Al-Mamoon University

College of Science

Department of Medical Physics

Lecture (1):

Fluid Mechanics

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Chapter One

Fluid Mechanics

Matter is normally classified as being in one of three states: solid, liquid, or gas. From everyday experience we know that a solid has a definite volume and shape, a liquid has a definite volume but no definite shape, and an unconfined gas has neither a definite volume nor a definite shape. These descriptions help us picture the states of matter, but they are somewhat artificial. For example, asphalt and plastics are normally considered solids, but over long time intervals they tend to flow like liquids. Likewise, most substances can be a solid, a liquid, or a gas (or a combination of any of these three), depending on the temperature and pressure. In general, the time interval required for a particular substance to change its shape in response to an external force determines whether we treat the substance as a solid, a liquid, or a gas.

A fluid is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container. Both liquids and gases are fluids.

In our treatment of the mechanics of fluids, we'll be applying principles and analysis models that we have already discussed. First, we consider the mechanics of a fluid at rest, that is, fluid statics, and then study fluids in motion, that is, fluid dynamics.

Pressure

Fluids do not sustain shearing stresses or tensile stresses, therefore, the only stress that can be exerted on an object submerged in a static fluid is one that tends to compress the object from all sides. In other words, the force exerted by a static fluid on an object is always perpendicular to the surfaces of the object as shown in Figure 1.

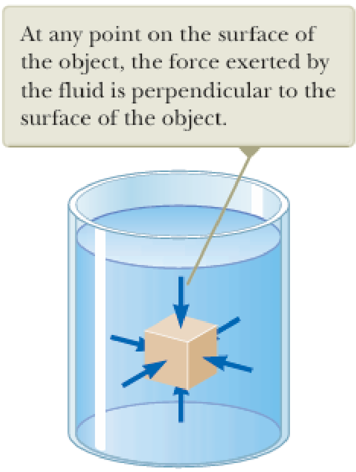


Figure 1 The forces exerted by a fluid on the surfaces of a submerged object.

The pressure in a fluid can be measured with the device pictured in Figure 2. The device consists of an evacuated cylinder that encloses a light piston connected to a spring. As the device is submerged in a fluid, the fluid presses on the top of the piston and compresses the spring until the inward force exerted by the fluid is balanced by the outward force exerted by the spring. The fluid pressure can be measured directly if the spring is calibrated in advance. If **F** is the magnitude of the force exerted on the piston and **A** is the surface area of the piston, the **pressure P** of the fluid at the level to which the device has been submerged is defined as the ratio of the force to the area:

$$P = \frac{F}{A} \qquad (1)$$

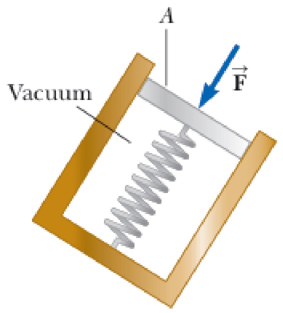


Figure 2 A simple device for measuring the pressure exerted by a fluid

Pressure is a scalar quantity because it is proportional to the magnitude of the force on the piston.

If the pressure varies over an area, the infinitesimal force dF on an infinitesimal surface element of area dA is

$$dF = PdA \qquad (2)$$

where P is the pressure at the location of the area dA. To calculate the total force exerted on a surface of a container, we must integrate Equation 2 over the surface. The units of pressure are newtons per square meter (N/m^2) in the SI system. Another name for the SI unit of pressure is the pascal (Pa):

$$1\ Pa = 1\ N/m^2 \qquad (3)$$

For a tactile demonstration of the definition of pressure, hold a tack between your thumb and forefinger, with the point of the tack on your thumb and the head of the tack on your forefinger. Now gently press your thumb and forefinger together. Your thumb will begin to feel pain immediately while your forefinger will not. The tack is exerting the same force on both your thumb and forefinger, but the pressure on your thumb is much larger because of the small area over which the force is applied.

Example 1

The mattress of a water bed is 2 m long by 2 m wide and 30 cm deep.

(A) Find the weight of the water in the mattress

(B) Find the pressure exerted by the water bed on the floor when the bed rests in its normal position. Assume the entire lower surface of the bed makes contact with the floor.

Solution

(A)

$$V = (2\,m)(2\,m)(0.3\,m) = 1.2\,m^3$$

$$M = \rho V = 1000\,kg/m^3 \times 1.2\,m^3 = 1.2 \times 10^3 kg$$

$$Mg = 1.2 \times 10^3 \times 9.8\,m/s^2 = 1.18 \times 10^4 N$$

(B)

$$P = \frac{1.18 \times 10^4 N}{4\,m^2} = 2.94 \times 10^3\ Pa$$

What iF ?

What if the water bed is replaced by a 300-lb regular bed that is supported by four legs? Each leg has a circular cross section of radius 2.00 cm. What pressure does this bed exert on the floor?

Answer

The weight of the regular bed is distributed over four circular cross sections at the bottom of the legs. Therefore, the pressure is

$$P = \frac{F}{A} = \frac{mg}{4\pi r^2} = \frac{300\ Ib}{4\pi (0.02\,m)^2}\left(\frac{1\,N}{0.225\ Ib}\right)$$

$$= 2.65 \times 10^5\ \text{Pa}$$

This result is almost 100 times larger than the pressure due to the water bed! The weight of the regular bed, even though it is much less than the weight of the water bed, is applied over the very small area of the four legs. The high pressure on the floor at the feet of a regular bed could cause dents in wood floors or permanently crush carpet pile.

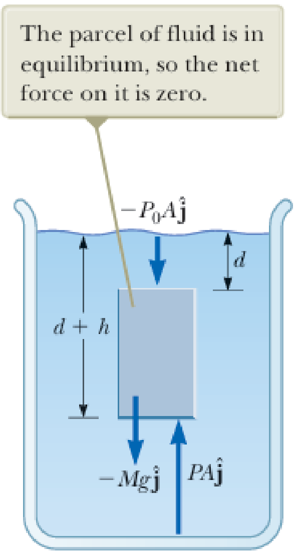
Variation of Pressure with Depth

As divers well know, water pressure increases with depth. Likewise, atmospheric pressure decreases with increasing altitude; for this reason, aircraft flying at high altitudes must have pressurized cabins for the comfort of the passengers. We now show how the pressure in a liquid increases with depth. The density of a substance is defined as its mass per unit volume; Table 1 lists the densities of various substances. These values vary slightly with temperature because the volume of a substance is dependent on temperature . Under standard conditions (at 0^{O}C and at atmospheric pressure), the densities of gases are about $\frac{1}{1000}$ the densities of solids and liquids. This difference in densities implies that the average molecular spacing in a gas under these conditions is about ten times greater than that in a solid or liquid.

Table 1 Densities of Some Common Substances at Standard Temperature (0 C) and Pressure (Atmospheric)

Substance	ρ (kg/m^3)	Substance	ρ (kg/m^3)
Air	1.29	Iron	7.86×10^3
Air (at 20°C and atmospheric pressure)	1.20	Lead	11.3×10^3
		Mercury	13.6×10^3
Aluminum	2.70×10^3	Nitrogen gas	1.25
Benzene	0.879×10^3	Oak	0.710×10^3
Brass	8.4×10^3	Osmium	22.6×10^3
Copper	8.92×10^3	Oxygen gas	1.43
Ethyl alcohol	0.806×10^3	Pine	0.373×10^3
Fresh water	1.00×10^3	Platinum	21.4×10^3
Glycerin	1.26×10^3	Seawater	1.03×10^3
Gold	19.3×10^3	Silver	10.5×10^3
Helium gas	1.79×10^{-1}	Tin	7.30×10^3
Hydrogen gas	8.99×10^{-2}	Uranium	19.1×10^3
Ice	0.917×10^3		

Now consider a liquid of density ρ at rest as shown in Figure 3. We assume ρ is uniform throughout the liquid, which means the liquid is incompressible. Let us select a parcel of the liquid contained within an imaginary block of cross-sectional area A extending from depth d to depth $d + h$. The liquid external to our parcel exerts forces at all points on the surface of the parcel, perpendicular to the surface. The pressure exerted by the liquid on the bottom face of the parcel is P, and the pressure on the top face is P_0. Therefore, the upward force exerted by the outside fluid on the bottom of the parcel has a magnitude PA, and the downward force exerted on the top has a magnitude P_0A. The mass of liquid in the parcel is $M = \rho V = \rho A h$; therefore, the weight of the liquid in the parcel is $Mg = \rho A h g$. Because the parcel is at rest and remains at rest, it can be modeled as a particle in equilibrium, so that the net force acting on it must be zero. Choosing upward to be the positive y direction, we see that



$$\sum \vec{F} = PA\hat{j} - P_o A\hat{j} - Mg\hat{j} = 0$$

Or

$$PA - P_o A - \rho Agh = 0$$

$$\boldsymbol{P = P_o + \rho gh} \qquad \textbf{(4)}$$

That is, the pressure P at a depth h below a point in the liquid at which the pressure is P_o is greater by an amount ρgh. If the liquid is open to the atmosphere and P_o is the pressure at the surface of the liquid, then P_o is atmospheric pressure. We usually take atmospheric pressure to be

$$P_o = \ 1\, atm = \ 1.013 \ \times \ 10^5\, Pa$$

Equation 4 implies that the pressure is the same at all points having the same depth, independent of the shape of the container.

Because the pressure in a fluid depends on depth and on the value of P_o, any increase in pressure at the surface must be transmitted to every other point in the fluid. This concept was first recognized by French scientist Blaise Pascal (1623 - 1662) and is called **Pascal's law**: a change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

An important application of Pascal's law is the hydraulic press illustrated in Figure 4a. A force of magnitude F_1 is applied to a small piston of surface area A_1. The pressure is transmitted through an incompressible liquid to a larger piston of surface area A_2. Because the pressure must be the same on both sides, $P = \frac{F1}{A1} = \frac{F2}{A2}$. Therefore, the force F_2 is greater than the force F_1 by a factor of $\frac{A2}{A1}$.

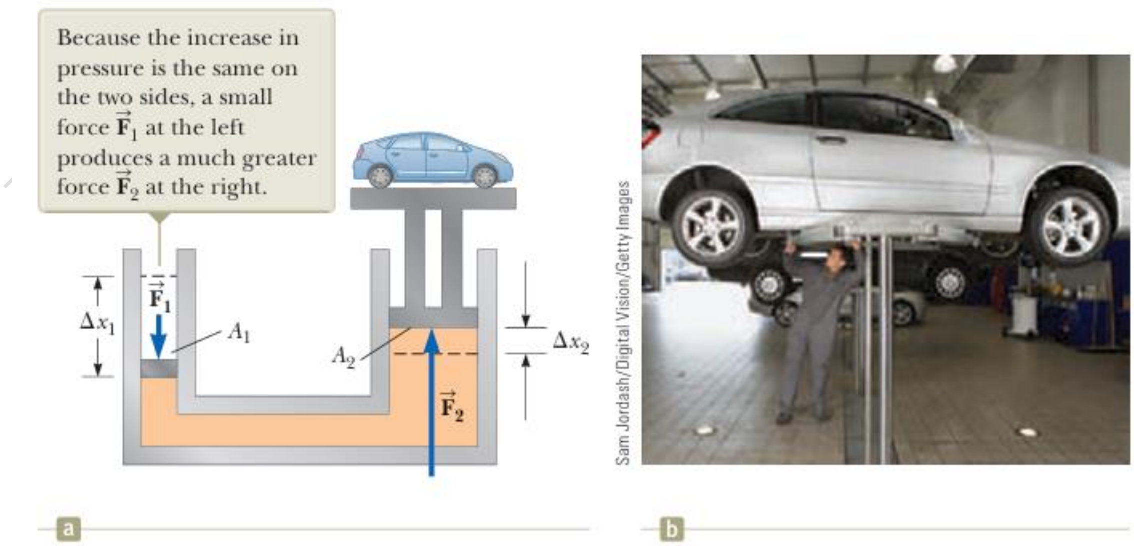


Figure 3 (a) Diagram of a hydraulic press. (b) A vehicle undergoing repair is supported by a hydraulic lift in a garage.

By designing a hydraulic press with appropriate areas A_1 and A_2, a large output force can be applied by means of a small input force. Hydraulic brakes, car lifts, hydraulic jacks, and forklifts all make use of this principle (Fig. 4b).

Because liquid is neither added to nor removed from the system, the volume of liquid pushed down on the left in Figure 14.4a as the piston moves downward through a displacement Δx_1 equals the volume of liquid pushed up on the right as the right piston moves upward through a displacement Δx_2. That is, $A_1 \Delta x_1 = A_2 \Delta x_2$; therefore, $\frac{A2}{A1} = \frac{\Delta x1}{\Delta x2}$. We have already shown that $\frac{A2}{A1} = \frac{F2}{F1}$. Therefore, $\frac{F2}{F1} = \frac{\Delta x1}{\Delta x2}$, so $F_1 \Delta x_1 \ = \ F2_2 \ \Delta x\ _2$. Each side of this equation is the work done by the force on its respective piston. Therefore, the work done by $\vec{F}_1$ on the input piston equals the work done by $\vec{F}_2$ on the output piston, as it must to conserve energy. (The process can be modeled as a special case of the non-isolated system model: the non-isolated system in steady state. There is energy transfer into and out of the system, but these energy transfers balance, so that there is no net change in the energy of the system.)

Quick Quiz 2

The pressure at the bottom of a filled glass of water ($\rho = 1000\ kg/m^3$) is P. The water is poured out, and the glass is filled with ethyl alcohol ($\rho = 806\ kg/m^3$). What is the pressure at the bottom of the glass?

(a) smaller than P (b) equal to P (c) larger than P (d) indeterminate

Example 2

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section of radius 5.00 cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0 cm.

(A) What force must the compressed air exert to lift a car weighing 13 300 N?
(B) What air pressure produces this force?

Solution

(A)

$$\frac{F1}{A1} = \frac{F2}{A2}$$

$$F1 = \left(\frac{A1}{A2}\right) F2 = \frac{\pi(5 \times 10^{-2} m)^2}{\pi(15 \times 10^{-2} m)^2}(1.33 \times 10^4 N) = 1.48 \times 10^3 N$$

(B)

$$P = \frac{F1}{A1} = \frac{1.48 \times 10^3 N}{\pi(5 \times 10^{-2} m)^2} = 1.88 \times 10^5 Pa$$

Example 3

Estimate the force exerted on your eardrum due to the water when you are swimming at the bottom of a pool that is 5 m deep.

Solution

The air inside the middle ear is normally at atmospheric pressure P0. Therefore, to find the net force on the eardrum, we must consider the difference between the total pressure at the bottom of the pool and atmospheric pressure. Let's estimate the surface area of the eardrum to be approximately 1 $cm^2 = 1 \times 10^{-4}\ m^2$.

$$P - P_o = \rho g h$$

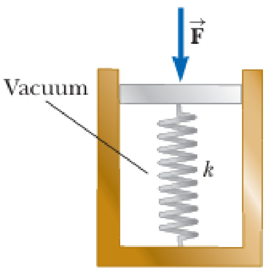
$$P - P_o = 1000\ kg/m^3 \times 9.8\,m/s^2 \times 5\,m = 4.9 \times 10^4\ Pa$$

$$F = (P - P_o)A = 4.9 \times 10^4\ Pa \times 10^{-4} m^2 \approx 5\ N$$

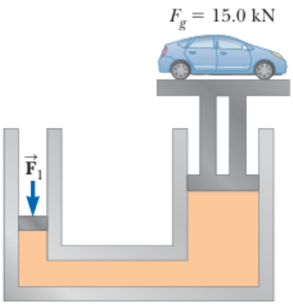
Problems

1-A 50.0-kg woman wearing high-heeled shoes is invited into a home in which the kitchen has vinyl floor covering. The heel on each shoe is circular and has a radius of 0.500 cm. (a) If the woman balances on one heel, what pressure does she exert on the floor? (b) Should the home owner be concerned? Explain your answer.

2-The spring of the pressure gauge shown in Figure has a force constant of 1250 N/m, and the piston has a diameter of 1.2 cm. As the gauge is lowered into water in a lake, what change in depth causes the piston to move in by 0.75 cm?



3-The small piston of a hydraulic lift (Fig. below) has a cross-sectional area of 3 cm^2, and its large piston has a cross-sectional area of 200 cm^2. What downward force of magnitude F1 must be applied to the small piston for the lift to raise a load whose weight is $Fg = 15\ kN$?

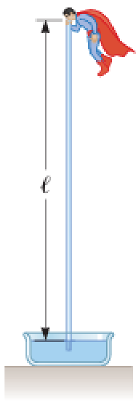


4-A swimming pool has dimensions 30 m × 10 m and a flat bottom. When the pool is filled to a depth of 2 m with fresh water, what is the force exerted by the water on (a) the bottom? (b) On each end? (c) On each side?

5-(a) Calculate the absolute pressure at the bottom of a freshwater lake at a point whose depth is 27.5 m. Assume the density of the water is 1.00×10^3 kg/m^3 and that the air above is at a pressure of 101.3 kPa.

(b) What force is exerted by the water on the window of an underwater vehicle at this depth if the window is circular and has a diameter of 35.0 cm?

6-Why is the following situation impossible? Superman attempting to drink cold water through a straw of length , l =12 m. The walls of the tubular straw are very strong and do not collapse. With his great strength, he achieves maximum possible suc tion and enjoys drinking the cold water.



7- A container is filled to a depth of 20.0 cm with water. On top of the water floats a 30.0-cm-thick layer of oil with specific gravity 0.700. What is the absolute pres sure at the bottom of the container?