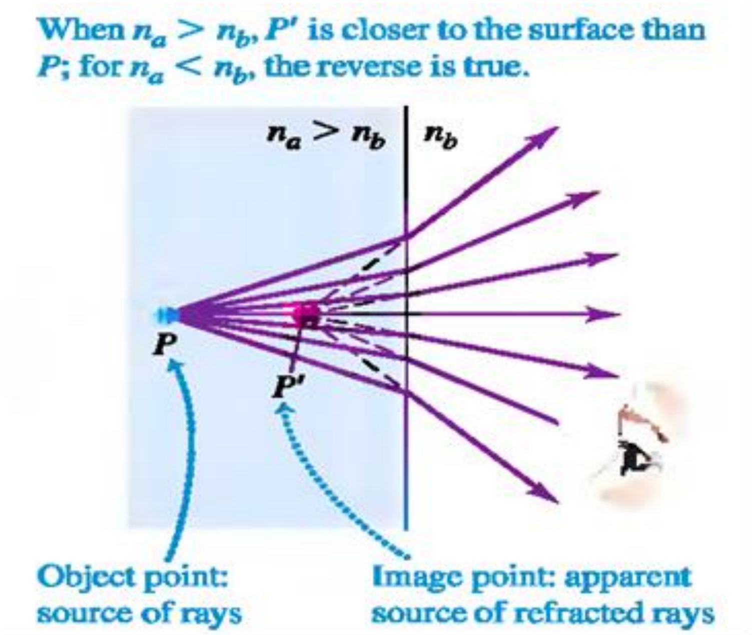


Fig. 2



5- In both Figs.1 and 2 the rays do not actually pass through the image point P'. Indeed, if the surface in Fig.1 is opaque, there is no light at all on its right side. If the outgoing rays don't actually pass through the image point, we call the image **a virtual image**. Later we will see cases in which the outgoing rays really do pass through an image point, and we will call the resulting image a **real image**.

3-2 Sign Rules:

1- Sign rule for the object distance: When the object is on the same side of the reflecting or refracting surface as the incoming light, the object distance s is positive; otherwise, it is negative .

2- Sign rule for the image distance: When the image is on the same side of the reflecting or refracting surface as the outgoing light, the image distance s' is positive; otherwise, it is negative .

3- Sign rule for the radius of curvature of a spherical surface: When the center of curvature C is on the same side as the outgoing light, the radius of curvature is positive; otherwise, it is negative.

The object distance s is positive because the object point P is on the incoming side (the left side) of the reflecting surface. The image distance s' is negative because the image point P' is not on the outgoing side (the left side) of the surface.

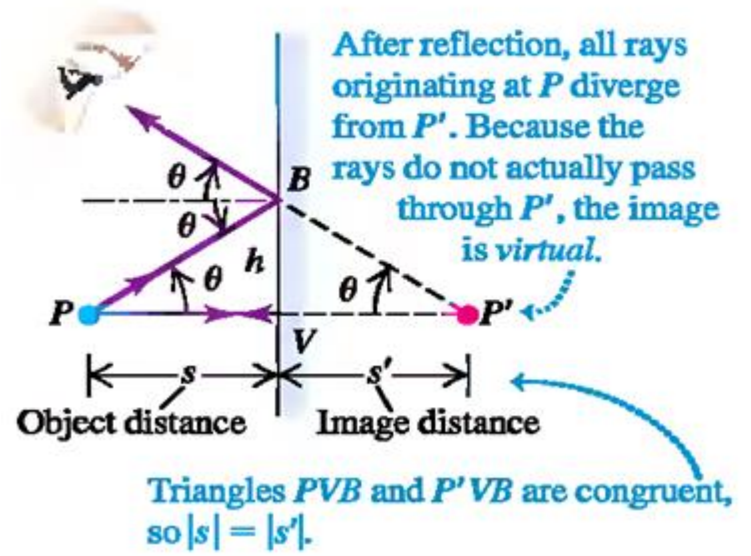
3-3 Image Formation by a Plane Mirror:

1- Fig.3. shows two rays diverging from an object point **P** at a distance **s** to the left of a plane mirror. We call **s** the **object distance**. The ray PV is incident normally on the mirror (that is, it is perpendicular to the mirror surface), and it returns along its original path .

2- The ray PB makes an angle θ with PV. It strikes the mirror at an angle of incidence θ and is reflected at an equal angle with the normal. When we extend the two reflected rays backward, they intersect at point **p'**, at a distance **s'** behind the mirror. We call **s'** the **image distance**.

3- The line between **P** and **P'** is perpendicular to the mirror. P and P' are at equal distances from the mirror, and **s** and **s'** have equal magnitudes. The image point P' is located exactly opposite the object point P as far behind the mirror as the object point is from the front of the mirror.

Fig. 3



The object and image distances s and s' are related simply by:

s = -s' **(plane mirror)**

3-4 Image of an Extended Object plane Mirror:

1 - The distance from the head to the tail of an arrow oriented in this way is called its height, the height is **y**. The image of the arrow is the line P'Q', with height y'. and **y = y'**

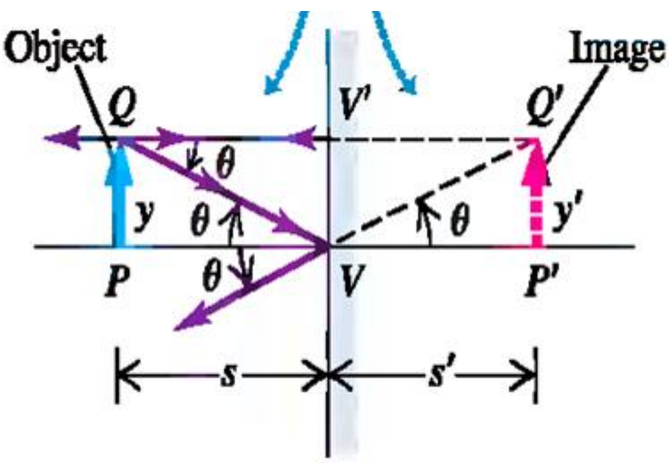
2-The ratio of image height to object height, y'/y, in any image-forming situation is called the lateral magnification m that is:

m= y' /y **(lateral magnification)**

3-The image arrow points in the same direction as the object arrow; we say that the image is **erect**. In this case, **y** and **y'** have the same sign, and the lateral magnification **m** is **positive**. The y and y' have both the same magnitude and the same sign; the lateral magnification of a plane mirror is always **m = + 1** .

4- Later we will encounter situations in which the image is inverted; that is, the image arrow points in the direction opposite to that of the object arrow. For **an inverted** image, y and y' have opposite signs, and the lateral magnification **m is negative**.

Fig.4



3-5 Reflection at a Spherical Surface:

A) Image of a Point Object: Spherical Mirror:

1- Fig.7 shows a spherical mirror with radius of curvature R, with its concave side facing the incident light .

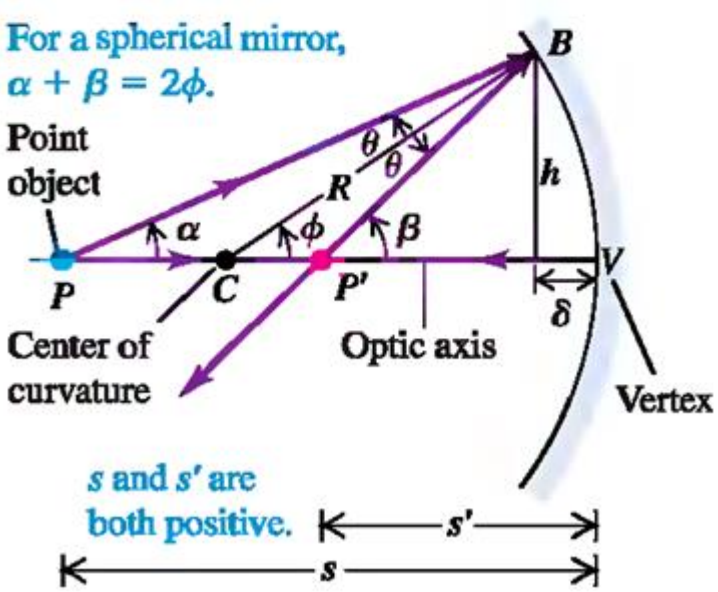
2-The center of curvature of the surface (the center of the sphere of which the surface is a part) is at C.

3 -The vertex of the mirror (the center of the mirror surface) is at V. The line CV is called the optic axis .

4-Point P is an object point that lies on the optic axis, we assume that the distance from P to V is greater than R .

5-Ray PV, passing through C, strikes the mirror normally and is reflected back on itself. Ray PB, at an angle α with the axis, strikes the mirror at B, where the angles of incidence and reflection are θ. The reflected ray intersects the axis at point P ', Thus P' is a real image.

Fig.7



We now use the following theorem from plane geometry: An exterior angle of a triangle equals the sum of the two opposite interior angles. Applying this theorem to triangles PBC and P'BC in Fig., we have

$$\phi = \alpha + \theta \qquad \beta = \phi + \theta \quad \text{……………1}$$

Eliminating θ between these equations gives:

$$\beta + \alpha = 2\phi \qquad \text{…………….2}$$

We may now compute the image distance s'. Let **h** represent the height of point B above the optic axis, and let **δ** represent the short distance from V to the foot of this vertical line. We now write expressions for the tangents of, β , α and ϕ, remembering that s, s', and R are all positive quantities:

$$\tan\alpha = \frac{h}{s-\delta} \qquad \tan\beta = \frac{h}{s'-\delta} \qquad \tan\phi = \frac{h}{R-\delta} \qquad \text{………..3}$$

$$\alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R} \qquad \text{……………4}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \qquad \text{(object–image relationship, spherical mirror)}$$

Concave mirror is also called **a converging mirror**.

Focal Point and Focal Length :

When the object point P is very far from the spherical mirror (s = co), the incoming rays are parallel. The image distance s' in this case is given by:

$$\frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R} \qquad s' = \frac{R}{2}$$

The beam of incident parallel rays converges, after reflection from the mirror, to a point F at a distance **R/2** from the vertex of the mirror. The point F at which the incident parallel rays converge is called the **focal point**. The distance from the vertex to the focal point, denoted by **f**, is called the **focal length**, We see that f is related to the radius of curvature R by:

$$f = \frac{R}{2} \qquad \text{(focal length of a spherical mirror)}$$

We will usually express the relationship between object and image distances for a mirror, f :

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \text{(object–image relationship, spherical mirror)}$$

B) Image of an Extended Object: Spherical Mirror:

Now suppose we have an object with finite size, represented by the arrow PQ in Fig.9.Not that the object and image arrows have different sizes, y and y', respectively, and that they have opposite orientation. we defined the lateral magnification m as the ratio of image size y' to object size y:

$$m = \frac{y'}{y}$$

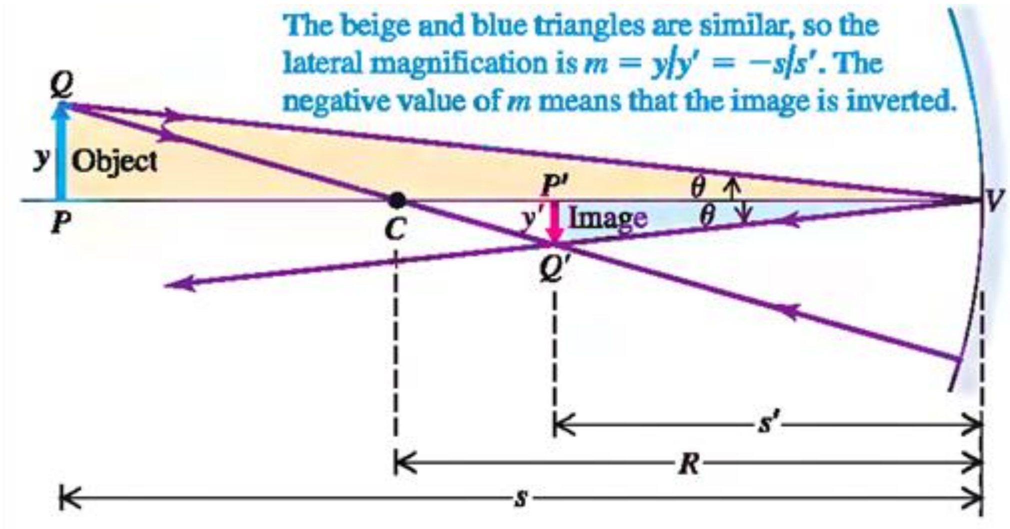


Fig.9

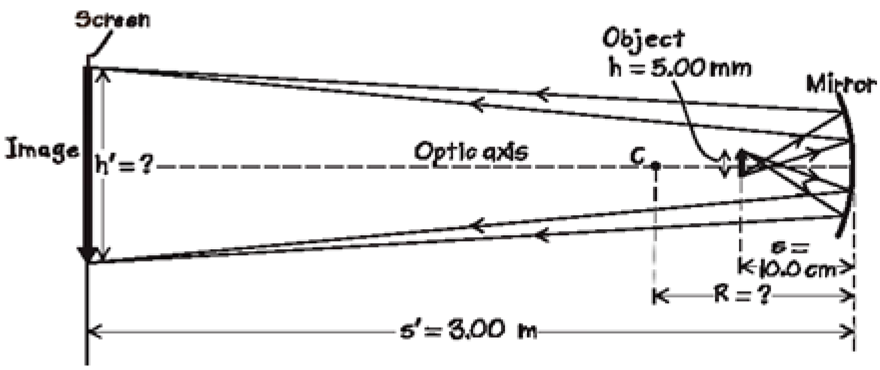
We also have the relationship y/s = -y'/s'. The negative sign is needed because object and image are on opposite sides of the optic axis; if y is positive, y' is negative. Therefore

$$m = \frac{y'}{y} = -\frac{s'}{s} \qquad \text{(lateral magnification, spherical mirror)}$$

If m is positive, the image is **erect** in comparison to the object; if m is negative, the image is **inverted** relative to the object.

Example :

A concave mirror forms an image, on a wall 3.00 m from the mirror, of the filament of a headlight lamp 10.0 cm in front of the mirror. (a) What are the radius of curvature and focal length of the mirror? (b) What is the height of the image if the height of the object is 5.00 mm?



Solution:

s = 10.0 cm and s' = 3.00 cm .

$$\frac{1}{10.0\ \text{cm}} + \frac{1}{300\ \text{cm}} = \frac{2}{R}$$

$$R = \frac{2}{0.100\ \text{cm}^{-1} + 3.33 \times 10^{-3}\ \text{cm}^{-1}} = 19.4\ \text{cm}$$

The focal length of the mirror is $f = R/2 = 9.7$ cm.

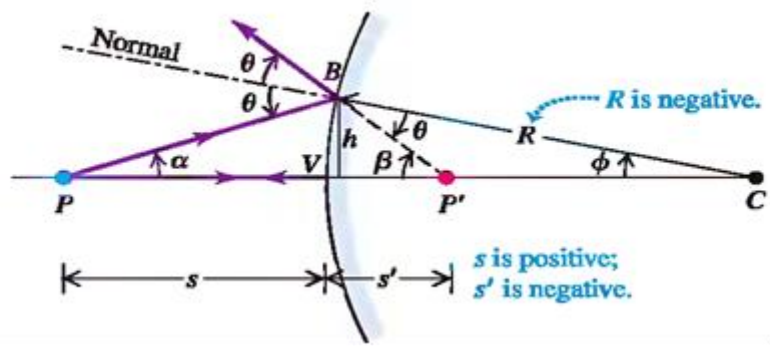
$$m = \frac{y'}{y} = -\frac{s'}{s} = -\frac{300\ \text{cm}}{10.0\ \text{cm}} = -30.0$$

Because m is negative, the image is inverted. The height of the image is 30.0 times the height of the object, or $(30.0)(5.00\ \text{mm}) = 150\ \text{mm}$.

3-6 Convex Mirrors:

In Fig.10 the convex side of a spherical mirror faces the incident light. The center of curvature is on the side opposite to the outgoing rays; R is negative. Therefore P' is the image of P. The object distance s is positive, the image distance s' is negative, and the radius of curvature R is negative for a convex mirror. $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$

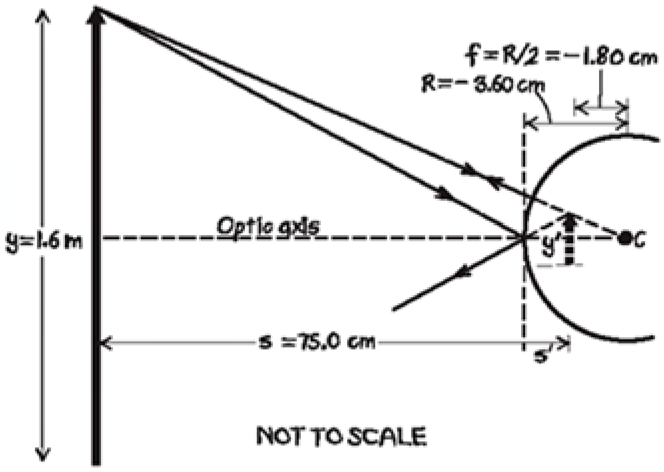
and the lateral magnification is: $m = \frac{y'}{y} = -\frac{s'}{s}$



(a) Construction for finding the position of an image formed by a convex mirror

Example 1:

Santa checks himself for soot, using his reflection in a shiny silvered Christmas tree ornament 0.750 m away .The diameter of the ornament is 7.20 cm. Standard reference works state that he is a "right jolly old elf," so we estimate his height to be 1.6 m. Where and how tall is the image of Santa formed by the ornament? Is it erect or inverted?



Solution:

is $R = -(7.20\,\text{cm})/2 = -3.60\,\text{cm}$, and the focal length is $f = R/2 = -1.80\,\text{cm}$.

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-1.80\,\text{cm}} - \frac{1}{75.0\,\text{cm}}$$

$$s' = -1.76\,\text{cm}$$

$$m = \frac{y'}{y} = -\frac{s'}{s} = -\frac{-1.76\,\text{cm}}{75.0\,\text{cm}} = 0.0234$$

$$y' = my = (0.0234)(1.6\,\text{m}) = 3.8 \times 10^{-2}\,\text{m} = 3.8\,\text{cm}$$

3-7 Image of a Point Object: Spherical Refracting Surface :

In Fig.11 a spherical surface with radius R forms an interface between two materials with different indexes of refraction **na** and **nb**. The surface forms an image P' of an object point P.

Applying this to the triangles **PBC** and **P'BC** gives:

$$\theta_a = \alpha + \phi \qquad \phi = \beta + \theta_b \qquad \text{……………1}$$

From the law of refraction,

$$n_a \sin\theta_a = n_b \sin\theta_b \qquad \text{…………..2}$$

Also, the tangents of α, β, and ϕ are

$$\tan\alpha = \frac{h}{s+\delta} \qquad \tan\beta = \frac{h}{s'-\delta} \qquad \tan\phi = \frac{h}{R-\delta} \qquad \text{…3}$$

$$n_a\theta_a = n_b\theta_b \qquad \text{………………4}$$

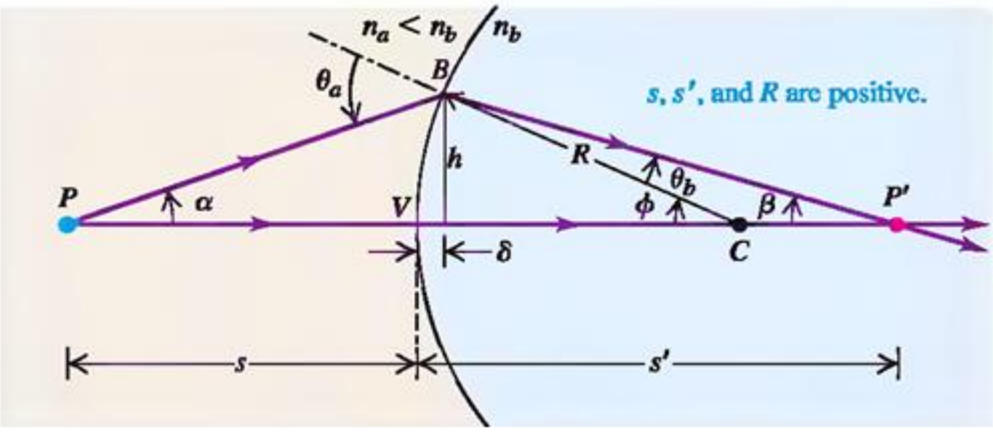
$$\theta_b = \frac{n_a}{n_b}(\alpha + \phi) \qquad \text{………………5}$$

$$n_a\alpha + n_b\beta = (n_b - n_a)\phi \qquad \text{……………6}$$

$$\alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R} \qquad \text{…………7}$$

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$ (object–image relationship, spherical refracting surface)

Fig11



To obtain the lateral magnification m for this situation From the triangles PQV and P'Q'V:

$$\tan\theta_a = \frac{y}{s} \qquad \tan\theta_b = \frac{-y'}{s'}$$

and from the law of refraction,

$$n_a \sin\theta_a = n_b \sin\theta_b$$

For small angles,

$$\tan\theta_a = \sin\theta_a \qquad \tan\theta_b = \sin\theta_b$$

so finally

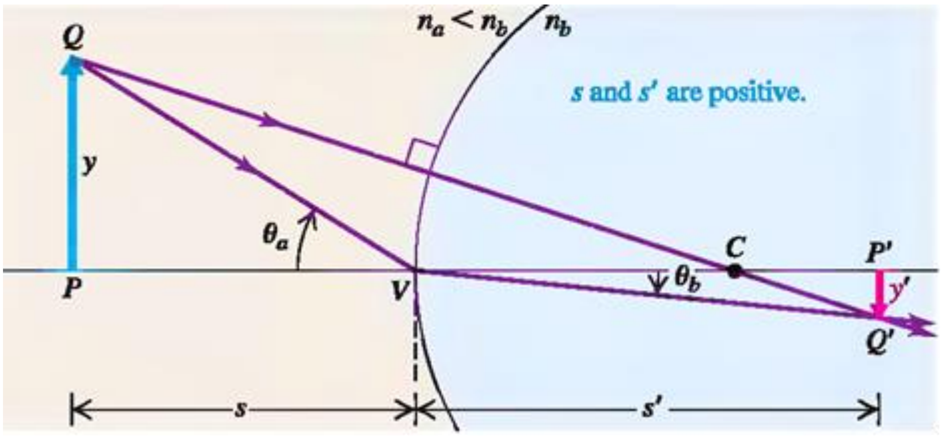
$$\frac{n_a y}{s} = -\frac{n_b y'}{s'} \qquad \text{or}$$

$$m = \frac{y'}{y} = -\frac{n_a s'}{n_b s} \qquad \text{(lateral magnification, spherical refracting surface)}$$

$$\frac{n_a}{s} + \frac{n_b}{s'} = 0 \qquad \text{(plane refracting surface)}$$

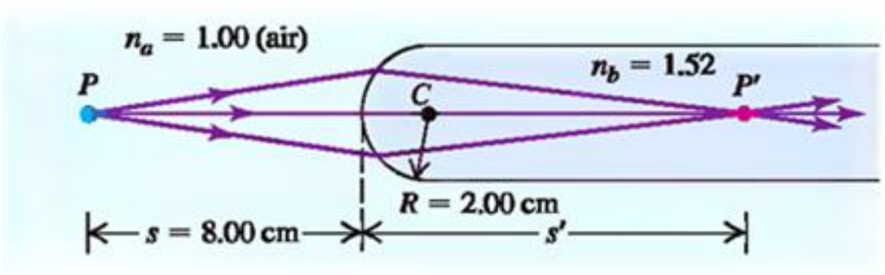
m=1

Fig.12



Example 2 :

A cylindrical glass rod in air has index of refraction 1.52. One end is ground to a hemispherical surface with radius R = 2.00 cm. (a) Find the image distance of a small object on the axis of the rod, 8.00 cm to the left of the vertex. (b) Find the lateral magnification.



Solution:

$$\frac{1.00}{8.00\text{ cm}} + \frac{1.52}{s'} = \frac{1.52 - 1.00}{+2.00\text{ cm}}$$

$$s' = +11.3\text{ cm}$$

$$m = -\frac{n_a s'}{n_b s} = -\frac{(1.00)(11.3\text{ cm})}{(1.52)(8.00\text{ cm})} = -0.929$$

Example3 :

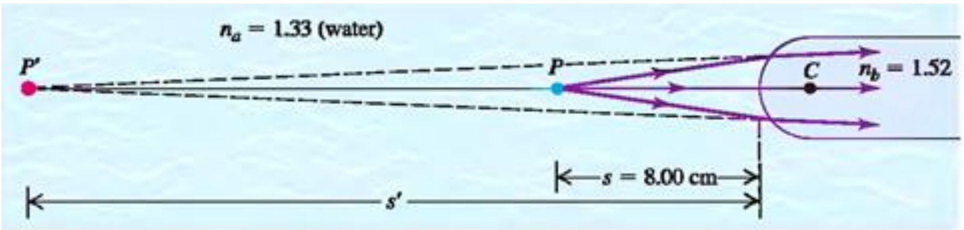
The glass rod in Example above is immersed in water (index of refraction n = 1.33), as shown in Fig. The other quantities have the same values as before. Find the image distance and lateral magnification.

Solution:

$$\frac{1.33}{8.00\text{ cm}} + \frac{1.52}{s'} = \frac{1.52 - 1.33}{+2.00\text{ cm}}$$

$$s' = -21.3\text{ cm}$$

$$m = -\frac{(1.33)(-21.3\text{ cm})}{(1.52)(8.00\text{ cm})} = +2.33$$



Chapter Four: Lenses and Optical Instrument

4-1 Image as Object :

The most familiar and widely used optical device (after the plane mirror) is the lens. A lens is an optical system with two refracting surfaces. The simplest lens has two spherical surfaces close enough together that we can neglect the distance between them (the thickness of the lens); we call this a **thin lens**.

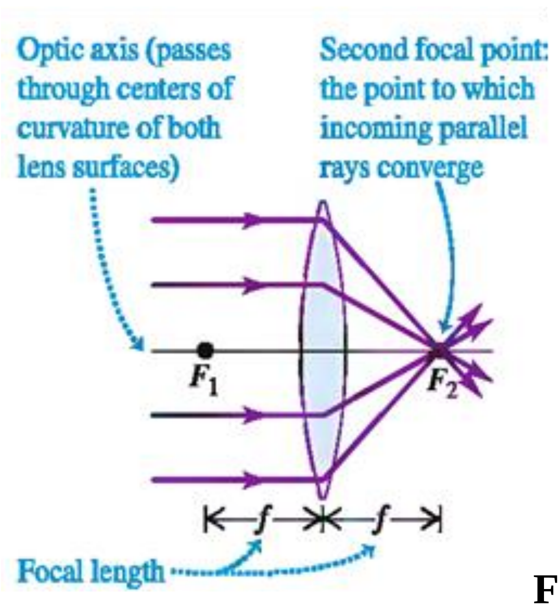
4-2 converging lens:

When a beam of rays parallel to the axis passes through the lens, the rays converge to a point **F2** and form a real image at that point. Such a lens is called a converging lens. Similarly, rays passing through point **F1**emerge from the lens as a beam of parallel rays Fig.13 .

The points Fl and F2 are called **the first and second focal points**.

The distance **f** (measured from the center of the lens) is called **the focal length**.

The focal length of a converging lens is defined to be a positive quantity. The lens is also called a **positive lens**.



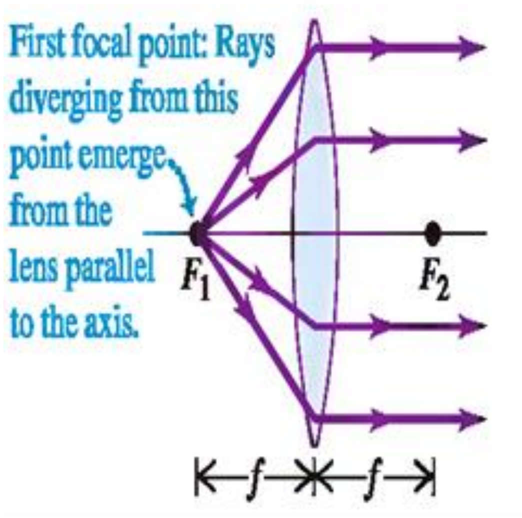


Fig.13

4- 3 Diverging Lenses :

Diverging lens; the beam of parallel rays incident on this lens diverges after refraction. The focal length of a diverging lens is a **negative** quantity, and the lens is also called a **negative lens**.

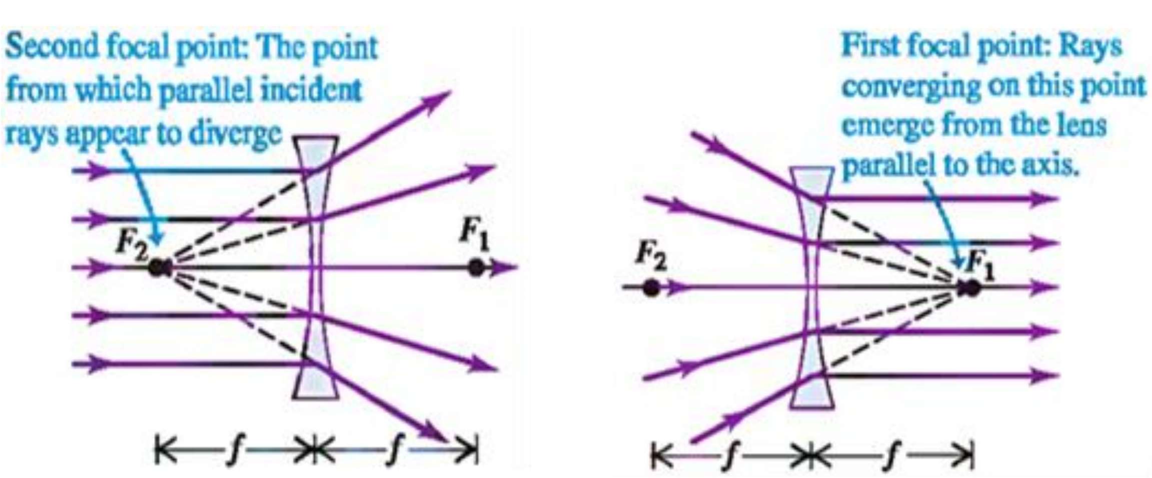


Fig. 14

4-4Image as Object for Lenses:

Image of an Extended Object: Converging Lens:

A converging lens can form an image of an extended object. Fig.15 shows how to find the position and lateral magnification of an image made by a thin converging lens.

Let y and y' be the object and image heights. Ray QA, parallel to the optic axis before refraction, passes through the second focal point F2 after refraction.

There is refraction where the ray enters and leaves the material but no net change in direction. The two angles labeled α are equal.

Therefore the two right triangles **PQO** and **P'Q'O** are similar, and ratios of corresponding sides are equal.

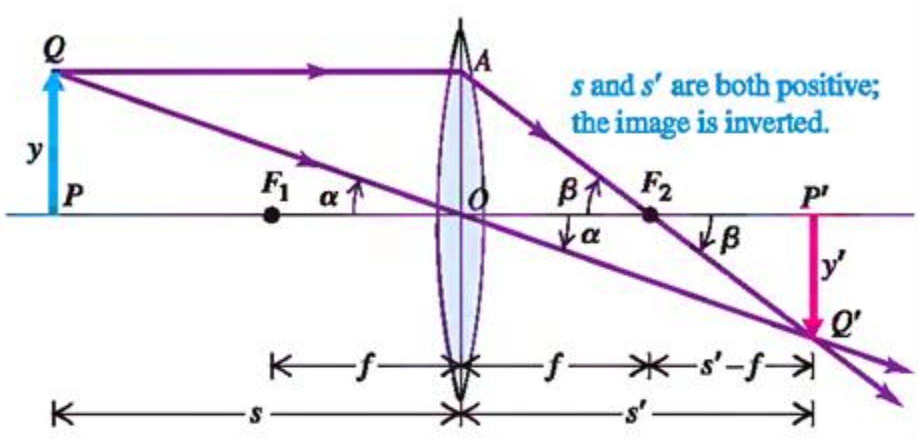


Fig.15

$$\frac{y}{s} = -\frac{y'}{s'} \quad \text{or} \quad \frac{y'}{y} = -\frac{s'}{s}$$

Also, the two angles labeled **β** are equal, and the two right triangles **OAF_2** and **$P'Q'F_2$** are similar, so

$$\frac{y}{f} = -\frac{y'}{s'-f} \quad \text{or}$$

Divide by s':

$$\frac{y'}{y} = -\frac{s'-f}{f}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \text{(object–image relationship, thin lens)}$$

$$m = -\frac{s'}{s} \quad \text{(lateral magnification, thin lens)}$$

4-5 The Lensmaker's Equation :

Lensmaker's equation, which is a relationship among the focal length **f**, the index of refraction n of the lens, and the radii of curvature **Rl** and $\mathbf{R_2}$ of the lens surfaces.

Two spherical interfaces separating three materials with indexes of refraction **na, nb and nc** , as shown in Fig.16. The object and image distances for the first surface are $\mathbf{s_l}$ and $\mathbf{s'_1}$and those for the second surface are $\mathbf{s_2}$ and $\mathbf{s'_2}$. We assume that the lens is thin, so that the distance t between the two surfaces is small in comparison with the object and

$$\frac{n_a}{s_1} + \frac{n_b}{s'_1} = \frac{n_b - n_a}{R_1}$$

$$\frac{n_b}{s_2} + \frac{n_c}{s'_2} = \frac{n_c - n_b}{R_2}$$

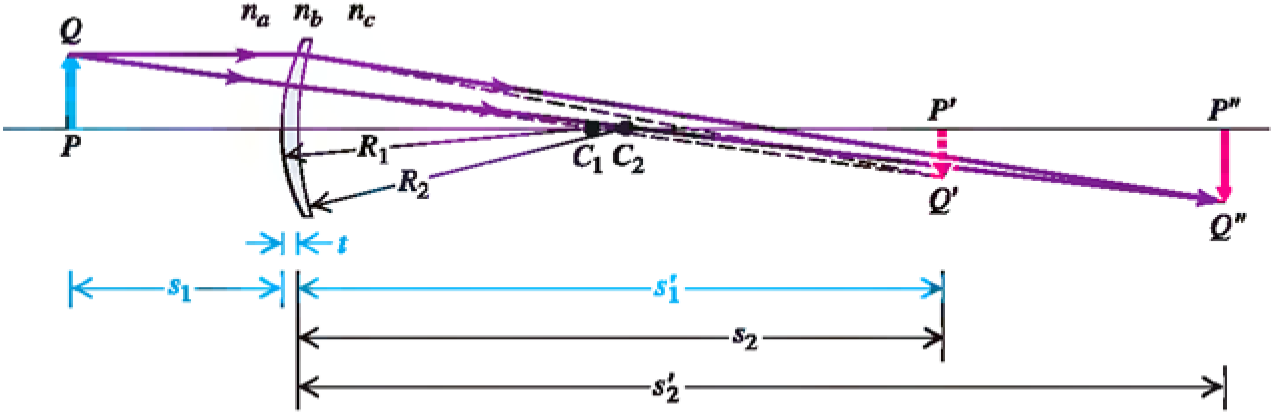


Fig.16

na = nc = 1. The second index nb is that of the lens, which we can call simply **n**. Substituting these values and the relationship $\mathbf{s_2=-s'_1}$ we get:

$$\frac{1}{s_1} + \frac{n}{s'_1} = \frac{n-1}{R_1}$$

$$-\frac{n}{s'_1} + \frac{1}{s'_2} = \frac{1-n}{R_2}$$

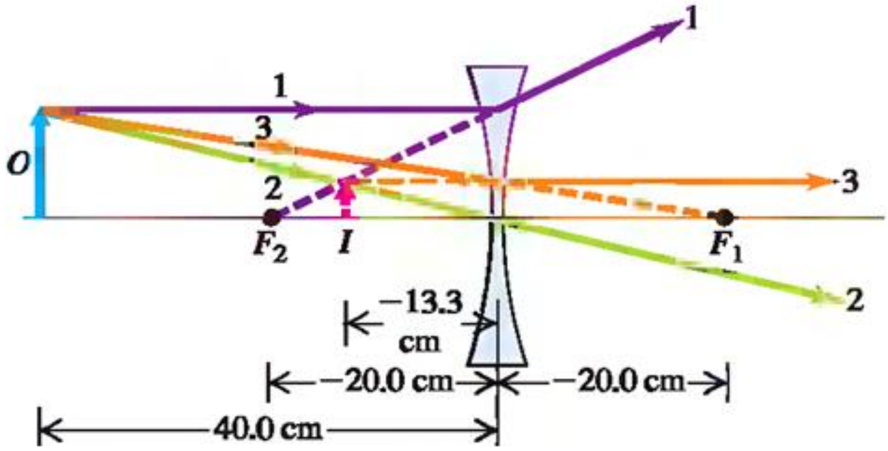
$$\frac{1}{s_1} + \frac{1}{s'_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{(lensmaker's equation for a thin lens)}$$

Example :

You are given a thin diverging lens. You find that a beam of parallel rays spreads out after passing through the lens, as though all the rays came from a point 20.0 cm from the center of the lens. You want to use this lens to form an erect virtual image that is l/3 the height of the object. Where the object should be placed?



m = +l = -s'/s
so s' = -s/3

$$\frac{1}{s} + \frac{1}{-s/3} = \frac{1}{-20.0\,\text{cm}}$$

$$s = 40.0\,\text{cm}$$

$$s' = -\frac{s}{3} = -\frac{40.0\,\text{cm}}{3} = -13.3\,\text{cm}$$

4-6 Cameras:

1-The basic elements of a camera are a light-tight box .

2-A converging lens.

3- Shutter to open the lens for a prescribed length of time .

4-Light-sensitive recording medium. In a digital camera this is an electronic detector called a chargecoupled device (CCD) array; in an older camera, this is photographic film .

5- The lens forms an inverted real image on the recording medium of the object being photographed. High-quality camera lenses have several elements.

Camera Lenses: Focal Length:

The choice of the focal length f for a camera lens depends on the film size and the desired angle of view. A lens of long focal length, called a **telephoto lens**, gives a small angle of view and a large image of a distant object .A lens of short focal length gives a small image and a wide angle of view and is called **a wide-angle lens**. The ratio of the image height y' to the object height y (the lateral magnification) is equal in absolute value to the ratio of image distance s' to the object distance s.

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Camera Lenses f –Number:

For the film to record the image properly, the total light energy per unit area reaching the film (the "**exposure**") must fall within certain limits. This is controlled by the shutter and the lens aperture.

The shutter controls the time interval during which light enters the lens. This is usually adjustable in steps corresponding to factors of about 2, often from 1 s to 1/1000 s.

The intensity of light reaching the film is proportional to the area viewed by the camera lens and to the effective area of the lens. The size of the area that the lens "sees" is proportional to the square of the angle of view of the lens, and so is roughly proportional to $\mathbf{l/f_2}$.

The effective area of the lens is controlled by means of an adjustable lens aperture, hence the effective area is proportional to **D2**.

the intensity of light reaching the film with a particular lens is proportional to $\mathbf{D_2/f_2}$. The ratio **f/D**, called the f-number of the lens:

$$f\text{-number} = \frac{\text{Focal length}}{\text{Aperture diameter}} = \frac{f}{D}$$

4-7 Lens Aberrations:

The aberration may be described in terms of the deviation of the deformed wave front from the ideal at various distances from the optical axis. High-quality camera lenses have several elements, permitting partial correction of various aberrations, including the dependence of index of refraction on wavelength and the limitations imposed by the paraxial approximation. There are different types for aberration :

1-Spherical Aberration

2-Coma Aberration

3-Astigmatism Aberration

4-Curvature of field Aberration

5-Distortion Aberration

6- Chromatic Aberration

4-8 The Magnifier:

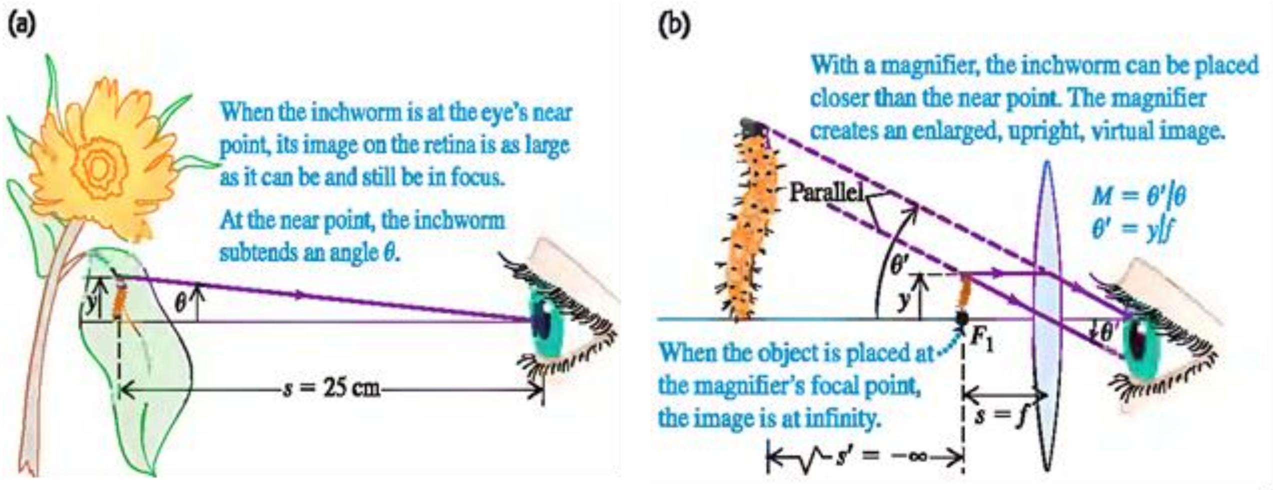
The apparent size of an object is determined by the size of its image on the retina, this size depends on the angle θ subtended by the object at the eye, called its **angular size** .

To look closely at a small object, such as an insect or a crystal, you bring it close to your eye, making the subtended angle and the retinal image as large as possible .

But your eye cannot focus sharply on objects that are closer than the near point, so the angular size of an object is greatest when it is placed at the near point. In the following discussion we will assume an average viewer for whom the near point is 25cm from the eye .

Aconverging lens can be used to form a virtual image that is larger and farther from the eye than the object itself, object can be moved closer to the eye, and the angular size of the image may be substantially larger than the angular size of the object at 25 cm without the lens .

A lens used in this way is called a **magnifier**, or a **simple magnifier**. The virtual image is most comfortable to view when it is placed at infinity , this means that the object is placed at the focal point **F1** of the magnifier. The object is at the near point, where it subtends an angle **θ** at the eye. A magnifier in front of the eye forms an image at infinity and the angle subtended at the magnifier is **θ'**.



$$M = \frac{\theta'}{\theta} \qquad \text{(angular magnification)}$$

$$\theta = \frac{y}{25\text{ cm}} \qquad \theta' = \frac{y}{f}$$

$$M = \frac{\theta'}{\theta} = \frac{y/f}{y/25\text{ cm}} = \frac{25\text{ cm}}{f} \qquad \text{(angular magnification for a simple magnifier)}$$

4-9 Microscopes:

The instrument that we usually use is the microscope.

The object to be viewed is placed just beyond the first focal point $\mathbf{F_1}$ of the objective, a converging lens that forms a real and enlarged image.

In a properly designed instrument this image lays just inside the first focal point $\mathbf{F'_1}$of a second converging lens called the eyepiece or ocular.

The eyepiece acts as a simple magnifier, and forms a final virtual image **I′** of I. The position of I′ may be anywhere between the near and far points of the eye.

The overall angular magnification of the microscope is the product of two factors. The first factor is the lateral magnification $\mathbf{m_l}$ of the objective, which determines the linear size of the real image **I**; the second factor is the angular magnification $\mathbf{M_2}$ of the eyepiece, which relates the angular size of the virtual image seen through the eyepiece to the angular size that the real image I would have if you viewed the eyepiece. The first of these factors is given by:

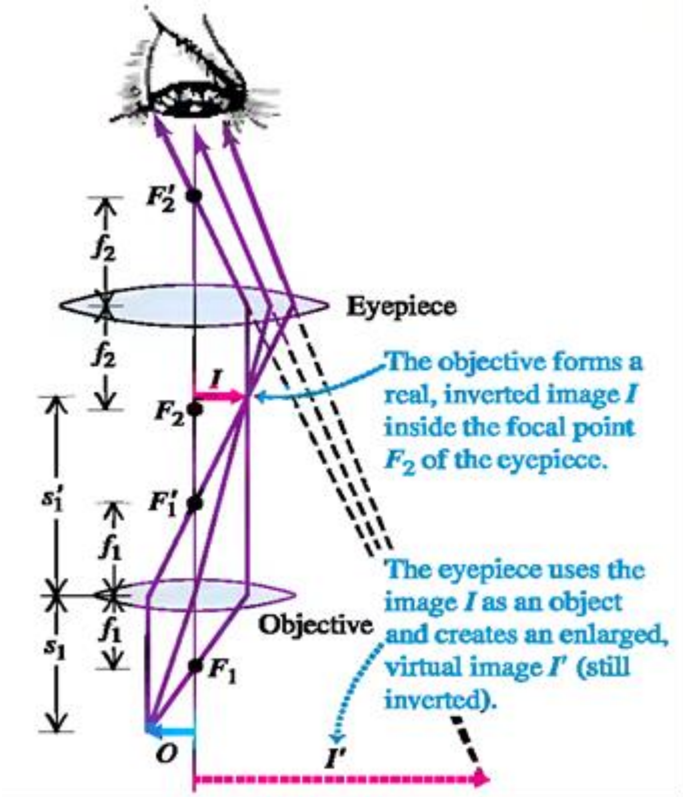
$$m_1 = -\frac{s'_1}{s_1}$$

$\mathbf{s'_1}$ is very great in comparison to the focal length $\mathbf{f_l}$ of the objective lens. Thus $\mathbf{S_l}$ is approximately equal to$\mathbf{f_1}$ and we can write

$$\mathbf{m_l = - s'_1/f_1}$$

$$M_2 = (25\text{ cm})/f_2$$

$$M = m_1 M_2 = \frac{(25\text{ cm})s'_1}{f_1 f_2} \qquad \text{(angular magnification for a microscope)}$$



4-10Telescopes :

The optical system of a telescope is similar to that of a microscope. The key difference is that the telescope is used to view large objects at large distances and the microscope is used to view small objects close at hand. Another difference is that many telescopes use a curved mirror, not a lens, as an objective.

Because this telescope uses a lens as an objective, it is called a **refracting telescope or refractor**.

The objective lens forms a real, reduced image **I** of the object. This image is the object for the eyepiece lens, which forms an enlarged, virtual image of I. If the final image **I'** formed by the eyepiece is at infinity the first image must also be at the first focal point of the eyepiece.

The distance between objective and eyepiece, which is the length of the telescope, is therefore the sum of the focal lengths of objective and eyepiece:

$d = f_1 + f_2$

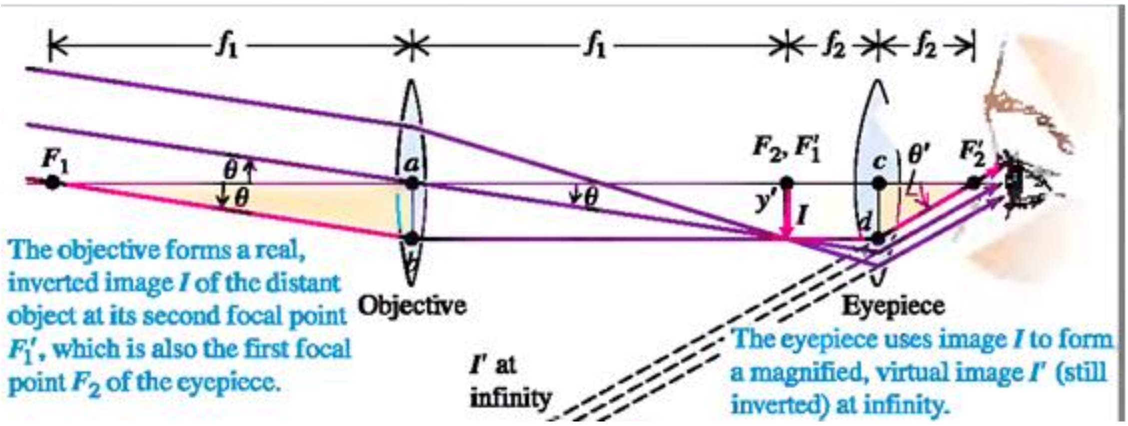
Because the angles **θ** and **θ'** are small, they may be approximated by their tangents. From the right triangles $\mathbf{F_1ab}$ and $\mathbf{F_2'cd}$:

$$\theta = \frac{-y'}{f_1} \qquad \theta' = \frac{y'}{f_2}$$

and the angular magnification *M* is

$$M = \frac{\theta'}{\theta} = -\frac{y'/f_2}{y'/f_1} = -\frac{f_1}{f_2} \qquad \text{(angular magnification for a telescope)}$$

The angular magnification **M** of a telescope is defined as the ratio of the focal length of the objective to that of the eyepiece.



In the reflecting telescope the objective lens is replaced by a concave mirror. In large telescopes this scheme has many advantages:

1- Mirrors are inherently free of chromatic aberrations dependence of focal length on wavelength
2- Spherical aberrations are easier to correct than with a lens.
3- The reflecting surface is sometimes parabolic rather than spherical.
4- The material of the mirror need not be transparent, and it can be made more rigid than a lens which has to be supported only at its edges.