Hay Bridge

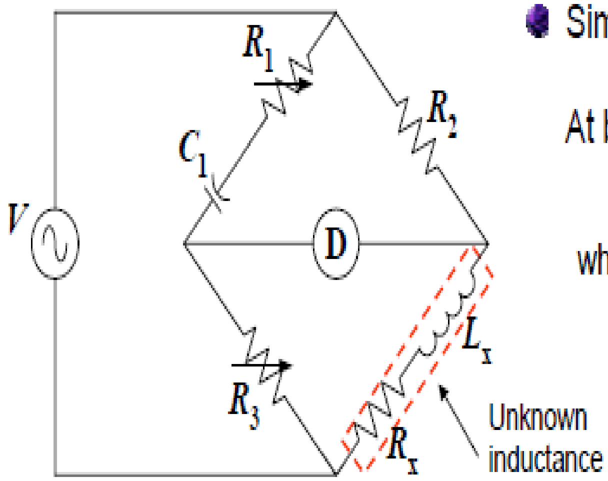


Diagram of Hay Bridge

- Similar to Maxwell bridge: but R_1 series with C_1

At balance point: $\mathbf{Z}_1\mathbf{Z}_x = \mathbf{Z}_2\mathbf{Z}_3$

where $\mathbf{Z}_1 = R_1 - \dfrac{j}{\omega C_1}$; $\mathbf{Z}_2 = R_2$; and $\mathbf{Z}_3 = R_3$

$$\left(R_1 + \frac{1}{j\omega C_1}\right)(R_x + j\omega L_x) = R_2 R_3$$

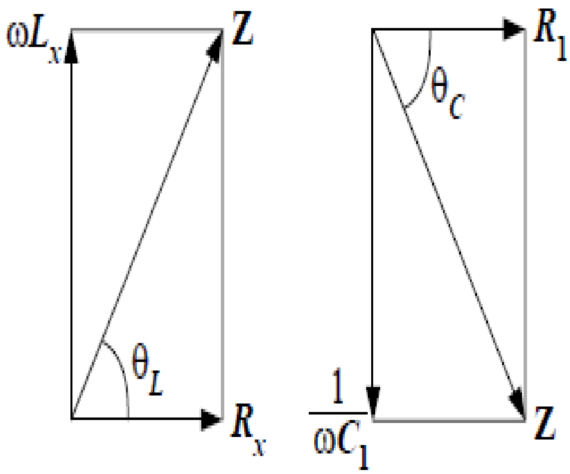
which expands to $R_1 R_x + \dfrac{L_x}{C_1} - \dfrac{jR_x}{\omega C_1} + j\omega L_x R_1 = R_2 R_3$

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \quad \text{.........(1)}$$

$$\frac{R_x}{\omega C_1} = \omega L_x R_1 \quad \text{..............(2)}$$

Solve the above equations simultaneously

$$R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1+\omega^2 C_1^2 R_1^2}$$ and $$L_x = \frac{R_2 R_3 C_1}{1+\omega^2 C_1^2 R_1^2}$$



Phasor diagram of arm 4 and 1

$$\tan\theta_L = \frac{X_L}{R} = \frac{\omega L_x}{R_x} = Q$$

$$\tan\theta_C = \frac{X_C}{R} = \frac{1}{\omega C_1 R_1}$$

$$\tan\theta_L = \tan\theta_C \text{ or } Q = \frac{1}{\omega C_1 R_1}$$

Thus, L_x can be rewritten as

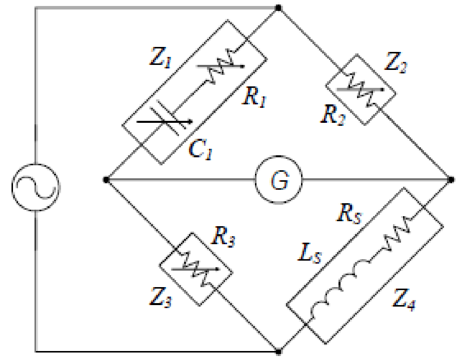
$$L_x = \frac{R_2 R_3 C_1}{1+(1/Q^2)}$$

For high Q coil (> 10), the term $(1/Q)^2$ can be neglected

$$L_x \approx R_2 R_3 C_1$$

EXAMPLE

Calculate the inductance and resistance of the network that causes a **Hay bridge** as shown in figure below to null with the following component values: **w=3000rad/s, C_1=0.1nF, R_1=20kΩ, R_2=10kΩ** and **R_3=1kΩ.**



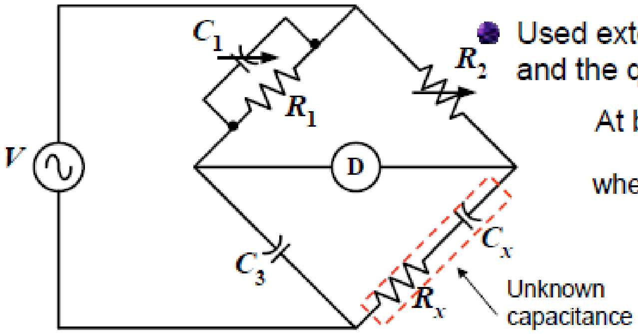
SOLUTION _ _

To find the series resistance and inductance, we use the above equations as:

$$L_S = \frac{C_1 R_2 R_3}{1 + w^2 R_1^2 C_1^2} = \frac{0.1n \times 10k \times 1k}{1 + (3000)^2 (20k)^2 (0.1n)^2} = 1mH$$

$$R_S = \frac{w^2 C_1^2 R_1 R_2 R_3}{1 + w^2 R_1^2 C_1^2} = \frac{(3000)^2 (0.1n)^2 \times 20k \times 10k \times 1k}{1 + (3000)^2 (20k)^2 (0.1n)^2} = 0.018\Omega$$

Schering Bridge



- Used extensively for the measurement of capacitance and the quality of capacitor in term of D

At balance point: $\mathbf{Z}_x = \mathbf{Z}_2\mathbf{Z}_3\mathbf{Y}_1$

where $\mathbf{Z}_2 = R_2$; $\mathbf{Z}_3 = \dfrac{1}{j\omega C_3}$; and $\mathbf{Y}_1 = \dfrac{1}{R_1} + j\omega C$

$$R_x - \frac{j}{\omega C_x} = R_2\left(\frac{-j}{\omega C_x}\right)\left(\frac{1}{R_1} + j\omega C_1\right)$$

Diagram of Schering Bridge

which expands to $R_x - \dfrac{j}{\omega C_x} = \dfrac{R_2 C_1}{C_3} - \dfrac{jR_2}{\omega C_3 R_1}$

Separation of the real and imaginary terms yields: $R_x = R_2 \dfrac{C_1}{C_3}$ and $C_x = C_3 \dfrac{R_1}{R_2}$

Dissipation factor of a series RC circuit: $D = \dfrac{R_x}{X_x} = \omega R_x C_x$

Dissipation factor tells us about the quality of a capacitor, how close the phase angle of the capacitor is to the ideal value of 90°

For Schering Bridge: $D = \omega R_x C_x = \omega R_1 C_1$

For Schering Bridge, R_1 is a fixed value, the dial of C_1 can be calibrated directly in D at one particular frequency

Wien Bridge

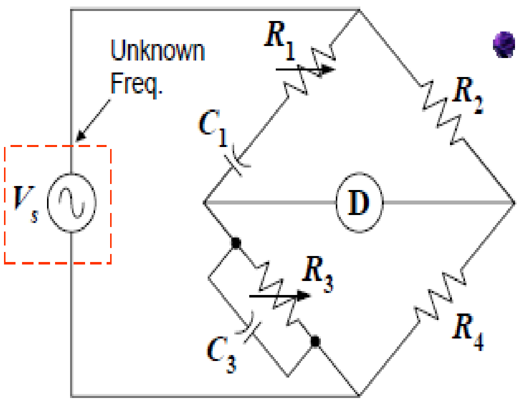


Diagram of Wien Bridge

- Measure frequency of the voltage source using series RC in one arm and parallel RC in the adjoining arm

At balance point: $\mathbf{Z}_2 = \mathbf{Z}_1\mathbf{Z}_4\mathbf{Y}_3$

$$\mathbf{Z}_1 = R_1 + \frac{1}{j\omega C_1}; \mathbf{Z}_2 = R_2; \mathbf{Y}_3 = \frac{1}{R_3} + j\omega C_3; \text{and } \mathbf{Z}_4 = R_4$$

$$R_2 = \left(R_1 - \frac{j}{\omega C_1}\right) R_4 \left(\frac{1}{R_3} + j\omega C_3\right)$$

which expands to $R_2 = \frac{R_1 R_4}{R_3} + j\omega C_3 R_1 R_4 - \frac{jR_4}{\omega C_1 R_3} + \frac{R_4 C_3}{C_1}$

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \quad \text{..........} \quad (1)$$

$$\omega C_3 R_1 = \frac{1}{\omega C_1 R_3} \quad \text{..........} \quad (2)$$

Rearrange Eq. (2) gives $f = \frac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}}$

In most, Wien Bridge, $R_1 = R_3$ and $C_1 = C_3$

(1) → $R_2 = 2R_4$ (2) → $f = \frac{1}{2\pi RC}$