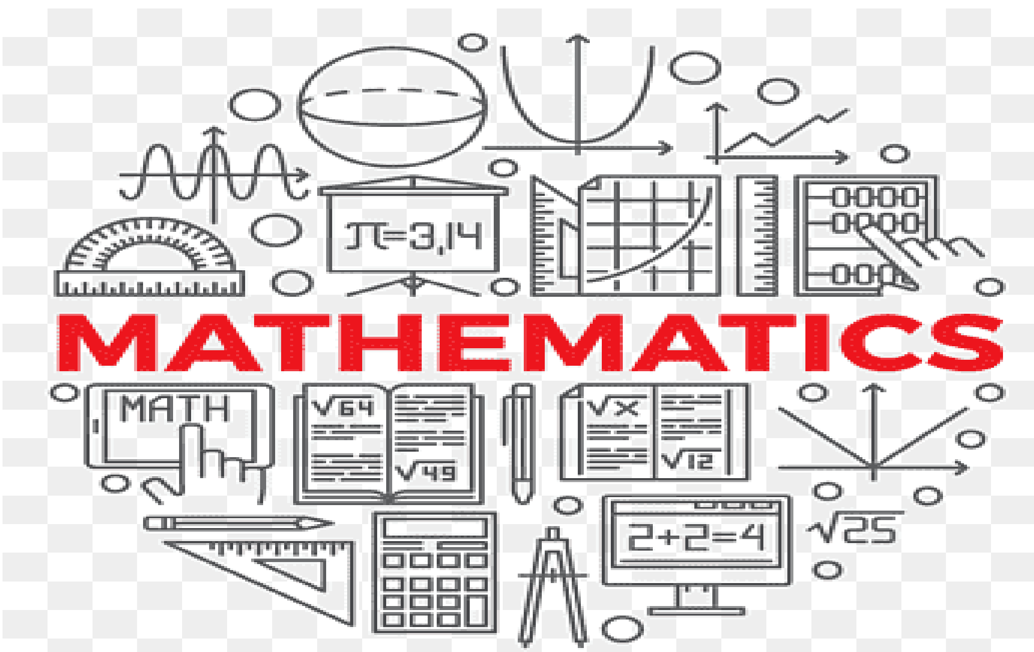
Cyber Security Engineering Department

قسم هندسة تقنيات الامن السيبراني

الرياضيات

المرحلة الاولى



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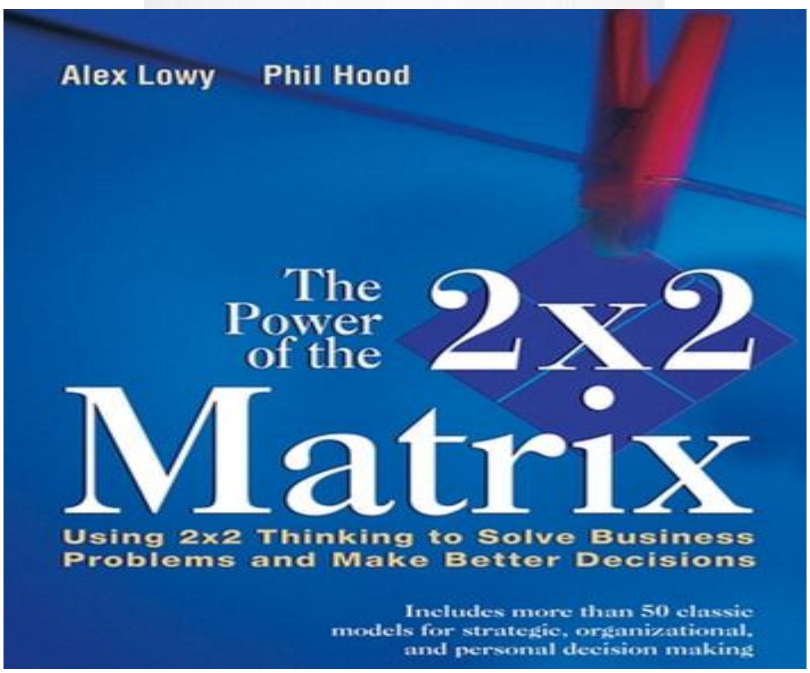

How to Find the Determinant of a 2×2 Matrix

Suppose we are given a square matrix A with four elements: a, b, c, and d.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det A = \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} =$$

$$\det A = ad - bc$$

Examples of How to Find the Determinant of a 2×2 Matrix

Example 1: Find the determinant of the matrix below.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\det\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = (1)(4) - (2)(3)$$
$$= 4 - 6$$
$$= -2 \checkmark$$

Example 2: Calculate the determinant of the matrix below.

$$B = \begin{bmatrix} -5 & -4 \\ -2 & -3 \end{bmatrix}$$

$$\det\begin{bmatrix} -5 & -4 \\ -2 & -3 \end{bmatrix} = (-5)(-3)-(-4)(-2)$$
$$= 15-8$$
$$= 7 \checkmark$$

Example 3: Evaluate the determinant of the matrix below.

$$C = \begin{bmatrix} -1 & -2 \\ 6 & 3 \end{bmatrix}$$

$$\det\begin{bmatrix} -1 & -2 \\ 6 & 3 \end{bmatrix} = (-1)(3)-(-2)(6)$$
$$= (-3)-(-12)$$
$$= -3+12$$
$$= 9 \checkmark$$

Example 4: Determine the determinant of the matrix below.

$$D = \begin{bmatrix} 5 & -3 \\ x & y \end{bmatrix}$$

$$\det\begin{bmatrix} 5 & -3 \\ x & y \end{bmatrix} = (5)(y)-(-3)(x)$$
$$= 5y-(-3x)$$
$$= 5y+3x \checkmark$$

Quick Examples of How to Find the Determinants of a 2 x 2 Matrix

Example 1: Find the determinant of the matrix A below.

$$A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (1)(4) - (2)(3) = 4 - 6 = -2$$

Example 2: Find the determinant of the matrix B below.

$$\mathbf{B} = \begin{bmatrix} 5 & -1 \\ 2 & -3 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 5 & -1 \\ 2 & -3 \end{vmatrix} = (5)(-3) - (-1)(2) = -15 - (-2) = -15 + 2 = -13$$

Cramer's Rules for Systems of Linear Equations with Two Variables

- Given a linear system

x-column   constant column

$$a_1 x + b_1 y = c_1$$
$$a_2 x + b_2 y = c_2$$

y-column

coefficient matrix:

$$D = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \qquad D_x = \begin{bmatrix} c_1 & b_1 \\ c_2 & b_2 \end{bmatrix}$$

$$D_y = \begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix}$$

To solve for the variable x:

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

To solve for the variable y:

$$y = \frac{D_y}{D} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Examples of How to Solve Systems of Linear Equations with Two Variables using Cramer’s Rule

Example 1: Solve the system with two variables by Cramer's Rule

$$4x - 3y = 11$$

$$6x + 5y = 7$$

- For **coefficient matrix**

$$D = \begin{bmatrix} 4 & -3 \\ 6 & 5 \end{bmatrix} \xrightarrow{\text{find its determinant}} |D| = \begin{vmatrix} 4 & -3 \\ 6 & 5 \end{vmatrix} = (4)(5) - (-3)(6)$$

$$= 20 - (-18)$$

$$= 20 + 18$$

$$= 38$$

- For **X – matrix**

$$D_x = \begin{bmatrix} 11 & -3 \\ 7 & 5 \end{bmatrix} \xrightarrow{\text{find its determinant}} |D_x| = \begin{vmatrix} 11 & -3 \\ 7 & 5 \end{vmatrix} = (11)(5) - (-3)(7)$$

$$= 55 - (-21)$$

$$= 55 + 21$$

$$= 76$$

- For **Y – matrix**

$$D_y = \begin{bmatrix} 4 & 11 \\ 6 & 7 \end{bmatrix} \xrightarrow{\text{find its determinant}} |D_y| = \begin{vmatrix} 4 & 11 \\ 6 & 7 \end{vmatrix} = (4)(7) - (11)(6)$$

$$= 28 - (66)$$

$$= -38$$

Once all three determinants are calculated, it's time to solve for the values of x and y using the formula above.

$$x = \frac{D_x}{D} = \frac{76}{38} = 2$$

$$y = \frac{D_y}{D} = \frac{-38}{38} = -1$$

I can write the final answer as: $(x, y) = (2, 1)$.

Example 2: Solve the system with two variables by Cramer's Rule

$$3x + 5y = -7$$
$$x + 4y = -14$$

$$D = \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix} \xrightarrow{\text{find its determinant}} |D| = \begin{vmatrix} 3 & 5 \\ 1 & 4 \end{vmatrix} = (3)(4) - (5)(1) = 7$$

- For the **X – matrix** (replace the x-column by the constant column)

$$D_x = \begin{bmatrix} -7 & 5 \\ -14 & 4 \end{bmatrix} \xrightarrow{\text{find its determinant}} |D_x| = \begin{vmatrix} -7 & 5 \\ -14 & 4 \end{vmatrix} = (-7)(4) - (5)(-14) = 42$$

- For the **Y – matrix** (replace the y-column by the constant column)

$$D_y = \begin{bmatrix} 3 & -7 \\ 1 & -14 \end{bmatrix} \xrightarrow{\text{find its determinant}} |D_y| = \begin{vmatrix} 3 & -7 \\ 1 & -14 \end{vmatrix} = (3)(-14) - (-7)(1) = -35$$

$$x = \frac{D_x}{D} = \frac{42}{7} = 6$$

$$y = \frac{D_y}{D} = \frac{-35}{7} = -5$$

Example 3: Solve the system with two variables by Cramer's Rule

$$x - 4y = -9$$
$$-x + 5y = 11$$

- For **coefficient matrix**

$$D = \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix} \xrightarrow{\text{find its determinant}} |D| = \begin{vmatrix} 1 & -4 \\ -1 & 5 \end{vmatrix} = (1)(5) - (-4)(-1) = 1$$

- For **X – matrix** (written as uppercase D with subscript x)

$$D_x = \begin{bmatrix} -9 & -4 \\ 11 & 5 \end{bmatrix} \xrightarrow{\text{find its determinant}} |D_x| = \begin{vmatrix} -9 & -4 \\ 11 & 5 \end{vmatrix} = (-9)(5) - (-4)(11) = -1$$

- For **Y – matrix** (written as uppercase D with subscript y)

$$D_y = \begin{bmatrix} 1 & -9 \\ -1 & 11 \end{bmatrix} \xrightarrow{\text{find its determinant}} |D_y| = \begin{vmatrix} 1 & -9 \\ -1 & 11 \end{vmatrix} = (1)(11) - (-9)(-1) = 2$$

calculate x and y as follows.

$$x = \frac{D_x}{D} = \frac{-1}{1} = -1$$

$$y = \frac{D_y}{D} = \frac{2}{1} = 2$$

Example 4: Solve by Cramer's Rule the system with two variables

$$-2x + 3y = -3$$

$$3x - 4y = 5$$

- For **coefficient matrix**

$$D=\begin{bmatrix}-2 & 3\\ 3 & -4\end{bmatrix}\xrightarrow{\text{find its determinant}}|D|=\begin{vmatrix}-2 & 3\\ 3 & -4\end{vmatrix}=(-2)(-4)-(3)(3)=-1$$

- For **X – matrix**

$$D_x=\begin{bmatrix}-3 & 3\\ 5 & -4\end{bmatrix}\xrightarrow{\text{find its determinant}}|D_x|=\begin{vmatrix}-3 & 3\\ 5 & -4\end{vmatrix}=(-3)(-4)-(3)(5)=-3$$

- For **Y – matrix**

$$D_y=\begin{bmatrix}-2 & -3\\ 3 & 5\end{bmatrix}\xrightarrow{\text{find its determinant}}|D_y|=\begin{vmatrix}-2 & -3\\ 3 & 5\end{vmatrix}=(-2)(5)-(-3)(3)=-1$$

$$x=\frac{D_x}{D}=\frac{-3}{-1}=3$$

$$y=\frac{D_y}{D}=\frac{-1}{-1}=1$$

Example 5: Solve the system with two variables by Cramer's Rule

$$5x + y = -13$$

$$3x - 2y = 0$$

- For **coefficient matrix**

$$D = \begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix} \xrightarrow{\text{find its determinant}} |D| = \begin{vmatrix} 5 & 1 \\ 3 & -2 \end{vmatrix} = (5)(-2) - (1)(3) = -13$$

- For **X – matrix**

$$D_x = \begin{bmatrix} -13 & 1 \\ 0 & -2 \end{bmatrix} \xrightarrow{\text{find its determinant}} |D_x| = \begin{vmatrix} -13 & 1 \\ 0 & -2 \end{vmatrix} = (-13)(-2) - (1)(0) = 26$$

- For **Y – matrix**

$$D_y = \begin{bmatrix} 5 & -13 \\ 3 & 0 \end{bmatrix} \xrightarrow{\text{find its determinant}} |D_y| = \begin{vmatrix} 5 & -13 \\ 3 & 0 \end{vmatrix} = (5)(0) - (-13)(3) = 39$$

The final solution to this problem is

$$x = \frac{D_x}{D} = \frac{26}{-13} = -2$$

$$y = \frac{D_y}{D} = \frac{39}{-13} = -3$$

Home Works

Problem 1: Find the determinant of the matrix below.

$$\begin{pmatrix} 3 & -32 \\ -32 & 3 \end{pmatrix}$$

Problem 2: Find the determinant of the matrix below.

$$\begin{pmatrix} 0.5 & -4 \\ -6 & 10 \end{pmatrix}$$

Problem 3: The determinant of a 2×2 matrix is 2. Find the value of x.

$$\det\begin{bmatrix} -2 & x \\ 4 & 5 \end{bmatrix} = 2$$

Problem 4: The determinant of a 2×2 matrix is -11. Find the value of y.

$$\det\begin{bmatrix} -1 & 2 \\ y & 3 \end{bmatrix} = -11$$

Problem 5: The determinant of a 2×2 matrix is -8. Find the value of k.

$$\det\begin{bmatrix} k & 3 \\ -9 & 7 \end{bmatrix} = -8$$

Problem 6: **Solve simultaneous equations 5x – 4y = 2 and 6x – 5y = 1 by using Cramer's rule.**

5x – 4y = 2 …………. (i)
6x – 5y = 1 …………. (ii)

Answer: x = 6 and y = 7

Problem 7: Use Cramer's rule to solve the equations: $x + 2y = 7$ and $2x – y = 4$.

$x + 2y = 7$ .... (i)
$2x – y = 4$ .... (ii)

Answer: $x = 3$ and $y = 2$

For the following exercises, solve the system of linear equations using Cramer's Rule.

8.

$2x-3y=-1$

$x+5y=9$

9.

$5x-4y=2$

$-4x+7y=6$

10.

$6x-3y=2$

$-8x+9y=-1$

11.

$2x+6y=12$

$5x-2y=13$

12.

$4x+3y=23$

$2x-y=-1$

13.

$10x-6y=2$

$-5x+8y=-1$

14.

$4x-3y=-3$

$2x+6y=-4$

15.

$4x-5y=7$

$-3x+9y=0$

16.

$4x+10y=180$

$-3x-5y=-105$

17.

$8x-2y=-3$

$-4x+6y=4$

18. Solve the following linear systems with Cramer's Rule

a. $3x+2y=3$
   $12x+3y=1$
b. $15x+3y=1$
   $2x+y=2$
c. $5y=3x+5$
   $-2y=4x-2$
d. $32x+4y=3$
   $64x+8y=6$

e. $3x^2 + 4y^2 = 91$

$6x^2 - y^2 = 38$

Solution:

Let $x^2 = a$, and $y^2 = b$

Substitute in the original equations

$3a + 4b = 91$

$6a - b = 38$

Solve for a and b

....

....

The answer is: X=3, and y = 4