Ac Bridge and their Application:

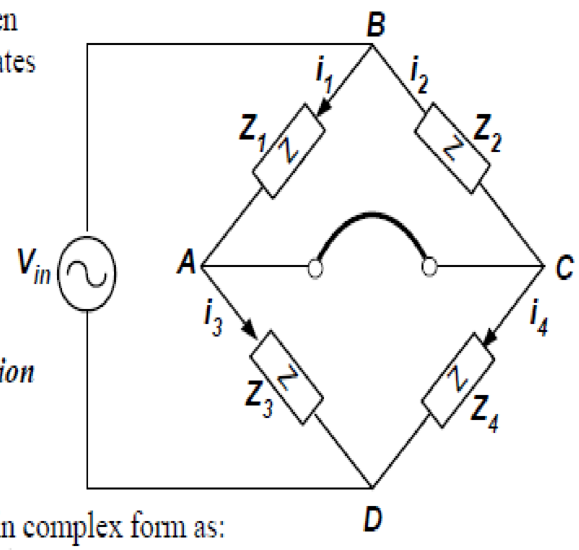
The ac bridge is a natural outgrowth of the dc bridge and in its basic form consists of four bridge arms, a source of excitation, and a null ac detector. For measurements at low frequencies, the power line may serve as the source of excitation; but at higher frequencies an oscillator generally supplies the excitation voltage. The null ac detector in its cheapest effective form consists of a pair of headphones or may be oscilloscope.

The balance condition is reached when the detector response is zero or indicates null. Then $V_{AC} = 0$ and $V_{Z1} = V_{Z2}$

$$V_{Z1} = Vin\frac{Z_1}{Z_1 + Z_3}$$

$$V_{Z2} = Vin\frac{Z_2}{Z_2 + Z_4} \quad \text{thus}$$

$$\boxed{Z_1Z_4 = Z_2Z_3}$$ *is the balance equation*



Or $\boxed{Y_1Y_4 = Y_2Y_3}$

The balance equation can be written in complex form as:

$$(Z_1\angle\theta_1)(Z_4\angle\theta_4) = (Z_2\angle\theta_2)(Z_3\angle\theta_3)$$

And $(Z_1Z_4\angle\theta_1 + \theta_4) = (Z_2Z_3\angle\theta_2 + \theta_3)$

So two conditions must be met simultaneously when balancing an ac bridge

Magnitude balance: $Z_1Z_4 = Z_2Z_3$

Phase balance: $\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$

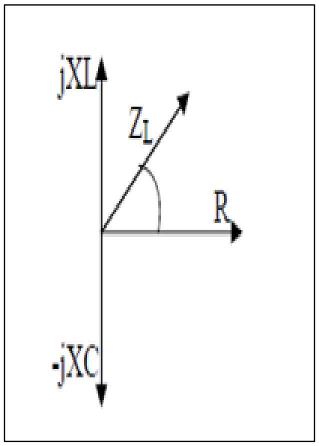
Review on Ac Impedance:

a) *In series connection*

Impedance = resistance ± j reactance

$Z_L = R + jXL$ and $Z_L = R + j\omega L$

$Z_C = R - jXC$ and $Z_C = R - j\dfrac{1}{\omega C}$



Conversion from polar to rectangular

$Z\angle\theta$ in polar form R= Z Cosθ X= Z Sinθ become $Z = R \pm jX$

Conversion from rectangular to polar

$Z = R \pm jX$ in rectangular form $Z = \sqrt{R^2 + X^2}$ $\theta = \tan^{-1}\dfrac{X}{R}$ $\tan\theta = \dfrac{X}{R}$

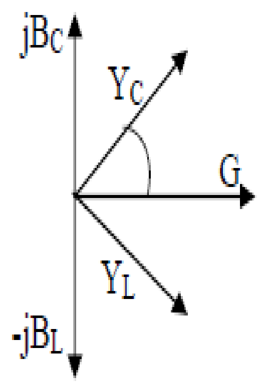
b) *In parallel connection*

Admittance = conductance ± j susceptance

$Y_L = G - jB_L$ and $Y_L = \dfrac{1}{R} - j\dfrac{1}{\omega L}$

$Y_C = G + jB_C$ and $Y_C = \dfrac{1}{R} + j\omega C$

$$\tan\theta = \frac{B_C}{G} = \frac{\frac{1}{Xc}}{\frac{1}{R}} = \frac{\omega C}{\frac{1}{R}} = \omega RC$$



Example The impedance of the basic ac bridge are given as follows:

$\mathbf{Z}_1 = 100\ \Omega\ \angle 80^\circ$ (inductive impedance) $\mathbf{Z}_3 = 400\ \angle 30^\circ \Omega$ (inductive impedance)

$\mathbf{Z}_2 = 250\ \Omega$ (pure resistance) $\mathbf{Z}_4 = $ unknown

Determine the constants of the unknown arm.

SOLUTION The first condition for bridge balance requires that

$$Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{250 \times 400}{100} = 1,000\ \Omega$$

The second condition for bridge balance requires that the sum of the phase angles of opposite arms be equal, therefore

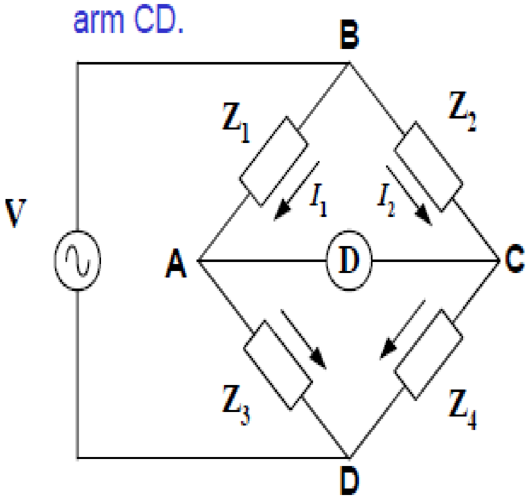
$$\angle\theta_4 = \angle\theta_2 + \angle\theta_3 - \angle\theta_1 = 0 + 30 - 80 = -50^\circ$$

Hence the unknown impedance $\mathbf{Z}_4$ can be written in polar form as

$$\mathbf{Z}_4 = 1,000\ \Omega\ \angle -50^\circ$$

Indicating that we are dealing with a capacitive element, possibly consisting of a series combination of at resistor and a capacitor.

Example an ac bridge is in balance with the following constants: arm AB, R = 200 Ω in series with L = 15.9 mH R; arm BC, R = 300 Ω in series with C = 0.265 μF; arm CD, unknown; arm DA, = 450 Ω. The oscillator frequency is 1 kHz. Find the constants of arm CD.



SOLUTION

$$\mathbf{Z}_1 = R + j\omega L = 200 + j100\ \Omega$$

$$\mathbf{Z}_2 = R + 1/j\omega C = 300 - j600\ \Omega$$

$$\mathbf{Z}_3 = R = 450\ \Omega$$

$$\mathbf{Z}_4 = \text{unknown}$$

The general equation for bridge balance states that $\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_2\mathbf{Z}_3$

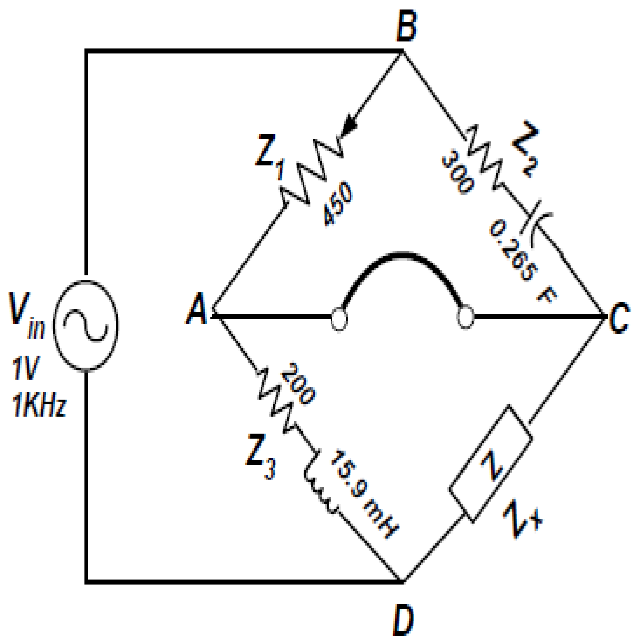
$$\mathbf{Z}_4 = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1} = \frac{450 \times (200 + j100)}{(300 - j600)} = j150\ \Omega$$

This result indicates that $\mathbf{Z}_4$ is a pure inductance with an inductive reactance of 150 Ω at at frequency of 1kHz. Since the inductive reactance $X_L = 2\pi f L$, we solve for L and obtain L = 23.9 mH

Example

For the following bridge find Zx?

The balance equation $Z_1 Z_4 = Z_2 Z_3$



$$Z_1 = R = 450\Omega$$

$$Z_2 = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C}$$

$$Z_2 = 300 - j600$$

$$Z_3 = R + j\omega L$$

$$Z_3 = 200 + j100$$

$$Z_4 = Z_x = unknown$$

$$Z_4 = \frac{Z_2 Z_3}{Z_1} \qquad Z_4 = \frac{(300 - j600)(200 + j100)}{450} = 266.6 - j200$$

$$R = 266.6\Omega \qquad C = \frac{1}{2\pi F \times 200} = 0.79\mu F$$

Comparison Bridge: Capacitance

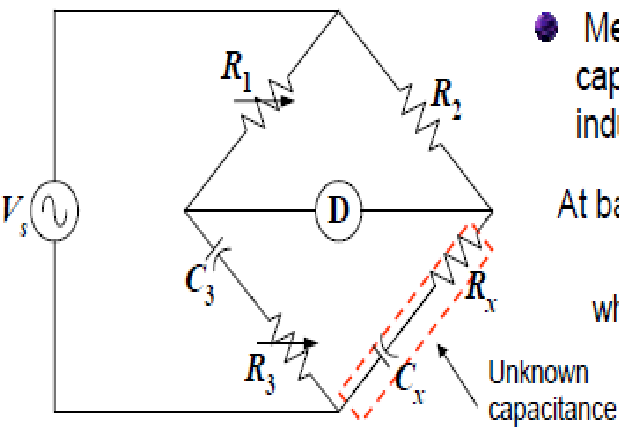


Diagram of Capacitance Comparison Bridge

- Measure an unknown inductance or capacitance by comparing with it with a known inductance or capacitance.

At balance point: $\mathbf{Z}_1\mathbf{Z}_x = \mathbf{Z}_2\mathbf{Z}_3$

where $\mathbf{Z}_1 = R_1;\ \mathbf{Z}_2 = R_2;$ and $\mathbf{Z}_3 = R_3 + \frac{1}{j\omega C_3}$

$$R_1\left(R_x + \frac{1}{j\omega C_x}\right) = R_2\left(R_3 + \frac{1}{j\omega C_3}\right)$$

Separation of the real and imaginary terms yields: $R_x = \frac{R_2 R_3}{R_1}$ and $C_x = C_3\frac{R_1}{R_2}$

- Frequency independent
- To satisfy both balance conditions, the bridge must contain two variable elements in its configuration.

Comparison Bridge: Inductance

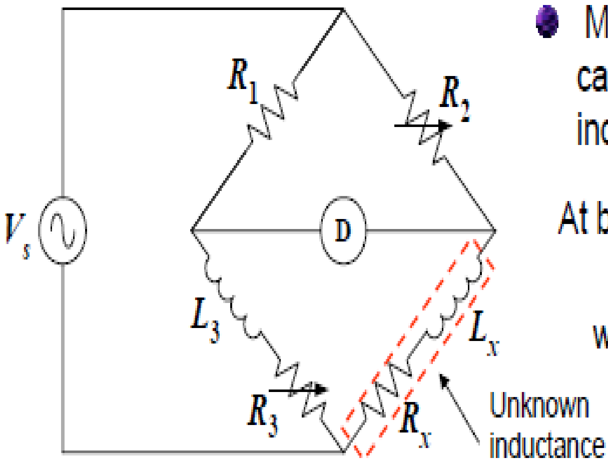


Diagram of Inductance Comparison Bridge

- Measure an unknown inductance or capacitance by comparing with it with a known inductance or capacitance.

At balance point: $\mathbf{Z}_1\mathbf{Z}_x = \mathbf{Z}_2\mathbf{Z}_3$

where $\mathbf{Z}_1 = R_1; \mathbf{Z}_2 = R_2;$ and $\mathbf{Z}_3 = R_3 + j\omega L_3$

$$R_1\left(R_x + j\omega L_x\right) = R_2\left(R_S + j\omega L_S\right)$$

Separation of the real and imaginary terms yields: $R_x = \dfrac{R_2 R_3}{R_1}$ and $L_x = L_3 \dfrac{R_2}{R_1}$

- Frequency independent
- To satisfy both balance conditions, the bridge must contain two variable elements in its configuration.