4.6 Kelvin Bridge

Fig. 4.6 shows the circuit diagram of a Kelvin Bridge. This circuit provides great accuracy in the measurement of low value resistance generally below 1 Ω. It is used for measuring resistance values ranging from microohms to 1 ohm.

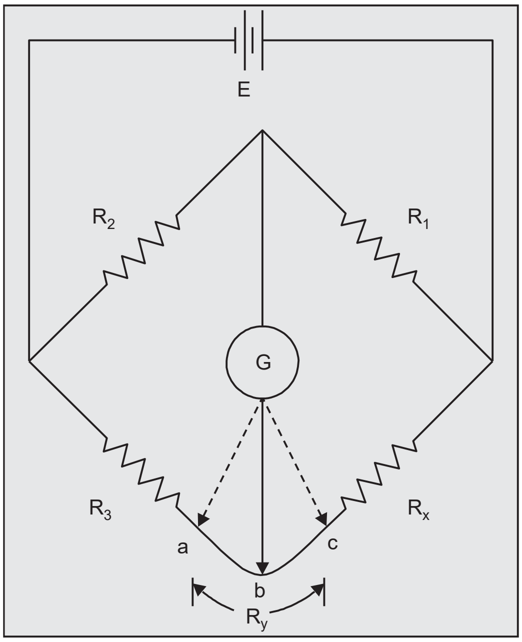

Fig. 4.6. Kelvin Bridge

The resistance R_y represents the resistance of the conducting lead from R_3 to R_x. The resistance R_x is the unknown resistance to be measured. The galvanometer can be connected either to point '*c*' or to point '*a*'. When it is connected to point '*a*', the resistance R_y of the connecting lead is added to the unknown resistance R_x. The measurement value of the resistance is too high than the actual value.

When the galvanometer is connected to the point '*c*', the resistance R_y of the connecting lead is added to the known resistance R_3. The actual value of R_3 is higher than the normal value by the resistance R_y and the resulting measurement of R_x is lower than the actual value.

If the galvanometer is connected to point '*b*', in between points '*c*' and '*a*', in such a way that the ratio of the resistance from '*c*' to '*b*' and that from '*a*' to '*b*' equals the ratio of resistance R_1 and R_2 then,

$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$$

Balance equation for the bridge is given by relation,

$$\frac{R_x + R_{cb}}{R_3 + R_{ab}} = \frac{R_1}{R_2}$$

$$(R_x + R_{cb}) = \frac{R_1}{R_2}(R_3 + R_{ab}) \qquad \ldots(i)$$

We know that

$$R_{ac} + R_{bc} = R_y$$

and $$\frac{R_{bc}}{R_{ac}} = \frac{R_1}{R_2} \qquad \ldots(ii)$$

Adding 1 on the both side of equation (*ii*) we get

$$\frac{R_{bc}}{R_{ac}} + 1 = \frac{R_1}{R_2} + 1$$

$$\frac{R_{bc} + R_{ac}}{R_{ac}} = \frac{R_1 + R_2}{R_2}$$

$$\frac{R_y}{R_{ac}} = \frac{R_1 + R_2}{R_2} \qquad \ldots(R_{ac} + R_{bc} = R_y)$$

$$R_{ac} = \frac{R_2 R_y}{R_1 + R_2} \qquad \ldots(iii)$$

$$R_{bc} = R_y - R_{ac}$$

$$= R_Y - \frac{R_2 R_Y}{R_1 + R_2}$$

$$= \frac{R_1 R_y}{R_1 + R_2} \qquad \ldots(iv)$$

Substituting the equation (*iii*) and (*iv*) in equation (*i*),

$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1}{R_2}\left(R_3 + \frac{R_2 R_y}{R_1 + R_2}\right)$$

$$R_x = \frac{R_1 R_3}{R_2}$$

This is the standard equation of the bridge balance. The equation does not depend on the resistance of connecting lead from R_3 to R_x. The effect of lead and contact resistances is completely eliminated by connecting the galvanometer to the intermediate position '*b*'.

4.7 Double Kelvin Bridge

Fig. 4.7 shows the circuit diagram of Kelvin double bridge. This bridge contains another set of ratio arms hence called double bridge. The second set of arms labeled '*l*' and '*m*'. The galvanometer is connected to point '*f*'. The ratio of the resistances of arms '*l*' and '*m*' is same as the ratio of R_1 and R_2.

The galvanometer indicates "zero" when the potential at '*a*' equals the potential at '*f*', i.e.,

$$E_{ab} = E_{bcf}$$

According to the *Voltage Divider Rule* the voltage across the '*ab*',

$$E_{ab} = \frac{R_2}{R_1 + R_2} \times E \qquad \ldots(i)$$

The value of *E* is given by,

$$E = I\,[R_3 + R_x + (l + m) \,||\, R_y]$$

$$E = I\left[R_3 + R_x + \frac{(l + m)\,R_y}{(l + m) + R_y}\right]$$

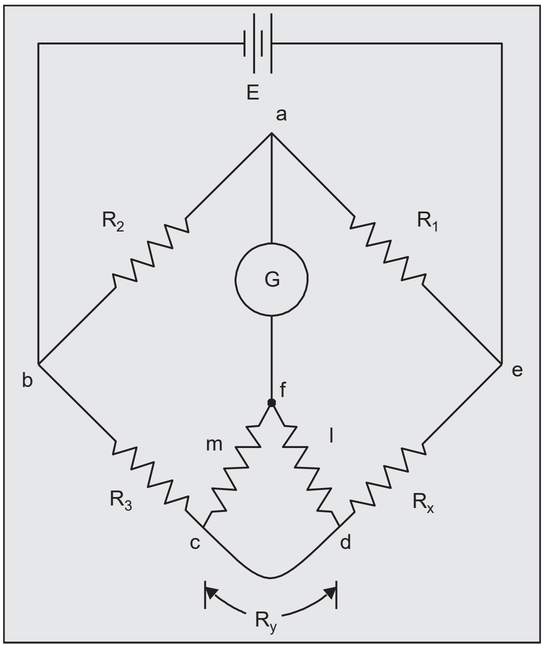

Fig. 4.7. Double Kelvin Bridge

Substituting the value of E in equation (*i*) we get,

$$E_{ab} = \frac{R_2}{R_1 + R_2} \times I \left[R_3 + R_x + \frac{(l+m) R_y}{(l+m) + R_y} \right]$$

Similarly,

$$E_{bcf} = I \left[R_3 + \frac{m}{l+m} \left\{ \frac{(l+m) R_y}{(l+m) + R_y} \right\} \right]$$

but,

$$E_{ab} = E_{bcf} = \frac{R_2}{R_1 + R_2} \times I \left[R_3 + R_x + \frac{(l+m) R_y}{(l+m) + R_y} \right]$$

Rearranging the above equation we get,

$$R_x = \frac{R_1 R_3}{R_2} + \frac{m R_1 R_y}{R_2 (a + b + R_y)} - \frac{l R_y}{l + m + R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{m R_y}{l + m + R_y} \left(\frac{R_1}{R_2} - \frac{l}{m} \right) \quad \ldots(ii)$$

The ratio of the resistances of arms '*l*' and '*m*' is same as the ratio of R_1 and R_2, i.e.,

$$\frac{R_1}{R_2} = \frac{l}{m} \quad \ldots(iv)$$

From equation (*iv*) and (*iii*) we get the value of R_x,

$$R_x = \frac{R_1 R_3}{R_2}$$

This is the equation for Kelvin Bridge. It indicates that the resistance of the connecting lead R_Y, has no effect on the measurement, provided that the ratios of the resistances of the two sets of ratio arms are equal. Fig. 4.8 shows the Kelvin double bridge. This bridge is mostly used for industrial and laboratory purpose.

Fig. 4.8. Kelvin double bridge used in industry

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Example 4.3. *In a Kelvin double bridge, there is error due to mismatch between the ratio of outer and inner arm resistances. The following data relates to this bridge,*

Standard resistance = 100.03 μΩ

Inner arms = 100.31 Ω and 200 Ω

Outer arms = 100.24 Ω and 200 Ω

The resistance of connecting leads from standard to unknown resistance is 680 μΩ. Determine the value of unknown resistance.

Solution. Given: $R_3 = 100.03\ \mu\Omega = 100.03 \times 10^{-6}\ \Omega$; $l = 100.31\ \Omega$; $m = 200\ \Omega$; $R_1 = 100.24\ \Omega$; $R_2 = 200\ \Omega$ and $R_y = 680\ \mu\Omega = 680 \times 10^{-6}\ \Omega$.

We know that the value of unknown resistance

$$R_x = \frac{R_1 R_3}{R_2} + \frac{m R_y}{l + m + R_y}\left(\frac{R_1}{R_2} - \frac{l}{m}\right)$$

$$= \frac{100.24 \times (100.03 \times 10^{-6})}{200} + \frac{200 \times (680 \times 10^{-6})}{100.31 + 200 + (680 \times 10^{-6})}\left(\frac{100.24}{200} - \frac{100.31}{200}\right)$$

$$= (50.135 \times 10^{-6}) + (4.528 \times 10^{-4}) \times (-3.5 \times 10^{-4})$$
$$= 49.97 \times 10^{-6}\ \Omega$$