but take at least three separate readings, preferably under conditions in which instruments are switched off-on.

The error may be originated from the sampling of the source, preparation of the samples

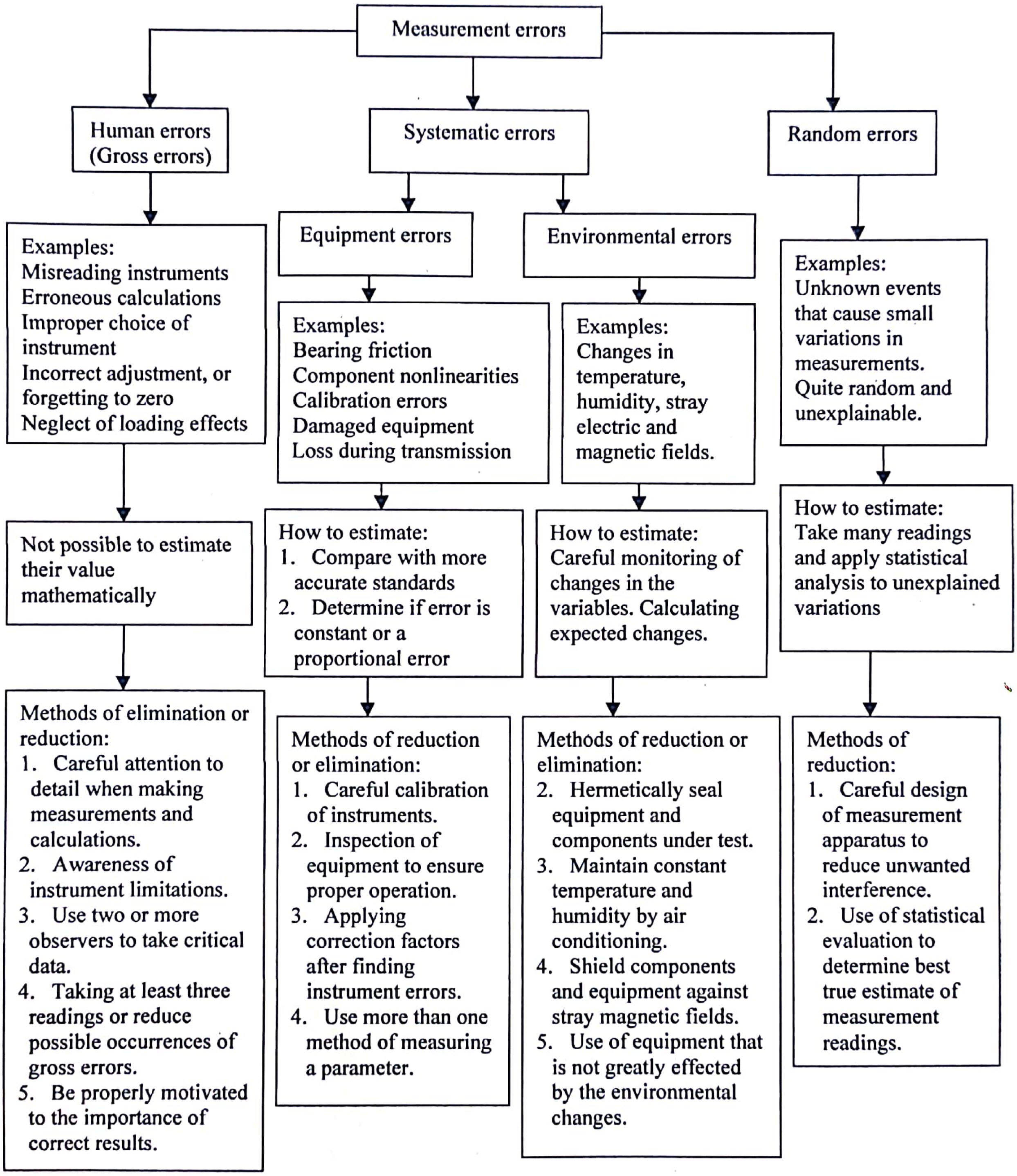


Figure 2.6. A schematic summary of measurement errors.

and measurement and analysis of the measurand. Care must be taken so that the sample is representative of the whole population (homogeneous vs. heterogeneous). No unwanted additions or deletions are allowed during the preparatory phase. Finally, calibration of the measuring instrument using standard measurands or standard solutions is done as frequent as defined by the equipment manufacturer. One way to assess total error is to treat a reference standard as a sample. The reference standard would be carried through the entire process to see how close the results are to the reference value.

2.2. ANALYSIS OF MEASUREMENT DATA

A statistical analysis of measurement data is common practice because it allows an analytical determination of the uncertainty of the final test result. The outcome of a certain measurement method may be predicted on the basis of sample data without having detailed information on all the disturbing factors. To make statistical methods and interpretations meaningful, a large number of measurements are usually required. Also, systematic errors should be small compared with residual or random errors, because statistical treatment of data cannot remove a fixed bias contained in all the measurements.

2.2.1. Arithmetic Mean

The most probable value of a measured variable is the arithmetic mean of the number of readings taken. The best approximation will be made when the number of readings of the same quantity is very large. Theoretically, an infinite number of readings would give the best result although in practice only a finite number of measurements can be made. The arithmetic mean is given by:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} = \frac{\sum x}{n} \qquad \text{......(2.7)}$$

where $\bar{x}$ = arithmetic mean, $x_1 \ldots x_n$ = readings taken, and n = number of readings.

Example 2.4

A set of independent current measurements was taken by six observers and recorded as 12.8 mA, 12.2 mA, 12.5 mA, 13.1 mA, 12.9 mA, and 12.4 mA. Calculate the arithmetic mean.

$$\bar{x} = \frac{12.8 + 12.2 + 12.5 + 13.1 + 12.9 + 12.4}{6} = 12.65 mA$$

2.2.2. Deviation from the Mean

In addition to knowing the mean value of a series of measurements, it is often informative to have some idea of their range about the mean. Deviation is the departure of a given reading from the arithmetic mean of the group of readings. If the deviation of the first reading x_1 is called d_1, and that of the second reading, x_2 is called d_2 and so on, then the deviations from the mean can be expressed as

Table 2.1. Deviations around mean

d_1 = 12.8 - 12.65 = 0.15 mA
d_2 = 12.2 - 12.65 = -0.45 mA
d_3 = 12.5 - 12.65 = -0.15 mA
d_4 = 13.1 - 12.65 = 0.45 mA
d_5 = 12.9 - 12.65 = 0.25 mA
d_6 = 12.4 - 12.65 = -0.25 mA

$$d_1 = x_1 - \bar{x} \; ; d_2 = x_2 - \bar{x} \; ; \ldots ; d_n = x_n - \bar{x} \qquad (2.8)$$

The deviation from the mean may have a positive or a negative value and that the algebraic sum of all the deviations must be zero. The computation of deviations for the previous example is given in Table 2.1.

2.2.2.1. Average Deviation

The average deviation is an indication of the precision at the instruments used in making the measurements. Highly precise instruments will yield a low average deviation between readings. By definition average deviation is the sum of the absolute values of the deviations divided by the number of readings. The absolute value of the deviation is the value without respect to sign. Average deviation may be expressed as

$$D = \frac{|d_1| + |d_2| + |d_3| + \cdots + |d_n|}{n} = \frac{\sum d}{n} \qquad (2.9)$$

Example 2.5

The average deviation for the data given in the above example:

$$D = \frac{0.15 + 0.45 + 0.15 + 0.45 + 0.25 + 0.25}{6} = 0.283 mA$$

2.2.2.2. Standard Deviation

The range is an important measurement. It indicates figures at the top and bottom around the average value. The findings farthest away from the average may be removed from the data set without affecting generality. However, it does not give much indication of the spread of observations about the mean. This is where the standard deviation comes in.

In statistical analysis of random errors, the root-mean-square deviation or standard deviation is a very valuable aid. By definition, the standard deviation σ of a finite number of data is the square root of the sum of all the individual deviations squared, divided by the number of readings minus one. Expressed mathematically:

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \cdots + d_n^2}{n-1}} = \sqrt{\frac{\sum d_i^2}{n-1}} \qquad (2.10)$$

Another expression for essentially the same quantity is the variance or mean square deviation, which is the same as the standard deviation except that the square root is not extracted. Therefore

$$\text{variance (V)} = \text{mean square deviation} = \sigma^2 \qquad (2.11)$$

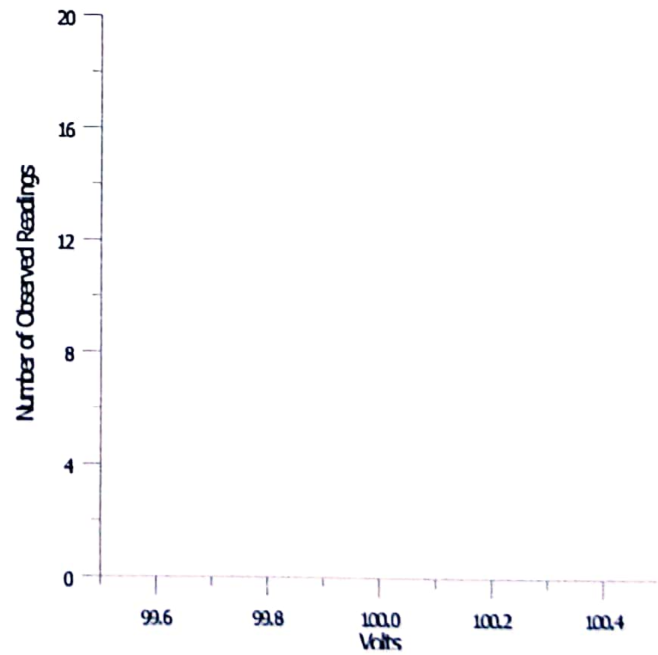
The variance is a convenient quantity to use in many computations because variances are additive. The standard deviation however, has the advantage of being of the same units as the variable making it easy to compare magnitudes. Most scientific results are now stated in terms of standard deviation.

2.2.3. Probability of Errors

2.2.3.1. Normal Distribution of Errors

A practical point to note is that, whether the calculation is done on the whole "population" of data or on a sample drawn from it, the population itself should at least approximately fall into a so called "normal (or Gaussian)" distribution.

For example, 50 readings of voltage were taken at small time intervals and recorded to



Limiting errors

In most indicating instruments, the accuracy is guaranteed to a certain percentage of full-scale reading. Circuit components (such as capacitors, resistors, etc.) are guaranteed within a certain percentage of their rated value.

The limits of these deviations from the specified values are known as limiting errors or guarantee errors. For example, if the resistance of a resistor is given as 500 Ω ± 10 per cent, the manufacturer guarantees that the resistance falls between the limits 450 Ω and 550 Ω. The maker is not specifying a standard deviation or a probable error, but promises that the error is no greater than the limits set.

Example-

A 0-150-V voltmeter has a guaranteed accuracy of 1 per cent full-scale reading. The voltage measured by this instrument is 83 V. Calculate the limiting error in per cent

Solution-

The magnitude of the limiting error is

$$0.01 \times 150\ V = 1.5\ V$$

The percentage error at a meter indication of 83 V is

$$1.5\ 83 \times 100 \text{ per cent} = 1.81\%$$

It is important to note in Example that a meter is guaranteed to have an accuracy of better than 1 per cent of the full-scale reading, but when the meter reads **83** V, the limiting error increases to 1.81 percent.

Correspondingly, when a smaller voltage is measured, the limiting error will increase further. If the meter reads **60** V, the per cent limiting error is 1.5 60 × 100 = 2.5 per cent; if the meter reads **30** V, the limiting error is 1.5 30 × 100 = 5 per cent. The increase in per cent limiting error, as smaller voltages are measured, occurs because the magnitude of the limiting error is a fixed quantity based on the full-scale reading of the meter.