

A.c Measuring Instrument

Review on Alternating Signal:

The instantaneous values of electrical signals can be graphed as they vary with time. Such graphs are known as the *waveforms* of the signal. If the value of waveform remains constant with time, the signal is referred to as *direct (d.c)* signal; such as the voltage of a battery. If a signal is time varying and has positive and negative instantaneous values, the waveform is known as *alternating (a.c)* waveform. If the variation of a.c signal is continuously repeated then the signal is known as *periodic* waveform.

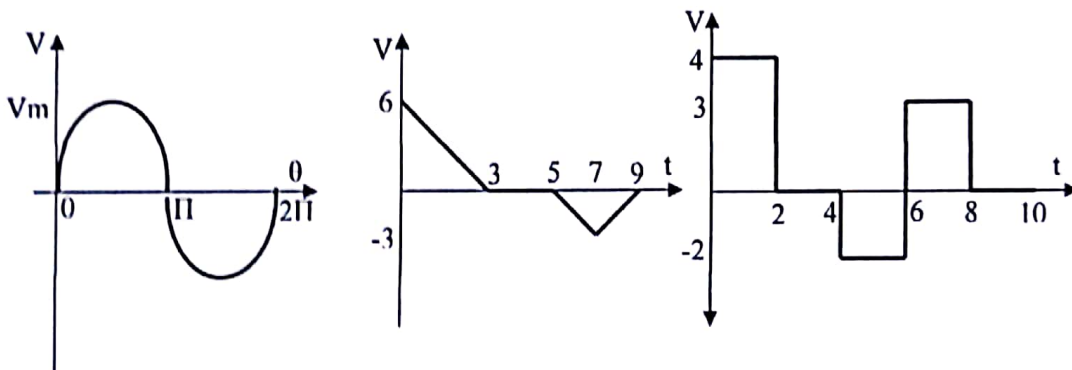
The *frequency of a.c signal* is defined as the *number of cycles traversed in one second*. Thus the time duration of *one cycle per second* for a.c signal is known as the *period (T)*. Where the complete variation of a.c signal before repeated itself is represent one *cycle*.

Average Values:

It is found by dividing the area under the curve of the waveform in one period (T) by the time of the period.

Average value = $\frac{\text{Algebraic sum of the areas under the curve}}{\text{Length of the curve}}$

$$Av = \frac{\sum \text{areas}}{T} \dots\dots\dots (1) \quad \text{or} \quad Av = \frac{1}{T} \int_0^T f(t) dt \dots\dots\dots (2)$$



$$Av = \frac{1}{2\pi} \int_0^{2\pi} Vm \sin \omega t \, d\theta$$

$$Av = -\frac{Vm}{2\pi} (\cos \theta \Big|_0^{2\pi})$$

$$Av = -\frac{Vm}{2\pi} (1 - 1) = 0$$

$$Av = \frac{\frac{1}{2} \times 3 \times 6 + \frac{1}{2} \times 4 \times (-3)}{9} \quad Av = \frac{4 \times 2 + (-2) \times 2 + 3 \times 2}{10}$$

The average value for the figure below by using equation (2) is:

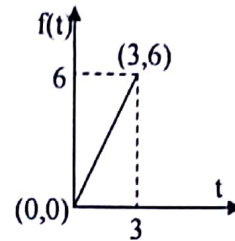
$Av = \frac{1}{T} \int_0^T f(t) dt$ we use the tangent equation for $(x_0, y_0) = (0, 0)$, and $(x_1, y_1) = (3, 6)$ to find the function of $f(t)$

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow \frac{y - 0}{x - 0} = \frac{6 - 0}{3 - 0} \Rightarrow \frac{y}{x} = \frac{6}{3} = 2 \Rightarrow y = 2x \quad \boxed{f(t) = 2t}$$

$$A_v = \frac{1}{3} \int_0^3 (2t) dt$$

$$A_v = \frac{2}{3} \left(\frac{t^2}{2} \Big|_0^3 \right)$$

$$A_v = \frac{1}{3} \left((3)^2 - (0)^2 \right) = \frac{9}{3} = 3$$



Root Mean Square Value (effective value of a.c signal):

The r.m.s value of a waveform refers to its power capability. It is refer to the effective value of a.c signal because the r.m.s value equal to the value of a d.c signal which would deliver the same power if it replaced with a.c signal.

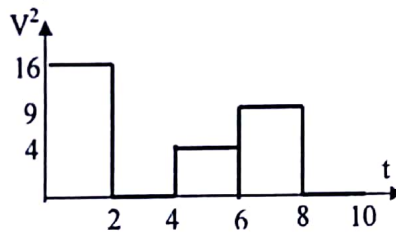
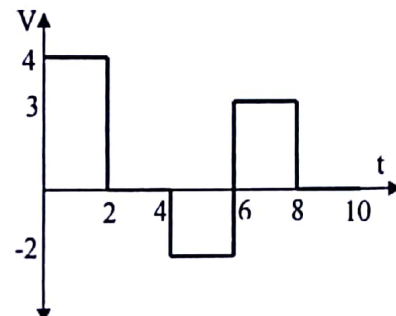
$$r.m.s = \sqrt{\frac{\sum \text{area}(V)^2}{T}} \quad (\text{for square waveform only})$$

$$1- r.m.s = \sqrt{\frac{16 \times 2 + 4 \times 2 + 9 \times 2}{10}}$$

In general form the r.m.s value has the following aqua.

$$r.m.s = \sqrt{\text{Average } f(t)^2}$$

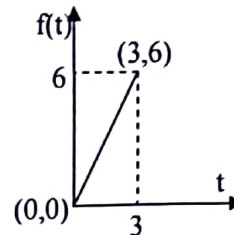
$$\boxed{r.m.s = \sqrt{\frac{1}{T} \int_0^T f(t)^2 dt}}$$



2- If $f(t) = 2t$ then its r.m.s value is:

$$r.m.s = \sqrt{\frac{1}{3} \int_0^3 (2t)^2 dt}$$

$$r.m.s = \sqrt{\frac{4}{3} \left(\frac{t^3}{3} \Big|_0^3 \right)} = \sqrt{\frac{4}{9} \left((3)^3 - (0)^3 \right)} = \sqrt{\frac{4 \times 27}{9}} = 3.46$$



3- If $f(t) = V_m \sin \theta$

$$r.m.s = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V_m^2 \sin^2 \theta d\theta}$$

$$r.m.s = \sqrt{\frac{V_m^2}{2\pi} \int_0^{2\pi} \frac{1 - \cos 2\theta}{2} d\theta}$$

$$r.m.s = \left\{ \frac{V_m^2}{4\pi} \left[\int_0^{2\pi} d\theta - \int_0^{2\pi} \cos 2\theta d\theta \right] \right\}^{\frac{1}{2}}$$

$$r.m.s = \sqrt{\frac{V_m^2}{4\pi} \left[\theta \Big|_0^{2\pi} - \frac{1}{2} \sin 2\theta \Big|_0^{2\pi} \right]}$$

$$r.m.s = \sqrt{\frac{Vm^2}{4\pi} [2\pi - 0]} = \sqrt{\frac{Vm^2}{2}} = \frac{Vm}{\sqrt{2}}$$

