

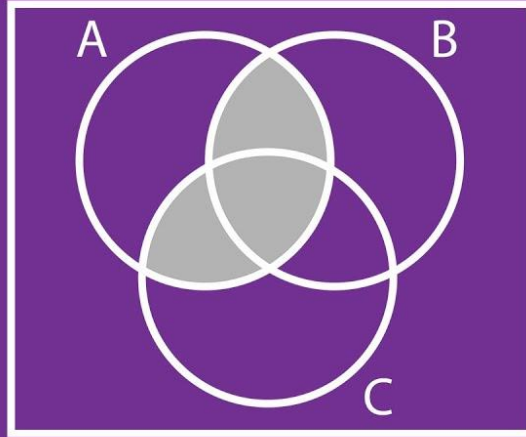
*Prepared by Assist. Prof. Imad Matti*

# هياكل متقطعة

## المرحلة الاولى

**MATHS ACADEMY**

## Set Theory



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# Basic Structures

A **set** is an unordered collection of objects.


The objects in a set are called the *elements*, or *members*, of the set. A set is said to contain its elements.

$$S = \{a, b, c, d\}$$

We write  $a \in S$  to denote that  $a$  is an element of the set  $S$ . The notation  $e \notin S$  denotes that  $e$  is not an element of the set  $S$ .

The set  $O$  of odd positive integers less than 10 can be expressed by  $O = \{1, 3, 5, 7, 9\}$ .

The set of positive integers less than 100 can be denoted by  $\{1, 2, 3, \dots, 99\}$ .



ellipses (...)

Another way to describe a set is to use **set builder** notation.

The set  $O$  of odd positive integers less than 10 can be expressed by  $O = \{1, 3, 5, 7, 9\}$ .

$$O = \{x \mid x \text{ is an odd positive integer less than } 10\},$$

$$O = \{x \in \mathbf{Z}^+ \mid x \text{ is odd and } x < 10\}.$$

$\mathbf{N} = \{0, 1, 2, 3, \dots\}$ , the set of all **natural numbers**

$\mathbf{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ , the set of all **integers**

$\mathbf{Z}^+ = \{1, 2, 3, \dots\}$ , the set of all **positive integers**

$\mathbf{Q} = \{p/q \mid p \in \mathbf{Z}, q \in \mathbf{Z}, \text{ and } q \neq 0\}$ ,  
the set of all **rational numbers**

$\mathbf{R}$ , the set of all **real numbers**

$\mathbf{R}^+$ , the set of all **positive real numbers**

$\mathbf{C}$ , the set of all **complex numbers**.

### Interval Notation

Closed interval  $[a, b]$

Open interval  $(a, b)$

$$[a, b] = \{x \mid a \leq x \leq b\}$$

$$[a, b) = \{x \mid a \leq x < b\}$$

$$(a, b] = \{x \mid a < x \leq b\}$$

$$(a, b) = \{x \mid a < x < b\}$$

If  $A$  and  $B$  are sets, then  $A$  and  $B$  are equal if and only if  $\forall x(x \in A \leftrightarrow x \in B)$ . We write  $A = B$ , if  $A$  and  $B$  are equal sets.

- The sets  $\{1, 3, 5\}$  and  $\{3, 5, 1\}$  are equal, because they have the same elements.
- $\{1, 3, 3, 5, 5, 5\}$  is the same as the set  $\{1, 3, 5\}$  because they have the same elements.

## Empty Set

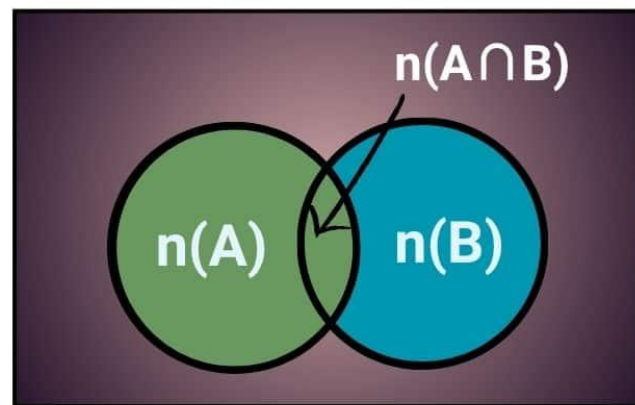
There is a special set that has no elements. This set is called the empty set, or null set, and is denoted by  $\emptyset$ .

The empty set can also be denoted by  $\{ \}$

# Cardinality

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Cardinal Number  
Of Some Sets



$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

The number of elements in a set is called the cardinality of a set. For example, let  $A = \{h, i, j, k, l\}$ . Then the cardinality of set  $A$  is denoted by  $n(A)$ . There are 5 elements in set  $A$ . So  $n(A) = 5$ .

## Cardinality

The cardinality is the number of distinct elements in  $S$ .  
The cardinality of  $S$  is denoted by  $|S|$ .

### Example1

$$S = \{a, b, c, d\}$$

$$|S| = 4$$

$$A = \{1, 2, 3, 7, 9\}$$

$$\emptyset = \{ \}$$

### Example2

$$S = \{a, b, c, d, \{2\}\}$$

$$|S| = 5$$

$$A = \{1, 2, 3, \{2,3\}, 9\}$$

$$|A| = 5$$

$$\{\emptyset\} = \{\{ \}\}$$

$$|\{\emptyset\}| = 1$$

If two or more sets are combined using the operations on sets, using the **formula based on the cardinality of sets**, we can calculate the cardinality.

Formulas based on the cardinality of sets are given below.

1. Let A and B be finite sets and  $(A \cap B) \neq \varnothing$ , then

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ .  $\cup$  denotes union and  $\cap$  denotes intersection.

To find the cardinality of  $A \cup B$ , we subtract the number of common elements of A and B from the sum of the cardinality of A and B.

2. If the sets A and B are disjoint sets (A and B do not have any common elements),  $n(A \cap B) = 0$

$n(A \cup B) = n(A) + n(B)$ .

**problem:** In a group of **30** students, **18** teach English (E) and **7** teach both English and Spanish (S). How many teach Spanish?.

Given  $n(E \cup S) = 30$ .

$n(E) = 18$ .

$n(E \cap S) = 7$ .

Formula:  $n(E \cup S) = n(E) + n(S) - n(E \cap S)$ .

Solution:  $30 = 18 + n(S) - 7$

$30 = 11 + n(S)$

$n(S) = 30 - 11 = 19$

**Problem:** In a party of **45** people, each one likes tea or coffee or both. **35** people like tea and **20** people like coffee. Find the number of people who like both tea and coffee.

**Answer = 10.**

**Example 1:**

In a college, there are 25 teachers who teach physics or mathematics. Of these, 15 teach physics, and 6 teach both physics and mathematics. How many teach mathematics?

**Solution:**

Let **A** denotes the number of teachers who teach physics, and **B** denotes the number of teachers who teach mathematics.

Total number of teachers who teach physics or math,  $n(A \cup B) = 25$

Number of physics teachers,  $n(A) = 15$

Number of teachers who teach both mathematics and physics,  $n(A \cap B) = 6$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\Rightarrow 25 = 15 + n(B) - 6$$

$$\Rightarrow n(B) = 16$$

**Exercises:**

1. (i) If  $n(A) = 25$ ,  $n(B) = 40$ ,  $n(A \cup B) = 50$ , find  $n(A \cap B)$ .

(ii) If  $n(A) = 300$ ,  $n(A \cup B) = 500$ ,  $n(A \cap B) = 50$ , find  $n(B)$ .

## Infinite

A set is said to be **infinite** if it is not finite.  
The set of positive integers is infinite.

$$\mathbb{Z}^+ = \{1, 2, 3, \dots\}$$

## Subset

The set  $A$  is said to be a subset of  $B$  if and only if every element of  $A$  is also an element of  $B$ .

We use the notation  $A \subseteq B$  to indicate that  $A$  is a subset of the set  $B$ .

$$(A \subseteq B) \equiv (B \supseteq A)$$

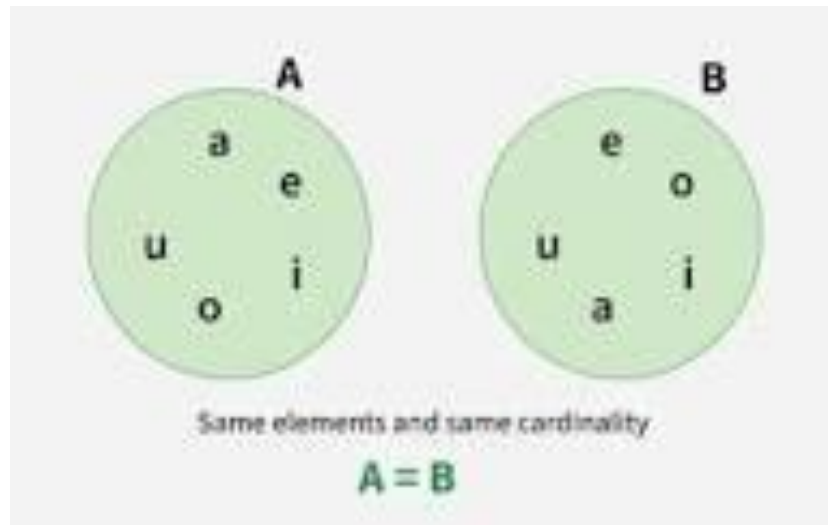
$$A \subseteq B \leftrightarrow \forall x(x \in A \rightarrow x \in B)$$

## Subset

For every set  $S$ ,

$$(i) \emptyset \subseteq S \quad \text{and} \quad (ii) S \subseteq S.$$

To show that two sets  $A$  and  $B$  are equal, show that  $A \subseteq B$  and  $B \subseteq A$ .



## Proper Subset

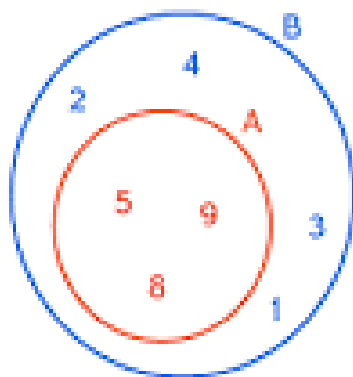
The set  $A$  is a subset of the set  $B$  but that  $A \neq B$ , we write  $A \subset B$  and say that  $A$  is a **proper subset** of  $B$ .

$$A \subset B \leftrightarrow (\forall x(x \in A \rightarrow x \in B) \wedge \exists x(x \in B \wedge x \notin A))$$

## Proper Subset



A subset that contains some, but not all, elements of another set



here,

$$A = \{5, 8, 9\}$$

$$B = \{1, 2, 3, 4, 5, 8, 9\}$$

$$A \subset B$$

## Example

For each of the following sets,  
determine whether 3 is an element of that set.

$\{1,2,3,4\}$

$\{\{1\}, \{2\}, \{3\}, \{4\}\}$

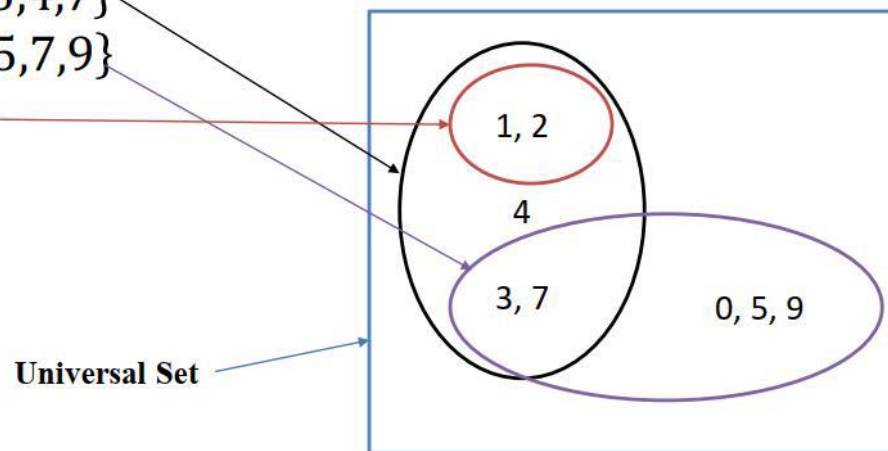
$\{1,2, \{1,3\}\}$

## Venn Diagram

$A = \{1,2,3,4,7\}$

$B = \{0,3,5,7,9\}$


$C = \{1,2\}$



## Power set

$\{a, b, c\}$

$\left\{ \begin{array}{l} \{\}, \\ \{a\}, \{b\}, \{c\}, \\ \{a, b\}, \{a, c\}, \{b, c\}, \\ \{a, b, c\} \end{array} \right\}$



# Counting Subsets with Induction

$$|A| = n$$
$$|P(A)| = 2^n$$

## Power Set

**The set of all subsets.**

If the set is  $S$ . The power set of  $S$  is denoted by  $P(S)$ .

The number of elements in the power set is  $2^{|S|}$

$$S = \{1,2,3\}$$

$$|P(S)| = 2^3 = 8 \text{ elements}$$

$$P(S) = 2^S$$

$$= \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

### **Example1**

What is the power set of the empty set?

$$\mathcal{P}(\emptyset) = \{\emptyset\}.$$

### **Example2**

What is the power set of the set  $\{\emptyset\}$ ?

$$\mathcal{P}(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}.$$

## Cartesian Products

Let  $A$  and  $B$  be sets.

The **Cartesian product** of  $A$  and  $B$ , denoted by  $A \times B$ , is the set of all ordered pairs  $(a, b)$ , where  $a \in A$  and  $b \in B$ . Hence,  $A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$ .

### Cartesian Products - Example

Let  $A = \{1, 2\}$ , and  $B = \{a, b, c\}$

$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$ .

$$|A \times B| = |A| * |B| = 2 * 3 = 6$$

Find  $B \times A$  ?

### The Cartesian product of more than two sets.

The Cartesian product of the sets  $A_1, A_2, \dots, A_n$ , denoted by  $A_1 \times A_2 \times \dots \times A_n$ , is the set of ordered  $n$ -tuples  $(a_1, a_2, \dots, a_n)$ , where  $a_i$  belongs to  $A_i$  for  $i = 1, 2, \dots, n$ . In other words,

$$A_1 \times A_2 \times \dots \times A_n = \{(a_1, a_2, \dots, a_n) \mid a_i \in A_i \text{ for } i = 1, 2, \dots, n\}.$$

#### Example:

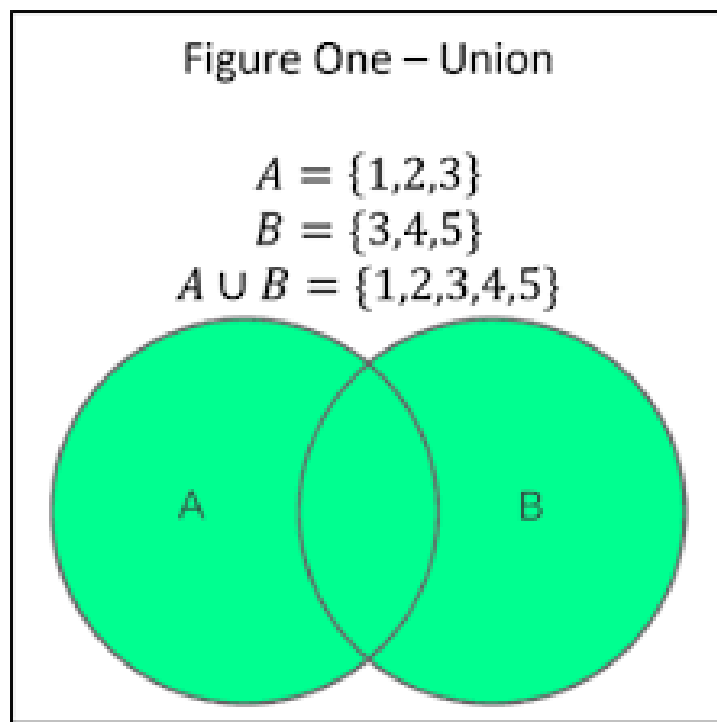
$A \times B \times C$ , where  $A = \{0, 1\}$ ,  $B = \{1, 2\}$ , and  $C = \{0, 1, 2\}$

$$A \times B \times C = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2)\}.$$

## Union

Let  $A$  and  $B$  be sets. The **union** of the sets  $A$  and  $B$ , denoted by  $A \cup B$ , is the set that contains those elements that are either in  $A$  or in  $B$ , or in both.

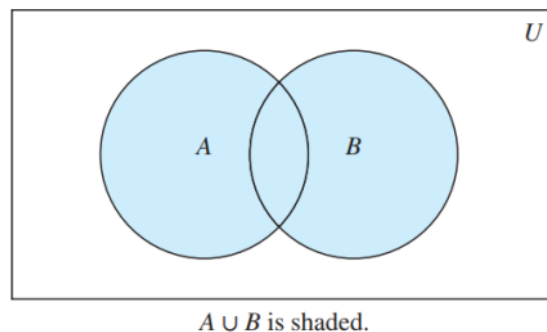
$$A \cup B = \{x \mid x \in A \vee x \in B\}$$



# Set Operations

## Union

Let  $A$  and  $B$  be sets. The **union** of the sets  $A$  and  $B$ , denoted by  $A \cup B$ , is the set that contains those elements that are either in  $A$  or in  $B$ , or in both.

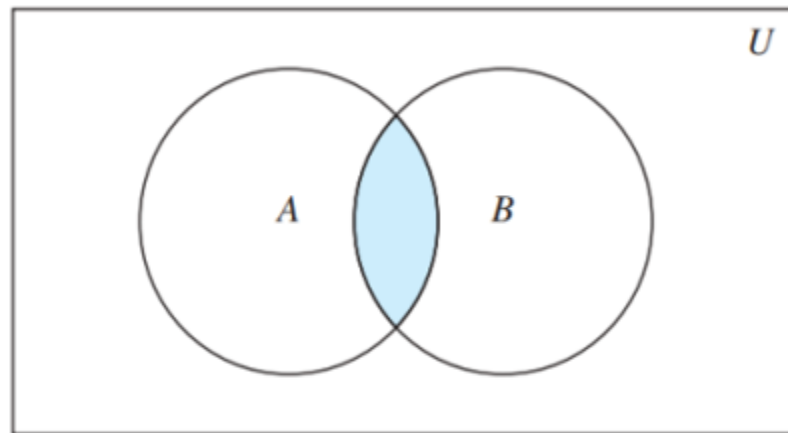


The union of the sets  $\{1, 3, 5\}$  and  $\{1, 2, 3\}$  is the set  $\{1, 2, 3, 5\}$

## Intersection

Let  $A$  and  $B$  be sets. The **intersection** of the sets  $A$  and  $B$ , denoted by  $A \cap B$ , is the set that contains those elements that are in both  $A$  and  $B$ .

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$



$A \cap B$  is shaded.

The intersection of the sets  $\{1, 3, 5\}$  and  $\{1, 2, 3\}$  is the set  $\{1, 3\}$

## Disjoint

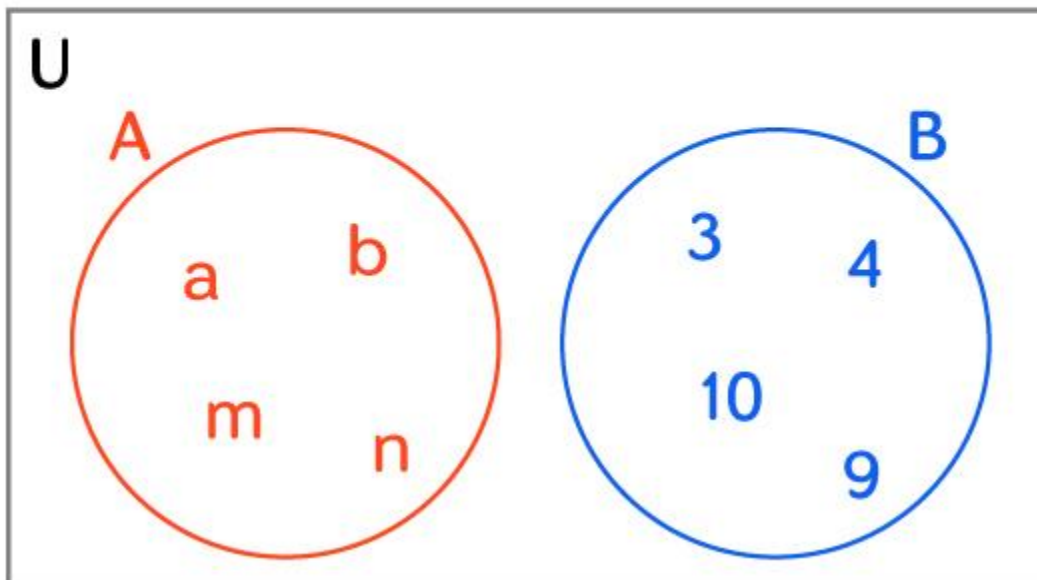
Two sets are called **disjoint** if their intersection is the empty set.

$$A \cap B = \emptyset$$

## Disjoint Sets



Sets that share no common elements between them



## Difference

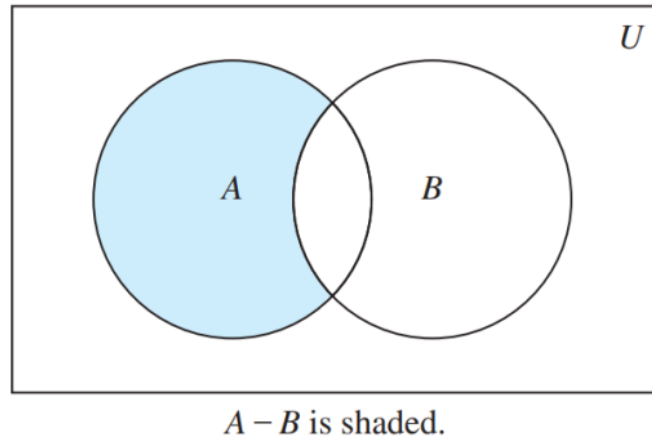
Let  $A$  and  $B$  be sets. The difference of  $A$  and  $B$ , denoted by  $A - B$ , is the set containing those elements that are in  $A$  but not in  $B$ .

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

$$A = \{1,3,5\}, \quad B = \{1,2,3\}$$

$$A - B = \{5\}$$

## Difference



## Complement

Let  $U$  be the universal set.

The complement of the set  $A$ , denoted by  $\bar{A}$

An element  $x$  belongs to  $U$  if and only if  $x \notin A$ .

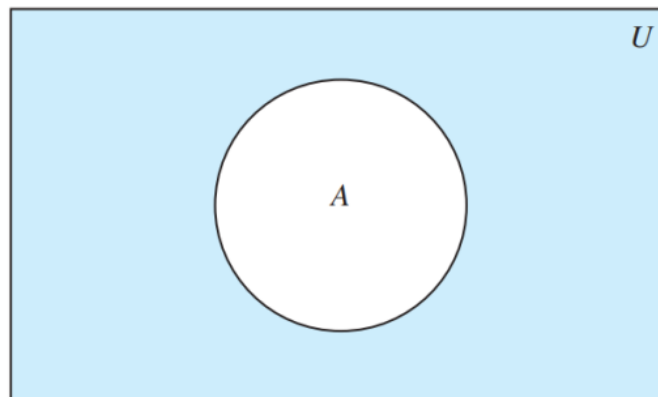
$$\bar{A} = \{x \in U \mid x \notin A\}$$

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$$U = \{1,2,3,4,5\}, \quad A = \{1,3\}$$

$$\bar{A} = \{2,4,5\}$$

## Complement

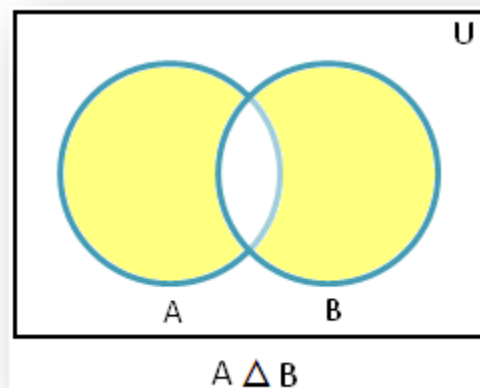


$\bar{A}$  is shaded.

## Symmetric Difference

### Venn Diagram For Symmetric Difference

The symmetric difference  $A \Delta B$  includes elements that are in either A or B but not in both.



**Example:** Let set  $A = \{1, 2, 3\}$ , and set  $B = \{3, 4, 5\}$ , then Find the symmetric difference

**Solution:**

$$A \Delta B = \{1, 2, 4, 5\}$$

The symmetric difference between two sets **A** and **B** is denoted as  **$A \Delta B$**  **and** is defined as the set of elements that are in either of set but not in their intersection. Mathematically, this can be expressed as:

$$A \Delta B = (A - B) \cup (B - A)$$

Where,

- $A - B$  is a set of elements in A, but not in B,
- $B - A$  is a set of elements in B, but not in A.

The **symmetric difference** can also be understood as union of both sets minus their intersection:

$$A \Delta B = (A \cup B) - (A \cap B)$$

## How to Find the Symmetric Difference?

To find Symmetric difference between the two sets **A** and **B**, follow these steps:

1. Find elements that are unique to set A, which is  $A - B$ .
2. Find elements that are unique to set B, which is  $B - A$ .
3. Take Union of these unique elements:  $A \Delta B = (A - B) \cup (B - A)$

**Example: Find symmetric difference between A and B ( $A \Delta B$ ) if  $A = \{1, 2, 3\}$  and  $B = \{2, 3, 4, 5\}$ .**

**Solution:**

If  $A = \{1, 2, 3\}$  and  $B = \{2, 3, 4, 5\}$ :

1.  $A - B = \{1\}$  (elements in A not in B)
2.  $B - A = \{4, 5\}$  (elements in B not in A)
3. Symmetric Difference:  $A \Delta B = \{1, 4, 5\}$

## Properties of Set Operations

### Commutative Property

- **Union:**  $A \cup B = B \cup A$
- **Intersection:**  $A \cap B = B \cap A$
- **Symmetric Difference:**  $A \Delta B = B \Delta A$

## Associative Property

- **Union:**  $(A \cup B) \cup C = A \cup (B \cup C)$
- **Intersection:**  $(A \cap B) \cap C = A \cap (B \cap C)$

## Distributive Property

- **Union over Intersection:**  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- **Intersection over Union:**  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

## Identity Property

- **Union:**  $A \cup \emptyset = A$
- **Intersection:**  $A \cap U = A$ , where  $U$  represents the universal set
- **Symmetric Difference:**  $A \Delta \emptyset = A$

## Complement Property

- **Union:**  $A \cup A' = U$ , where  $U$  is the universal set
- **Intersection:**  $A \cap A' = \emptyset$  (the empty set)

## Absorption Property

- **Union over Intersection:**  $A \cup (A \cap B) = A$
- **Intersection over Union:**  $A \cap (A \cup B) = A$

## Solved Question of Operations on Sets

**Question 1:** Find the union of two sets  $A = \{8, 10, 14\}$  and  $B = \{7, 16\}$

**Solution:**

$$(A \cup B) = \{8, 10, 14\} \cup \{7, 16\}$$

$$(A \cup B) = \{7, 8, 10, 14, 16\}$$

**Question 2:** Find the intersection of sets  $P = \{a, n, x\}$  and  $Q = \{x, y, z\}$

**Solution:**

$$(P \cap Q) = \{a, n, x\} \cap \{x, y, z\}$$

$$(P \cap Q) = \{x\}$$

**Question 3:** Find the complement of set  $X = \{4, 6, 9\}$  where Universal set  $U = \{1, 2, 3, 4, 6, 9\}$

**Solution:**

$$X' = U - X$$

$$X' = \{1, 2, 3, 4, 6, 9\} - \{4, 6, 9\}$$

$$X' = \{1, 2, 3\}$$

**Question 4:** Given two sets  $A = \{5, 6, 9, 10\}$  and  $B = \{3, 6, 12\}$  then find,  $A - B$  and  $B - A$

**Solution:**

$$A - B = \{5, 6, 9, 10\} - \{3, 6, 12\}$$

$$A - B = \{5, 9, 10\}$$

$$B - A = \{3, 6, 12\} - \{5, 6, 9, 10\}$$

$$B - A = \{3, 12\}$$

**Question 5:** Find the number of elements in the set  $(A \cup B)$  given that  $n(A) = 10$ ,  $n(B) = 4$ , and  $n(A \cap B) = 5$ .

**Solution:**

To find  $n(A \cup B)$  we use formula:

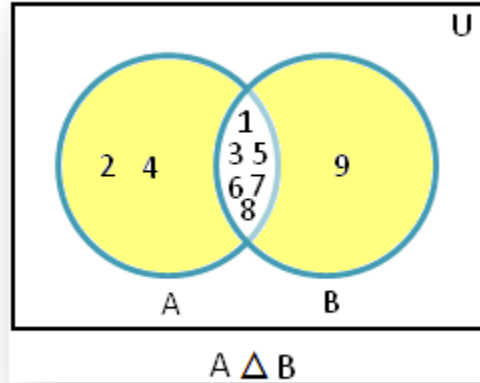
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B) = 10 + 4 - 5$$

$$n(A \cup B) = 9$$

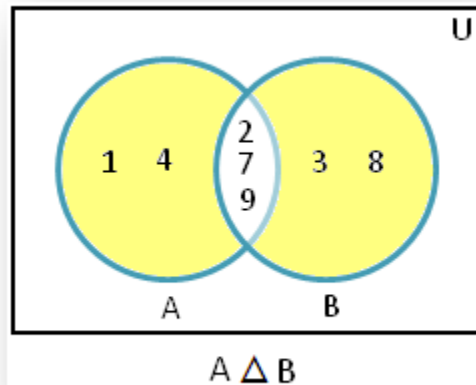
**Example to find the symmetric difference using Venn diagram:**

1. If  $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$  and  $B = \{1, 3, 5, 6, 7, 8, 9\}$ , then  $A - B = \{2, 4\}$ ,  $B - A = \{9\}$  and  $A \Delta B = \{2, 4, 9\}$ .



Therefore, the shaded part of the Venn diagram represents  $A \Delta B = \{2, 4, 9\}$ .

2. If  $A = \{1, 2, 4, 7, 9\}$  and  $B = \{2, 3, 7, 8, 9\}$  then  $A \Delta B = \{1, 3, 4, 8\}$



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Therefore, the shaded part of the Venn diagram represents  $A \Delta B = \{1, 3, 4, 8\}$ .

## What are De Morgan's Laws?

(i)  $(A \cup B)' = A' \cap B'$

(ii)  $(A \cap B)' = A' \cup B'$ .

### De Morgan's Law Example

Let us understand De Morgan's law with the help of a simple example. Let the universal set  $U = \{7, 8, 9, 10, 11, 12, 13\}$ .

The two subsets are given by  $A = \{11, 12, 13\}$  and  $B = \{7, 8\}$ .

- **De Morgan's Law of Union Example:**

- $(A \cup B) = \{7, 8, 11, 12, 13\}$ ,
- $(A \cup B)' = \{9, 10\}$ .
- $A' = \{7, 8, 9, 10\}$  and
- $B' = \{9, 10, 11, 12, 13\}$ .
- $A' \cap B' = \{9, 10\}$ .

- Thus,  $(A \cup B)' = A' \cap B'$

- **De Morgan's Law of Intersection Example:**

- $(A \cap B) = \emptyset$ ,
- $(A \cap B)' = \{7, 8, 9, 10, 11, 12, 13\}$ .
- $A' \cup B' = \{7, 8, 9, 10, 11, 12, 13\}$ .

- Hence,  $(A \cap B)' = A' \cup B'$

## **De Morgan's Law Proof**

### **Proof of $(A \cup B)' = A' \cap B'$ :**

The first **De Morgan's theorem** or law of Union can be proved as follows:

Let  $R = (A \cup B)'$  and  $S = A' \cap B'$ . Our intention now is to prove  $R = S$ . For this, it is sufficient to prove that  $R \subset S$  and  $S \subset R$ .

Suppose we choose an element  $y$  that belongs to  $R$ . This is denoted as  $y \in R$ .

$$\Rightarrow y \in (A \cup B)'$$

$$\Rightarrow y \notin (A \cup B)$$

$$\Rightarrow y \notin A \text{ and } y \notin B$$

$$\Rightarrow y \in A' \text{ and } y \in B'$$

$$\Rightarrow y \in A' \cap B'$$

$$\Rightarrow y \in S$$

Thus, we conclude that  $R \subset S$  ( $R$  is a subset of  $S$ ) ...(1)

Now suppose we have an arbitrary element  $z$  that belongs to set  $S$ . Then  $z \in S$

$$\Rightarrow z \in A' \cap B'$$

$$\Rightarrow z \in A' \text{ and } z \in B'$$

$$\Rightarrow z \notin A \text{ and } z \notin B$$

$$\Rightarrow z \notin (A \cup B)$$

$$\Rightarrow z \in (A \cup B)'$$

$$\Rightarrow z \in R$$

Hence,  $S \subset R$  ...(2)

From (1) and (2) we infer that  $S = R$  or  $(A \cup B)' = A' \cap B'$ . Thus, this theorem is proved.

**Proof of  $(A \cap B)' = A' \cup B'$ :**

The second De Morgan's theorem or law of Intersection can be mathematically proved using the following steps:

Let  $G = (A \cap B)'$  and  $H = A' \cup B'$ . We would now prove that  $G = H$  by proving  $G \subset H$  and  $H \subset G$ .

Let an element  $y$  belong to  $G$ .  $y \in G$ .

$$\Rightarrow y \in (A \cap B)'$$

$$\Rightarrow y \notin (A \cap B)$$

$$\Rightarrow y \notin A \text{ or } y \notin B$$

$$\Rightarrow y \in A' \text{ or } y \in B'$$

$$\Rightarrow y \in A' \cup B'$$

$$\Rightarrow y \in H$$

This implies that  $G \subset H$  ...(1)

If  $z$  is an arbitrary element of  $H$  then  $z \in H$

$$\Rightarrow z \in A' \cup B'$$

$$\Rightarrow z \in A' \text{ or } z \in B'$$

$$\Rightarrow z \notin A \text{ or } z \notin B$$

$$\Rightarrow z \notin (A \cap B)$$

$$\Rightarrow z \in (A \cap B)'$$

$$\Rightarrow z \in G$$

Therefore,  $H \subset G$  ...(2)

Now when we combine (1) and (2) we can say that  $G = H$  or  $(A \cap B)' = A' \cup B'$ . Hence, we have successfully proved the second theorem.

## Examples Using De Morgan's Law

- **Example 1:** If  $U = \{1, 3, 5, 7, 9, 11\}$ ,  $A = \{3, 5\}$  and  $B = \{5, 7, 9\}$ , then prove De Morgan's first law.

**Solution:** According to De Morgan's First law,  $(A \cup B)' = A' \cap B'$

$$(A \cup B) = \{3, 5, 7, 9\}, (A \cup B)' = \{1, 11\}$$

$$A' = \{1, 7, 9, 11\} \text{ and } B' = \{1, 3, 11\}$$

$$A' \cap B' = \{1, 11\}.$$

**Answer:** Hence,  $(A \cup B)' = A' \cap B'$

## Solved Examples of De Morgan's First Law

**Example 1:**

Let  $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ ,  $A = \{3, 4, 5\}$ ,  $B = \{4, 5, 6\}$ . Show that  $(A \cup B)' = A' \cap B'$ .

**Solution:**

Given that:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$A = \{3, 4, 5\}$$

$$B = \{4, 5, 6\}$$

We know that, Demorgan's first law is  $(A \cup B)' = A' \cap B'$ .

**L.H.S:**

$$A \cup B = \{3, 4, 5\} \cup \{4, 5, 6\}$$

$$A \cup B = \{3, 4, 5, 6\}$$

$$\text{Hence, } (A \cup B)' = \{1, 2, 7, 8\}$$

**R.H.S:**

$$\text{We know that } A = \{3, 4, 5\}$$

$$\text{Hence, } A' = \{1, 2, 6, 7, 8\}$$

$$\text{Similarly, } B = \{4, 5, 6\}$$

$$\text{Thus, } B' = \{1, 2, 3, 7, 8\}$$

$$\text{Therefore, } A' \cap B' = \{1, 2, 6, 7, 8\} \cap \{1, 2, 3, 7, 8\}$$

$$A' \cap B' = \{1, 2, 7, 8\}$$

$$\text{Thus, L.H.S} = \text{R.H.S}$$

Hence,  $(A \cup B)' = A' \cap B'$  is proved.

**Example 2:**

If  $U = \{a, b, c, d, e, f, g, h\}$ ,  $P = \{a, c, d\}$ ,  $Q = \{a, b, f, g\}$ . Prove the Demorgan's first law.

**Solution:**

Given:

$$U = \{a, b, c, d, e, f, g, h\}$$

$$P = \{a, c, d\}$$

$$Q = \{a, b, f, g\}$$

According to Demorgan's first law, we have to prove,  $(P \cup Q)' = P' \cap Q'$

**To prove  $(P \cup Q)'$ :**

$$PUQ = \{a, c, d\} \cup \{a, b, f, g\}$$

$$PUQ = \{a, b, c, d, f, g\}$$

$$\text{Hence, } (PUQ)' = \{e, h\}$$

**To prove  $P' \cap Q'$ :**

$$\text{Given that } P = \{a, c, d\}$$

$$\text{Hence, } P' = \{b, e, f, g, h\}$$

$$\text{Also given that, } Q = \{a, b, f, g\}$$

$$Q' = \{c, d, e, h\}$$

$$\text{Hence, } P' \cap Q' = \{b, e, f, g, h\} \cap \{c, d, e, h\} = \{e, h\}$$

Hence,  $(PUQ)' = P' \cap Q'$  is proved.

## HOME WORKS

➤ Answer the following questions:

1.  $\{2,3,4\} \times \{1,3,4\}$

2.  $\{0,1\} \times \{0,1\}$

3.  $|\{1,2,3\} \times \{0\}|$

4.  $|\{1,1\} \times \{2,3,4\}|$

□ Write the answers to following questions.

1.  $|\{1,2,3,4,5,6,7,8,9,0\}|$

**1** Let  $U = \{1, \dots, 10\}$ ,  $A = \{1, 2, 3, 4, 5\}$ ,  $B = \{3, 4, 5, 7, 8, 9\}$ ,  $C = \{1, 2, 3, 4, 9\}$ ,  $D = \{2, 4, 6, 8\}$ .

Calculate each of the following:

1.  $(A \cup D) \cap C$ .
2.  $(A \cap B) \cup C$ .
3.  $B^c$ .
4.  $C - D$ .
5.  $(B - A)^c$ .

## Practice Questions on Set Operations

**Question 1.** Find the union of two sets  $A = \{2, 4, 5, 9\}$  and  $B = \{2, 6, 10, 12\}$ .

**Question 2.** Find the intersection of sets  $P = \{5, 6, 7, 9\}$  and  $Q = \{6, 9\}$ .

**Question 3.** Find the complement of set  $X = \{p, q, r\}$  where Universal set  $U = \{e, f, h, p, q, r\}$ .

**Question 4.** Given two sets  $A = \{1, 2, 4, 7, 10\}$  and  $B = \{1, 2, 8, 15\}$ , then find,  $B - A$ .

**Question 5.** Find the number of elements in set A, given that  $n(A \cup B) = 12$ ,  $n(B) = 7$ , and  $n(A \cap B) = 4$ .

**Q6:** Show that  $(A \cup B)' = A' \cap B'$ .

1. if  $U = \{11, 12, 13, 14, 15, 16, 17\}$ ,  $A = \{11, 12, 13, 15\}$ , and  $B = \{13, 15, 16, 17\}$ .
2. Let  $U = \{p, q, r, s, t, u, v, w\}$ ,  $A = \{p, r, s, t\}$  and  $B = \{t, v, w\}$ .