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EX (4):

Boolean algebra and De Morgan Theorems

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Boolean Algebra

There are some important rules must be considered to understand the Boolean algebra which are:

- 1- $A + 1 = 1$, which does not mean addition operation, it means OR operation.
- 2- $A + 0 = A$
- 3- $\overline{\overline{A}} = A$
- 4- $A + A = A$
- 5- $A \cdot A = A$
- 6- $\overline{A} \cdot A = 0$
- 7- $\overline{A} + A = 1$
- 8- $A + \overline{A}B = A + B$

Ex: Prove that $A + \overline{A}B = A + B$

Sol: take the right side

$$A(1 + B) + \overline{A}B = A + AB + \overline{A}B \{ \text{since } (1 + B) = 1 \}$$

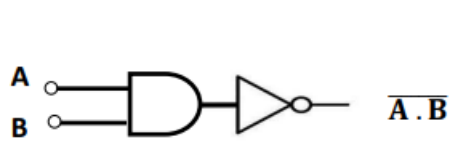
$$\rightarrow A + AB + \overline{A}B = A + B(A + \overline{A})$$

$$= A + B$$

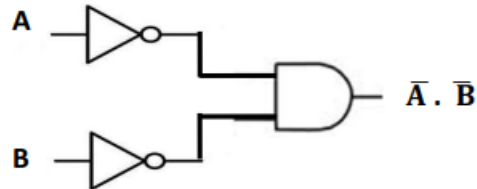
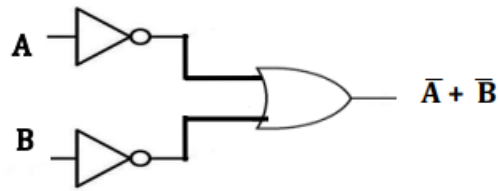
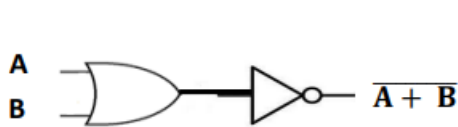
De- Morgan's Theorem

De-Morgan's Theorems are two additional simplification techniques, which used to simplify Boolean expressions. It is important to note that the more simple Boolean expression gives simple logic circuit.

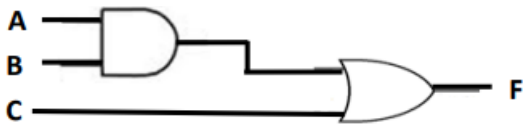
$$1- \overline{(A \cdot B)} = \overline{(A)} + \overline{(B)}$$



$$2- \overline{A + B} = \overline{A} \cdot \overline{B}$$



Ex: Write the Boolean expression for the following logic circuit.



Sol: $F = A \cdot B + C$

Ex : Simplify the following Boolean algebra

$$Y = \overline{A} + AB + ACB + CA + \overline{B} + \overline{C}$$

Sol:

$$\begin{aligned} \overline{A} + AB + ACB + CA + \overline{B} + \overline{C} &= \overline{A}(1 + B) + AB + AC(1 + B) + \overline{B} + \overline{C} \\ &= \overline{A} + \overline{A}B + AB + AC + \overline{B} + \overline{C} = \overline{A} + B(\overline{A} + A) + AC + \overline{B} + \overline{C}(A + 1) \\ &= \overline{A} + B + AC + \overline{C} + \overline{C}A + \overline{B} = \overline{A} + B + A(C + \overline{C}) + \overline{C} + \overline{B} \\ &= \overline{A} + B + A + \overline{C} + \overline{B} = (\overline{A} + A) + (\overline{B} + B) + \overline{C} \\ &= 1 + \overline{C} = 1 \end{aligned}$$

Ex: using NAND and NOR gates to build other logic gates.

Sol:

First using NAND gates:

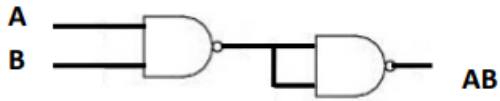
1-NOT gate

NOT gate can be built easily by shorting the two terminal of the NAND gate.



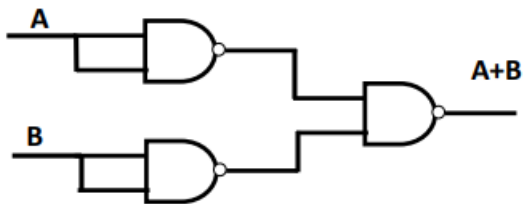
2- AND gate

AND gate can be obtained only by adding inverter NAND gate to the NAND gate as shown below.



3- OR gate

OR gate can be represented by three NAND gates

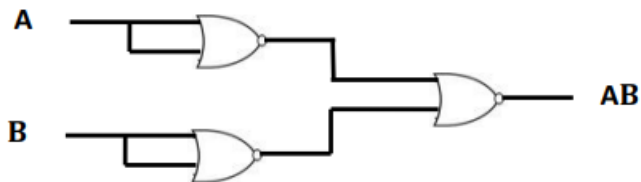


Second using NOR gates:

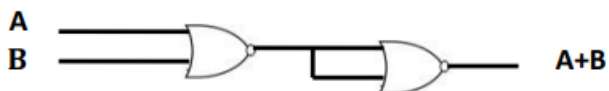
1- NOT gate



2- AND gate



3- OR gate



Ex: Represent the following Boolean expression using only NAND gates.

$$F = AB + CD$$

Sol:

$$F = \overline{\overline{AB + CD}} \implies F = \overline{\overline{AB} \cdot \overline{CD}}$$

the logic circuit is shown below

