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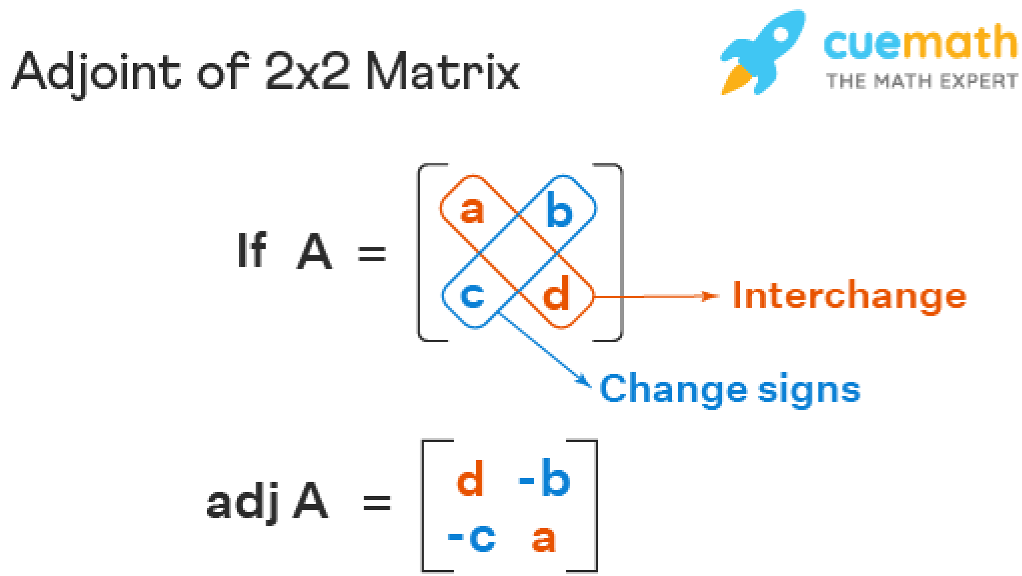
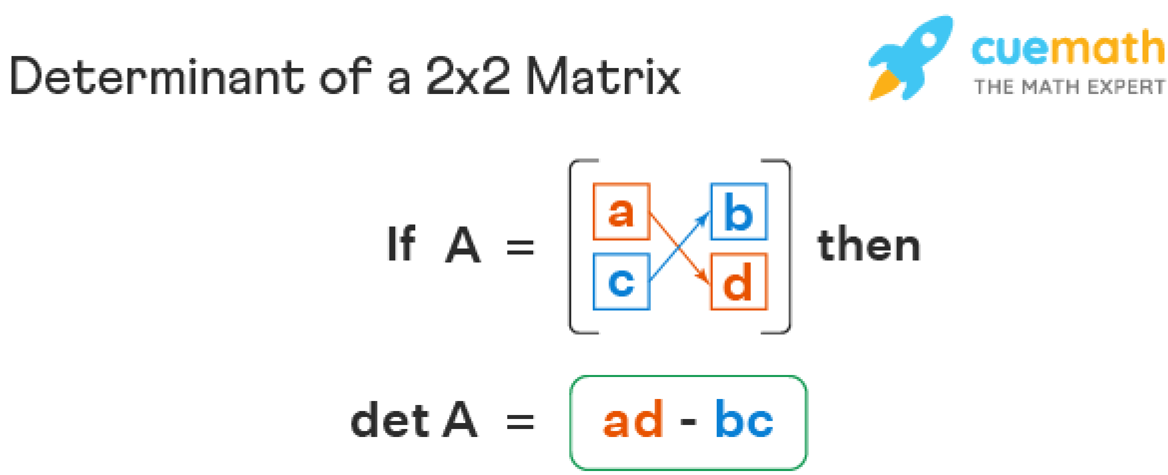
Matrix inverse

Chapter 6

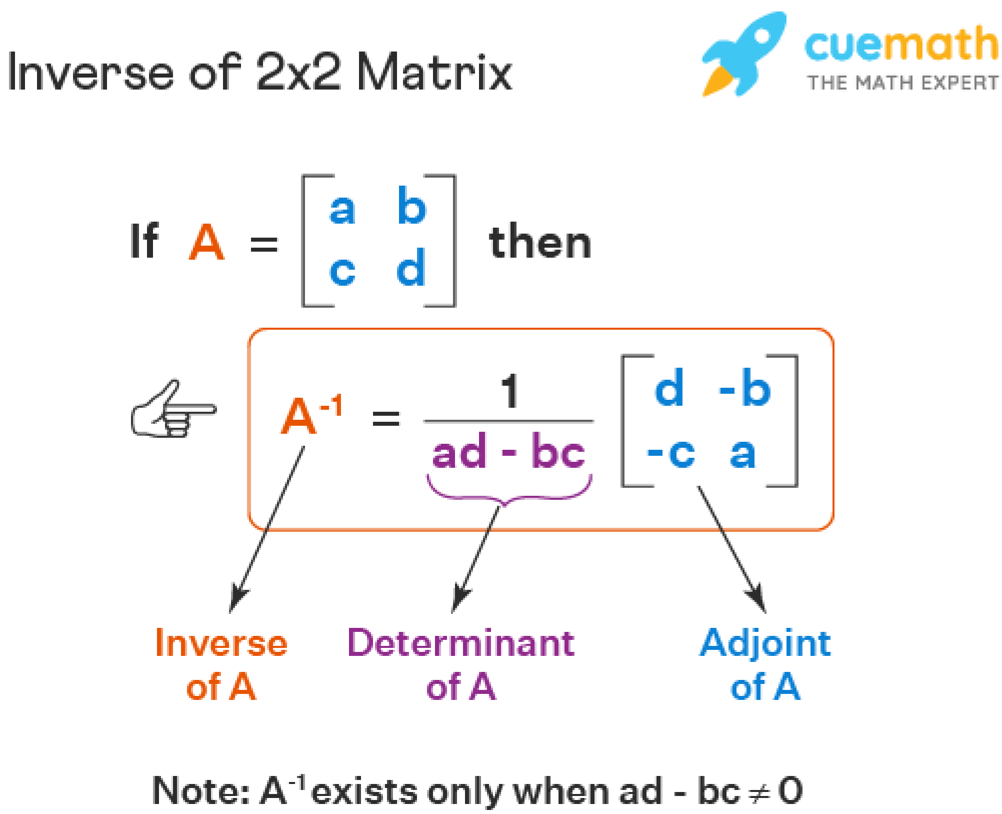
$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A$$


Adjoint of 2x2 Matrix
cuemath
THE MATH EXPERT
If A =
a b
c d
Interchange
Change signs
adj A =
d -b
-c a


Determinant of a 2x2 Matrix
cuemath
THE MATH EXPERT
If A =
a b
c d
then
det A =
ad - bc

Inverse of a Matrix

$$A^{-1} = \frac{1}{|A|} \text{Adj } A$$



Example 1: **Find the inverse of**

$$A = \begin{bmatrix} 2 & -3 \\ 4 & -7 \end{bmatrix}$$

<u>Solution</u>:

The following method to find the inverse is only applicable for 2 × 2 matrices.

1. Interchange leading diagonal elements:

-7 → 2; 2 → -7

$$\begin{bmatrix} -7 & -3 \\ 4 & 2 \end{bmatrix}$$

2. Change signs of the other 2 elements:

-3 → 3; 4 → -4

$$\begin{bmatrix} -7 & 3 \\ -4 & 2 \end{bmatrix}$$

3. Find the determinant $|A|$

$$\begin{vmatrix} 2 & -3 \\ 4 & -7 \end{vmatrix} = -14 + 12 = -2$$

4. Multiply result of [2] by 1/ |*A*|

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} -7 & 3 \\ -4 & 2 \end{bmatrix}$$
$$= \frac{1}{-2}\begin{bmatrix} -7 & 3 \\ -4 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 3.5 & -1.5 \\ 2 & -1 \end{bmatrix}$$

Finding Inverses 2x2

Example: Find the inverse of A.

$$A = \begin{bmatrix} 2 & 4 \\ -4 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{(2)(-10)-(-4)(4)}\begin{bmatrix} -10 & -4 \\ 4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-4}\begin{bmatrix} -10 & -4 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & 1 \\ -1 & -\frac{1}{2} \end{bmatrix}$$

Example: Find the inverse of matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}$

1. The determinant of $A = \begin{vmatrix} 3 & 2 \\ 1 & 5 \end{vmatrix} = (3 \times 5) - (2 \times 1) = 15 - 2 = 13.$

2. The adjoint of a matrix A

$$\text{adj } A = \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}$$

As we know, the formula of inverse for 2×2 matrix is given by:

$A^{-1} = (\text{Adj } A) / \det(A)$

$$A^{-1} = 1/13 \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}$$

Therefore $$A^{-1} = \begin{bmatrix} 5/13 & -2/13 \\ -1/13 & 3/13 \end{bmatrix}$$

Example: Find the inverse of

$$A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

Solution:

Now, we have to find

$$|A| = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

$\det(A) = 3(7) – 2(10) = 21 – 20 = 1 \neq 0$

$$\text{Adj } A = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

As we know, the formula of inverse for 2×2 matrix is given by:

$A^{-1} = (\text{Adj } A) / \det(A)$

$$A^{-1} = 1/1 \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

Therefore $A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$

Example 1.14

If $A=\begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix}$ then, find A^{-1}.

Solution

$$A = \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 4 \\ -3 & 2 \end{vmatrix}$$

$$= 16 \neq 0$$

Since A is a nonsingular matrix, A^{-1} exists

Now adj A $=\begin{bmatrix} 2 & -4 \\ 3 & 2 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|} adjA$$

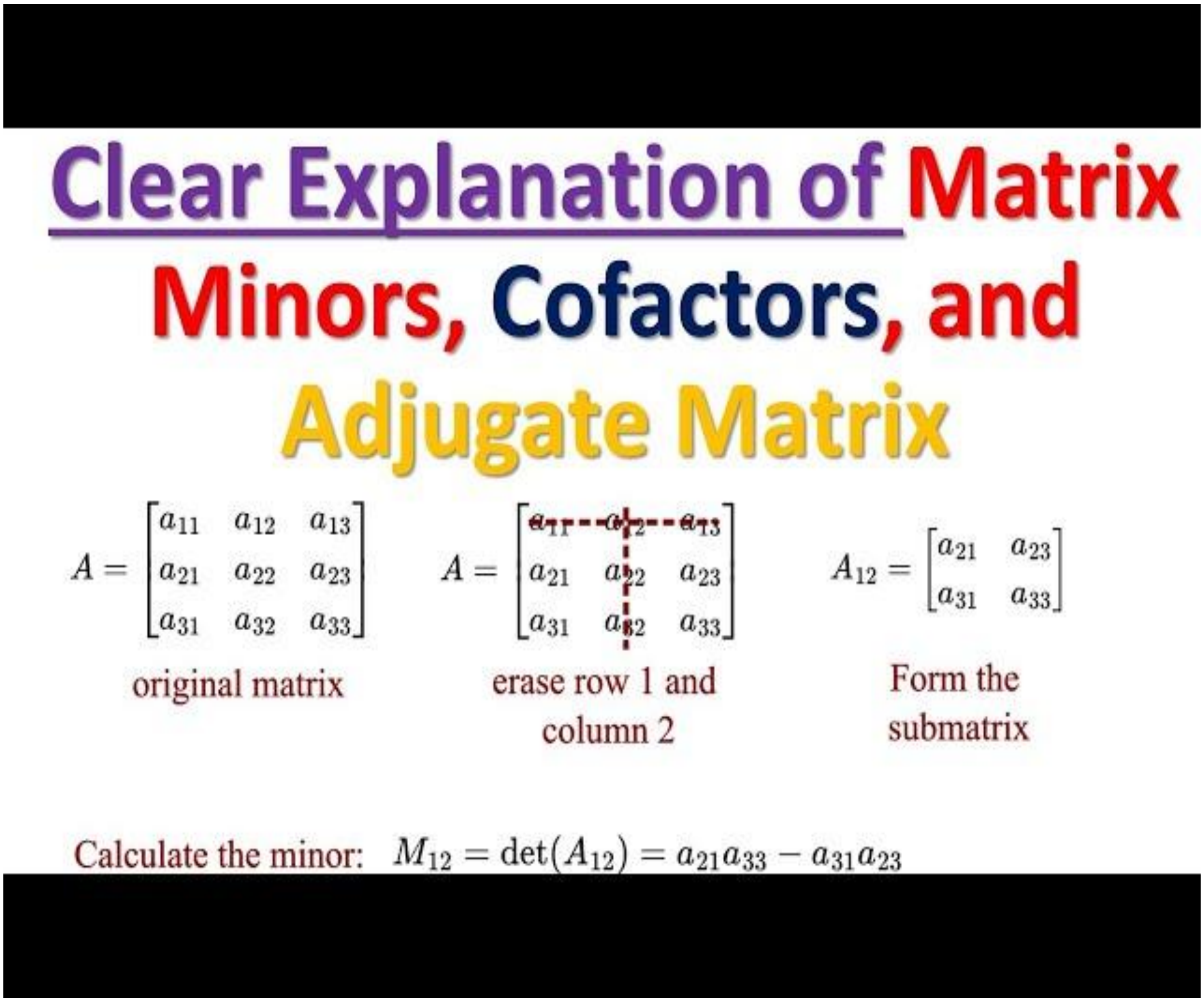
$$= \frac{1}{16}\begin{bmatrix} 2 & -4 \\ 3 & 2 \end{bmatrix}$$

Example 1.15

If $A = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$ then, find A^{-1}

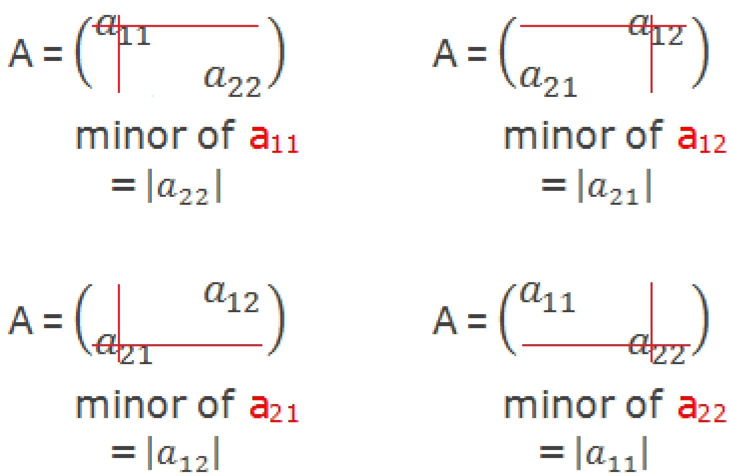
Since det A = (-2)(-9)-(18)=18-18=0

Therefore A^{-1} does not exist.



Minors For 2×2 Matrix

Let $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ be a 2×2 matrix. Then the minor of the element a_{11} is the determinant of matrix formed by omitting 1st row and 1st column of A. i.e. $M_{11} = |a_{22}|$. Similarly, minor of a_{12} is $M_{12} = |a_{21}|$ and so on.



Thus,

M_{11} = minor of a_{11} = $|a_{22}|$ = a_{22}

M_{12} = minor of a_{12} = $|a_{21}|$ = a_{21}

M_{21} = minor of a_{21} = $|a_{12}|$ = a_{12}

M_{22} = minor of a_{22} = $|a_{11}|$ = a_{11}

Cofactors:

Let A be a square matrix and a_{ij} is the element in i^{th} row and j^{th} column of A. Then the **cofactor** of the element a_{ij} is given by the number $(-1)^{i+j} M_{ij}$, where M_{ij} is the minor of the element a_{ij}. The cofactor of element a_{ij} is denoted by A_{ij}.

Cofactors For 2×2 Matrix

Let $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ be a 2×2 matrix. Thus, the cofactors of the elements of A are as follows:

$A_{11} = \text{cofactor of } a_{11} = (-1)^{1+1} M_{11} = |a_{22}| = a_{22}$

$A_{12} = \text{cofactor of } a_{12} = (-1)^{1+2} M_{12} = -|a_{21}| = -a_{21}$

$A_{21} = \text{cofactor of } a_{21} = (-1)^{2+1} M_{21} = -|a_{12}| = -a_{12}$

$A_{22} = \text{cofactor of } a_{22} = (-1)^{2+2} M_{22} = |a_{11}| = a_{11}$

Adj of matrix A = **Transpose** of matrix A

$$\textbf{Adj } A = \textbf{Transpose} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$\textbf{Adj A} = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix}$$

Inverse of a Matrix

$$A^{-1} = \frac{1}{|A|} \text{ Adj A}$$

Example 1: Find the minor and cofactor of matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

Solution: Here,

Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

Then,

$a_{11} = 1$, $a_{12} = 2$, $a_{21} = 3$ and $a_{22} = 4$

Thus,

M_{11} = minor of $a_{11} = |4| = 4$

M_{12} = minor of $a_{12} = |3| = 3$

M_{21} = minor of $a_{21} = |2| = 2$

M_{22} = minor of $a_{22} = |1| = 1$

And,

A_{11} = cofactor of $a_{11} = (-1)^{1+1} M_{11} = 4$

A_{12} = cofactor of $a_{12} = (-1)^{1+2} M_{12} = -3$

A_{21} = cofactor of $a_{21} = (-1)^{2+1} M_{21} = -2$

A_{22} = cofactor of $a_{22} = (-1)^{2+2} M_{22} = 1$

Now to find the inverse of the matrix A:

The matrix of cofactors is = $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$

Adj of matrix A = **Transpose** of matrix A

$$\textbf{Adj } A = \textbf{Transpose} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$= \text{transpose of} \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$$

$$\textbf{Adj A} = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

Inverse of a Matrix

$$A^{-1} = \frac{1}{|A|} \text{ Adj A}$$

$$A^{-1} = 1 / ((4 \times 1) - (-2 \times -3)) \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= 1 / -2 \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3/2 & 1/-2 \end{bmatrix}$$

Minors For 3×3 Matrix

Let $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ be a 3×3 matrix. Then the minor of the element a_{11} is the determinant of the matrix formed by omitting 1^{st} row and 1^{st} column of A. i.e. $M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$.

Similarly, minor of a_{12} is $M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$ and so on.

$$\begin{pmatrix} a_{11} & & \\ & a_{22} & a_{23} \\ & a_{32} & a_{33} \end{pmatrix}$$
minor of a_{11} $= \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

$$\begin{pmatrix} & a_{12} & \\ a_{21} & & a_{23} \\ a_{31} & & a_{33} \end{pmatrix}$$
minor of a_{12} $= \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

$$\begin{pmatrix} & & a_{13} \\ a_{21} & a_{22} & \\ a_{31} & a_{32} & \end{pmatrix}$$
minor of a_{13} $= \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

$$\begin{pmatrix} & a_{12} & a_{13} \\ a_{21} & & \\ & a_{32} & a_{33} \end{pmatrix}$$
minor of a_{21} $= \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$

$$\begin{pmatrix} a_{11} & & a_{13} \\ & a_{22} & \\ a_{31} & & a_{33} \end{pmatrix}$$
minor of a_{22} $= \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$

$$\begin{pmatrix} a_{11} & a_{12} & \\ & & a_{23} \\ a_{31} & a_{32} & \end{pmatrix}$$
minor of a_{23} $= \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$

$$\begin{pmatrix} & a_{12} & a_{13} \\ & a_{22} & a_{23} \\ a_{31} & & \end{pmatrix}$$
minor of a_{31} $= \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$

$$\begin{pmatrix} a_{11} & & a_{13} \\ a_{21} & & a_{23} \\ & a_{32} & \end{pmatrix}$$
minor of a_{32} $= \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$

$$\begin{pmatrix} a_{11} & a_{12} & \\ a_{21} & a_{22} & \\ & & a_{33} \end{pmatrix}$$
minor of a_{33} $= \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

Minors for 3 x 3 matrix

$$M_{11} = \text{minor of } a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}a_{33} - a_{23}a_{32}$$

$$M_{12} = \text{minor of } a_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = a_{21}a_{33} - a_{23}a_{31}$$

$$M_{13} = \text{minor of } a_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = a_{21}a_{32} - a_{22}a_{31}$$

$$M_{21} = \text{minor of } a_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = a_{12}a_{33} - a_{13}a_{32}$$

$$M_{22} = \text{minor of } a_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} = a_{11}a_{33} - a_{13}a_{31}$$

$$M_{23} = \text{minor of } a_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} = a_{11}a_{32} - a_{12}a_{31}$$

$$M_{31} = \text{minor of } a_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} = a_{12}a_{23} - a_{13}a_{22}$$

$$M_{32} = \text{minor of } a_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} = a_{11}a_{23} - a_{13}a_{21}$$

$$M_{33} = \text{minor of } a_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

Cofactors For 3×3 Matrix

Let $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ be a 3×3 matrix. Thus, the cofactors of the elements of A are as follows:

$A_{11} = \text{cofactor of } a_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

$A_{12} = \text{cofactor of } a_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

$A_{13} = \text{cofactor of } a_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

$A_{21} = \text{cofactor of } a_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$

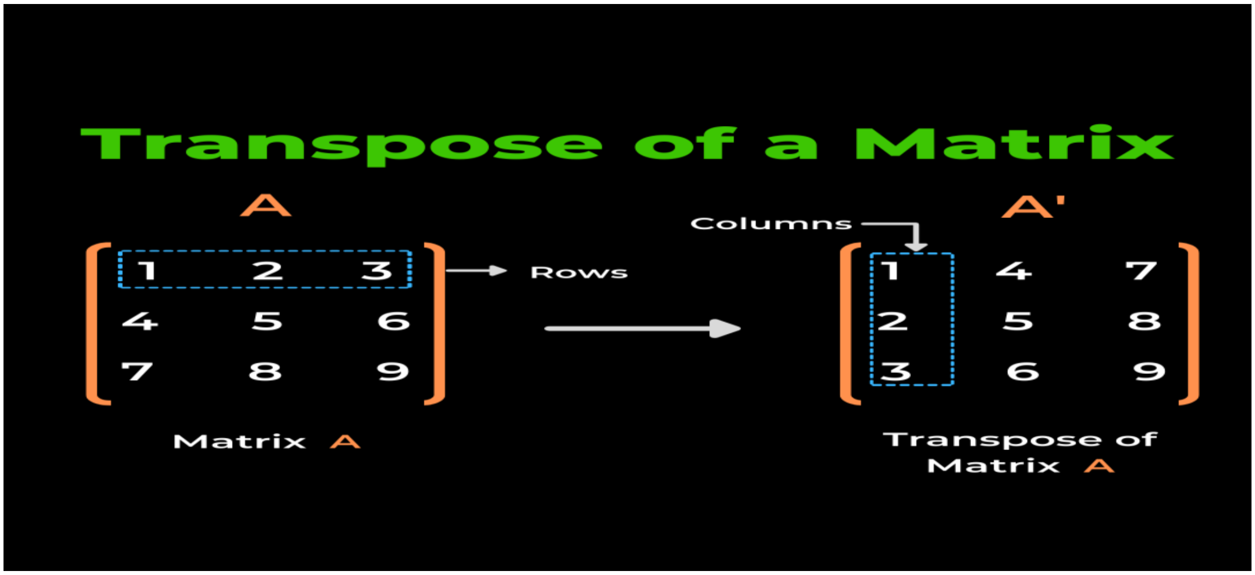
$A_{22} = \text{cofactor of } a_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$

$A_{23} = \text{cofactor of } a_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$

$A_{31} = \text{cofactor of } a_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$

$A_{32} = \text{cofactor of } a_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$

$A_{33} = \text{cofactor of } a_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$


Transpose of a Matrix
A
A'
Columns
1 2 3
4 5 6
7 8 9
Rows
1 4 7
2 5 8
3 6 9
Matrix A
Transpose of
Matrix A

The **transpose** of a matrix A is denoted by A' and is formed by **swapping the rows and columns** of the matrix A.

The first **row** of the matrix A becomes the first **column** of the matrix A', and so on.

Example

If $A = \begin{pmatrix} 3 & 4 & 5 \\ 2 & -1 & 3 \end{pmatrix}$, then $A' = \begin{pmatrix} 3 & 2 \\ 4 & -1 \\ 5 & 3 \end{pmatrix}$.

Note that the matrix A is of order 2×3, whereas the matrix A' is of order 3×2.

This is true in general; if the matrix A is of order $m \times n$, then the matrix A' will be of order $n \times m$.

[*The transpose has the following properties:*

(1) $(A')' = A$ for any matrix A.

(2) $(A+B)' = A' + B'$ for any matrices A and B of the same order.

(3) $(kA)' = kA'$ for any matrix A and scalar k.]

Adjoint of a Matrix

$$\textbf{adj A} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}^{T}$$

$$= \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{31} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

Let $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ be a 3×3 matrix, and A_{ij} be the cofactor of the element a_{ij}, then **adjoint** or **adjugate of matrix** A is defined by,

$$\textbf{adj A} = \text{transpose of} \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$$

$$= \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{31} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

$$\text{adj } A = \begin{bmatrix} +\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & +\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & +\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \\ +\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & +\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix}$$

Example 2: Find the minors and cofactors of matrix $\begin{pmatrix} 1 & 2 & -3 \\ 2 & 0 & 4 \\ 3 & 2 & 1 \end{pmatrix}$.

Solution: Here,

$$Let\ A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 0 & 4 \\ 3 & 2 & 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Then,

$a_{11} = 1$ $\quad a_{12} = 2$ $\quad a_{13} = -3$

$a_{21} = 2$ $\quad a_{22} = 0$ $\quad a_{23} = 4$

$a_{31} = 3$ $\quad a_{32} = 2$ $\quad a_{33} = 1$

Thus,

$M_{11} = \text{minor of } a_{11} = \begin{vmatrix} 0 & 4 \\ 2 & 1 \end{vmatrix} = 0 - 8 = -8$

$M_{12} = \text{minor of } a_{12} = \begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} = 2 - 12 = -10$

$M_{13} = \text{minor of } a_{13} = \begin{vmatrix} 2 & 0 \\ 3 & 2 \end{vmatrix} = 4 - 0 = 4$

$M_{21} = \text{minor of } a_{21} = \begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = 2 + 4 = 8$

$M_{22} = \text{minor of } a_{22} = \begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix} = 1 + 9 = 10$

$M_{23} = \text{minor of } a_{23} = \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = 2 - 6 = -4$

$M_{31} = \text{minor of } a_{31} = \begin{vmatrix} 2 & -3 \\ 0 & 4 \end{vmatrix} = 8 - 0 = 8$

$M_{32} = \text{minor of } a_{32} = \begin{vmatrix} 1 & -3 \\ 2 & 4 \end{vmatrix} = 4 + 6 = 10$

$M_{33} = \text{minor of } a_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} = 0 - 4 = -4$

And,

$A_{11} = \text{cofactor of } a_{11} = (-1)^{1+1} M_{11} = -8$

$A_{12} = \text{cofactor of } a_{12} = (-1)^{1+2} M_{12} = -(-10) = 10$

$A_{13} = \text{cofactor of } a_{13} = (-1)^{1+3} M_{13} = 4$

$A_{21} = \text{cofactor of } a_{21} = (-1)^{2+1} M_{21} = -8$

$A_{22} = \text{cofactor of } a_{22} = (-1)^{2+2} M_{22} = 10$

$A_{23} = \text{cofactor of } a_{23} = (-1)^{2+3} M_{23} = -(-4) = 4$

$A_{31} = \text{cofactor of } a_{31} = (-1)^{3+1} M_{31} = 8$

$A_{32} = \text{cofactor of } a_{32} = (-1)^{3+2} M_{32} = -10$

$A_{33} = \text{cofactor of } a_{33} = (-1)^{3+3} M_{33} = -4$

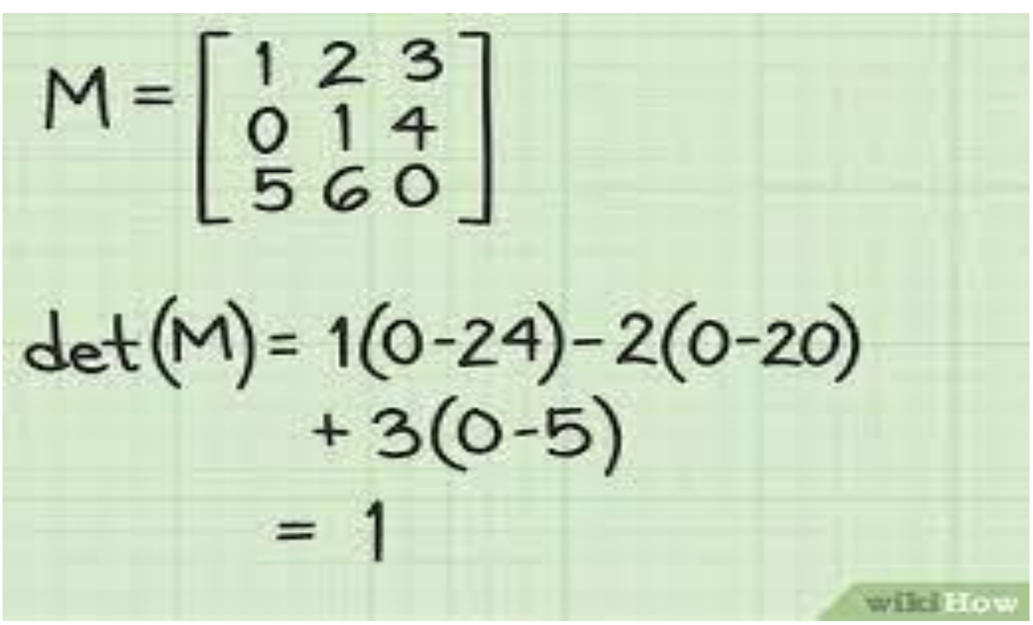
$$\text{Matrix of Cofactors} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} = \begin{pmatrix} -8 & -10 & 4 \\ 8 & 10 & -4 \\ 8 & -10 & -4 \end{pmatrix}$$

$$\textbf{AdJ(A)} = \mathbf{A^T} = \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} = \begin{pmatrix} -8 & 8 & 8 \\ -10 & 10 & -10 \\ 4 & -4 & -4 \end{pmatrix}$$

Find det(A), then apply the following to find the inverse of A

$$A^{-1} = (1 / \det A) \,.\, \text{AdJ}(A)$$

Example:



$$M^T = \begin{bmatrix} 1 & 0 & 5 \\ 2 & 1 & 6 \\ 3 & 4 & 0 \end{bmatrix}$$

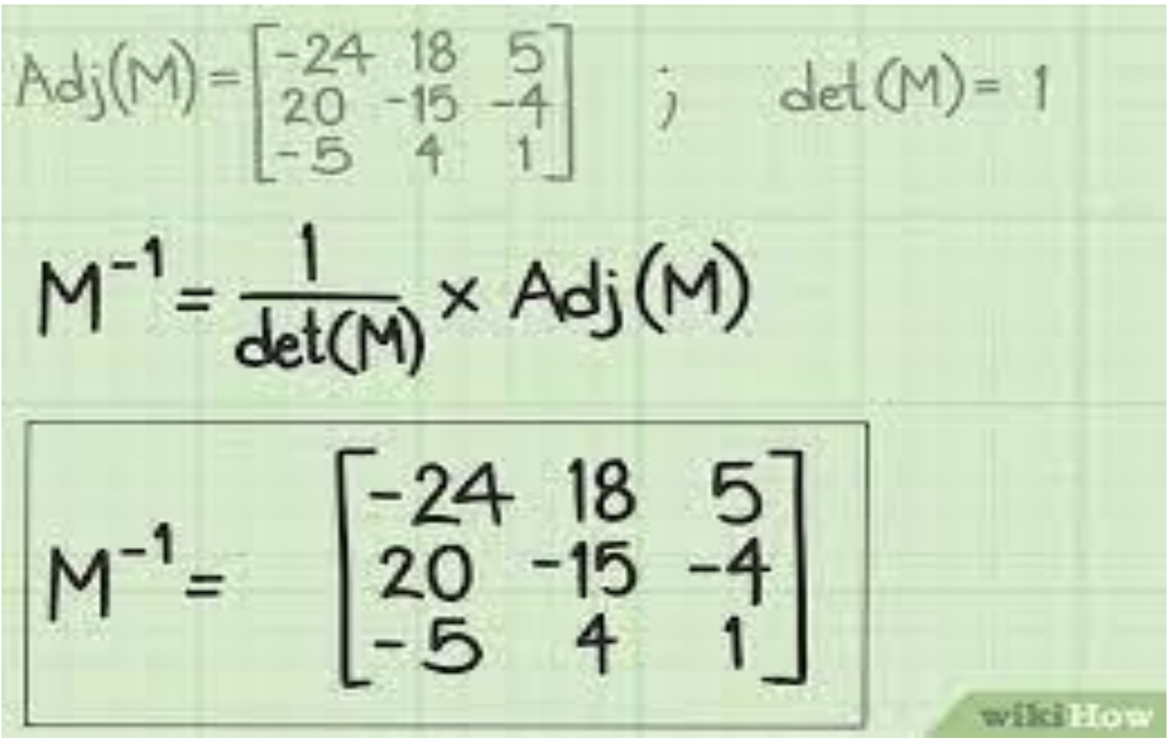
$$\begin{vmatrix} 1 & 6 \\ 4 & 0 \end{vmatrix} = -24 \qquad \begin{vmatrix} 2 & 6 \\ 3 & 0 \end{vmatrix} = -18 \qquad \begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} = 5$$

$$\begin{vmatrix} 0 & 5 \\ 4 & 0 \end{vmatrix} = -20 \qquad \begin{vmatrix} 1 & 5 \\ 3 & 0 \end{vmatrix} = -15 \qquad \begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix} = 4$$

$$\begin{vmatrix} 0 & 5 \\ 1 & 6 \end{vmatrix} = -5 \qquad \begin{vmatrix} 1 & 5 \\ 2 & 6 \end{vmatrix} = -4 \qquad \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1$$

$$Adj(M) = \begin{bmatrix} -24 & -18 & 5 \\ -20 & -15 & 4 \\ -5 & -4 & 1 \end{bmatrix} \times \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$Adj(M) = \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}$$


Adj(M) = [-24 18 5; 20 -15 -4; -5 4 1] ; det(M) = 1
M^-1 = 1/det(M) × Adj(M)
M^-1 = [-24 18 5; 20 -15 -4; -5 4 1]
wikiHow

Exercise 1.2

1. Find the adjoint of the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

2. If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ then verify that $A\,(adj\ A) = |A|\, I$ and also find A^{-1}

3. Find the inverse of each of the following matrices

   (i) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & 1 \\ -1 & 3 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$ (iv) $\begin{bmatrix} -3 & -5 & 4 \\ -2 & 3 & -1 \\ 1 & -4 & -6 \end{bmatrix}$

4. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 4 \\ 1 & -2 \end{bmatrix}$, then verify $adj\,(AB) = (adj\ B)(adj\ A)$

5. If $A = \begin{bmatrix} 2 & -2 & 2 \\ 2 & 3 & 0 \\ 9 & 1 & 5 \end{bmatrix}$ then, show that $(adj\ A)A = O$

6. If $A = \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{bmatrix}$ then, show that the inverse of A is A itself.

7. If $A^{-1} = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ then, find A.

8. Show that the matrices $A=\begin{bmatrix}2&2&1\\1&3&1\\1&2&2\end{bmatrix}$ and $B=\begin{bmatrix}\frac{4}{5}&-\frac{2}{5}&-\frac{1}{5}\\-\frac{1}{5}&\frac{3}{5}&-\frac{1}{5}\\-\frac{1}{5}&-\frac{2}{5}&\frac{4}{5}\end{bmatrix}$ are inverses of each other.

9. If $A=\begin{bmatrix}3&7\\2&5\end{bmatrix}$ and $B=\begin{bmatrix}6&8\\7&9\end{bmatrix}$ then, verify that $(AB)^{-1}=B^{-1}A^{-1}$

10. Find λ if the matrix $\begin{bmatrix}1&1&3\\2&\lambda&4\\9&7&11\end{bmatrix}$ has no inverse.

11. If $X=\begin{bmatrix}8&-1&-3\\-5&1&2\\10&-1&-4\end{bmatrix}$ and $Y=\begin{bmatrix}2&1&-1\\0&2&1\\5&p&q\end{bmatrix}$ then, find p, q if $Y=X^{-1}$