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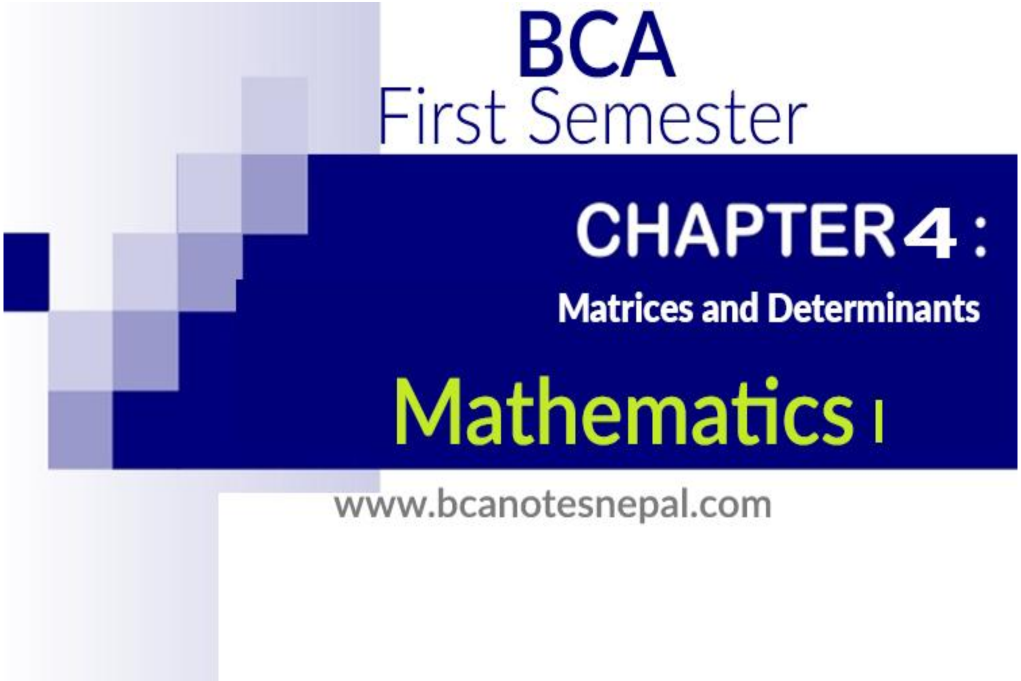

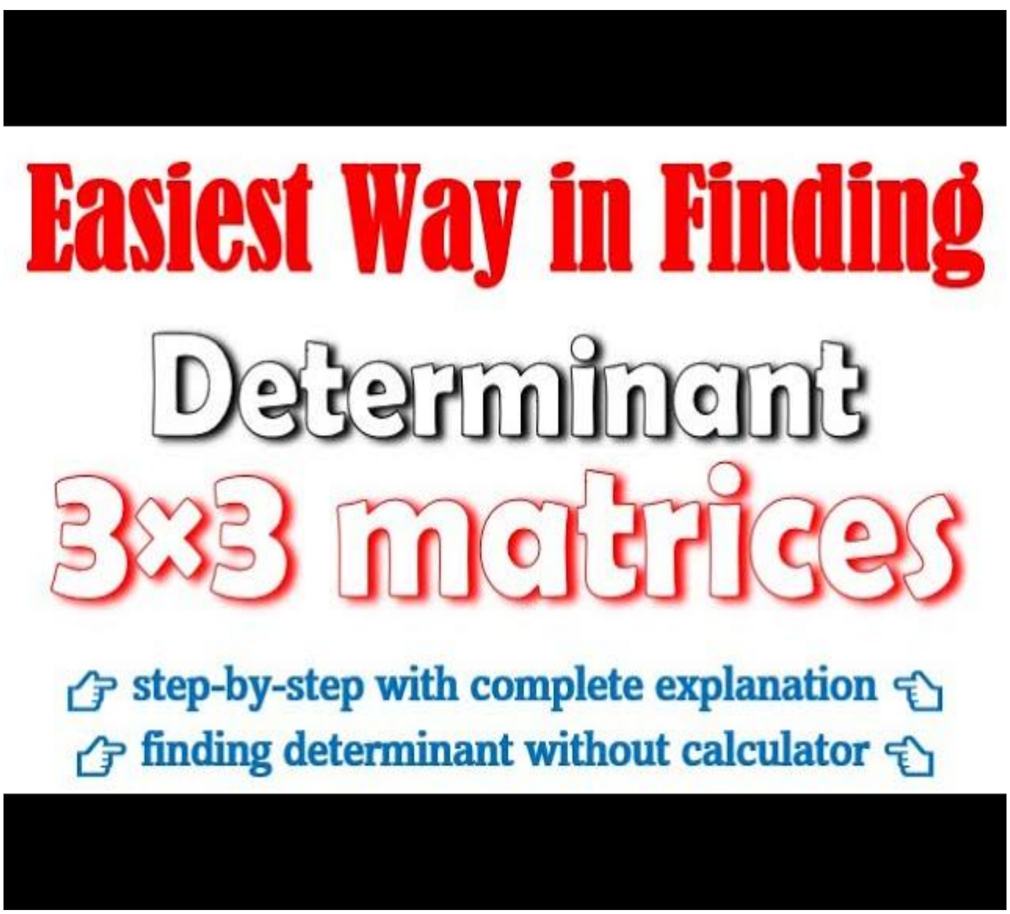

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Determinant of 3×3 Matrix

(First Method)

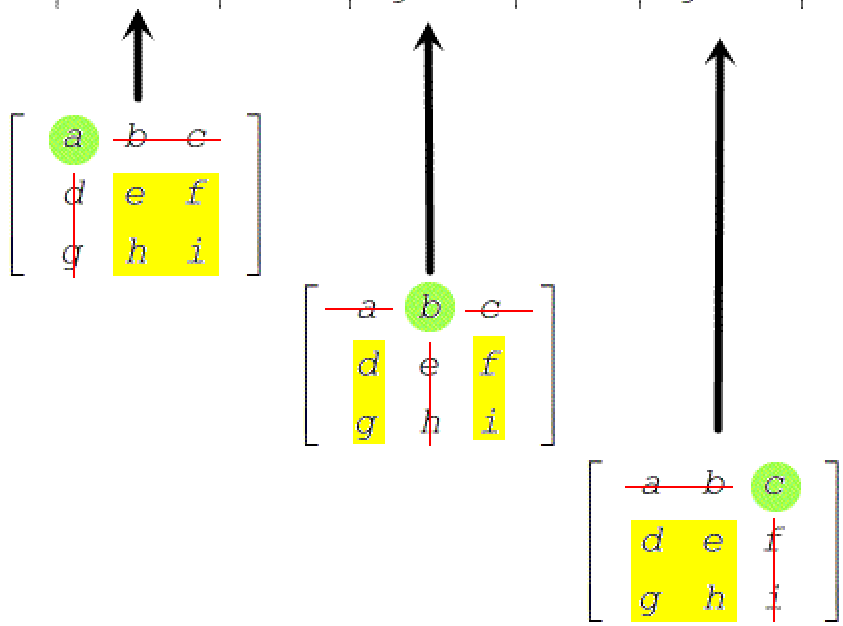
Suppose we are given a square matrix A where,

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

The **determinant of matrix A** is calculated as

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \bullet \det \begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \bullet \det \begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \bullet \det \begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \cdot \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \cdot \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \cdot \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$



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Examples of How to Find the Determinant of a 3×3 Matrix

Example 1: Find the determinant of the 3×3 matrix below.

$$\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{bmatrix}$$

Applying the formula,

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \bullet \det\begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \bullet \det\begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \bullet \det\begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

$$\det\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{bmatrix} = 2\bullet\det\begin{bmatrix} 0 & -1 \\ 4 & 5 \end{bmatrix} - (-3)\bullet\det\begin{bmatrix} 2 & -1 \\ 1 & 5 \end{bmatrix} + 1\bullet\det\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= 2\left[0-(-4)\right] + 3\left[10-(-1)\right] + 1\left[8-0\right]$$

$$= 2(0+4) + 3(10+1) + 1(8)$$

$$= 2(4) + 3(11) + 8$$

$$= 8 + 33 + 8$$

$$= 49 \checkmark$$

Example 2: Evaluate the determinant of the 3×3 matrix below.

$$\begin{bmatrix} 1 & 3 & 2 \\ -3 & -1 & -3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 \\ -3 & -1 & -3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \bullet \det\begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \bullet \det\begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \bullet \det\begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

$$\det\begin{bmatrix} 1 & 3 & 2 \\ -3 & -1 & -3 \\ 2 & 3 & 1 \end{bmatrix} = 1 \bullet \det\begin{bmatrix} -1 & -3 \\ 3 & 1 \end{bmatrix} - (3) \bullet \det\begin{bmatrix} -3 & -3 \\ 2 & 1 \end{bmatrix} + 2 \bullet \det\begin{bmatrix} -3 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= 1[-1-(-9)] - 3[-3-(-6)] + 2[-9-(-2)]$$

$$= 1(-1+9) - 3(-3+6) + 2(-9+2)$$

$$= 1(8) - 3(3) + 2(-7)$$

$$= 8 - 9 - 14$$

$$= -15$$ ✓

Example 3: Solve for the determinant of the 3×3 matrix below.

$$\begin{bmatrix} -5 & 0 & -1 \\ 1 & 2 & -1 \\ -3 & 4 & 1 \end{bmatrix}$$

Using the formula, we have…

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \bullet \det\begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \bullet \det\begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \bullet \det\begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

$$\det\begin{bmatrix} -5 & 0 & -1 \\ 1 & 2 & -1 \\ -3 & 4 & 1 \end{bmatrix} = -5\bullet\det\begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix} - (0)\bullet\det\begin{bmatrix} 1 & -1 \\ -3 & 1 \end{bmatrix} + (-1)\bullet\det\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$$

$$= -5[2-(-4)] - 0[1-(3)] - 1[4-(-6)]$$

$$= -5(2+4) - 0 - 1(4+6)$$

$$= -5(6) - 1(10)$$

$$= -30 - 10$$

$$= -40 \checkmark$$

Example 4: Solve for the determinant of the 3×3 matrix below.

$$\begin{pmatrix} 1 & -2 & 3 \\ 2 & 0 & 3 \\ 1 & 5 & 4 \end{pmatrix}$$

Solution:

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 3 \\ 1 & 5 & 4 \end{bmatrix} = \begin{bmatrix} \boxed{1} & -2 & 3 \\ 2 & 0 & 3 \\ 1 & 5 & 4 \end{bmatrix} - \begin{bmatrix} 1 & \boxed{-2} & 3 \\ 2 & 0 & 3 \\ 1 & 5 & 4 \end{bmatrix} + \begin{bmatrix} 1 & -2 & \boxed{3} \\ 2 & 0 & 3 \\ 1 & 5 & 4 \end{bmatrix}$$

$$= \boxed{1} \times \begin{vmatrix} 0 & 3 \\ 5 & 4 \end{vmatrix} - \boxed{-2} \times \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} + \boxed{3} \times \begin{vmatrix} 2 & 0 \\ 1 & 5 \end{vmatrix}$$

$$= 1 \times (0 - 15) + 2 \times (8 - 3) + 3 \times (10 - 0)$$

$$= 1(-15) + 2(5) + 3(10)$$

$$= -15 + 10 + 30$$

$$= 25$$

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Example 5: Calculate the determinant of the three-by-three matrix below.

$$\begin{pmatrix} -5 & -5 & -5 \\ 3 & -1 & -2 \\ 4 & 2 & 1 \end{pmatrix}$$

Solution:

$$\begin{bmatrix} -5 & -5 & -5 \\ 3 & -1 & -2 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} \boxed{-5} & -5 & -5 \\ 3 & -1 & -2 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -5 & \boxed{-5} & -5 \\ 3 & -1 & -2 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -5 & -5 & \boxed{-5} \\ 3 & -1 & -2 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \boxed{-5} \times \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} - \boxed{-5} \times \begin{vmatrix} 3 & -2 \\ 4 & 1 \end{vmatrix} + \boxed{-5} \times \begin{vmatrix} 3 & -1 \\ 4 & 2 \end{vmatrix}$$

$$= -5(3) + 5(11) - 5(10)$$

$$= -10$$

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Example 6: Find the determinant of the 3×3 matrix below.

$$\begin{pmatrix} 7 & -4 & 2 \\ 3 & 1 & -5 \\ 2 & 2 & -5 \end{pmatrix}$$

Answer:

$$\begin{bmatrix} 7 & -4 & 2 \\ 3 & 1 & -5 \\ 2 & 2 & -5 \end{bmatrix} = \begin{bmatrix} \boxed{7} & -4 & 2 \\ 3 & 1 & -5 \\ 2 & 2 & -5 \end{bmatrix} - \begin{bmatrix} 7 & \boxed{-4} & 2 \\ 3 & 1 & -5 \\ 2 & 2 & -5 \end{bmatrix} + \begin{bmatrix} 7 & -4 & \boxed{2} \\ 3 & 1 & -5 \\ 2 & 2 & -5 \end{bmatrix}$$

$$= \boxed{7} \times \begin{vmatrix} 1 & -5 \\ 2 & -5 \end{vmatrix} - \boxed{-4} \times \begin{vmatrix} 3 & -5 \\ 2 & -5 \end{vmatrix} + \boxed{2} \times \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix}$$

$$= 7(5) + 4(-5) + 2(4)$$

$$= 23$$

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Example 7: Find the determinant of the 3×3 matrix below.

$$\begin{pmatrix} \mathbf{1} & \mathbf{-6} & \mathbf{-7} \\ 1 & -4 & 7 \\ -1 & -3 & -6 \end{pmatrix}$$

Answer:

$$\begin{bmatrix} 1 & -6 & -7 \\ 1 & -4 & 7 \\ -1 & -3 & -6 \end{bmatrix} = \begin{bmatrix} \boxed{1} & -6 & -7 \\ 1 & -4 & 7 \\ -1 & -3 & -6 \end{bmatrix} - \begin{bmatrix} 1 & \boxed{-6} & -7 \\ 1 & -4 & 7 \\ -1 & -3 & -6 \end{bmatrix} + \begin{bmatrix} 1 & -6 & \boxed{-7} \\ 1 & -4 & 7 \\ -1 & -3 & -6 \end{bmatrix}$$

$$= \boxed{1} \times \begin{vmatrix} -4 & 7 \\ -3 & -6 \end{vmatrix} - \boxed{-6} \times \begin{vmatrix} 1 & 7 \\ -1 & -6 \end{vmatrix} + \boxed{-7} \times \begin{vmatrix} 1 & -4 \\ -1 & -3 \end{vmatrix}$$

$$= 1(45) + 6(1) - 7(-7)$$

$$= 100$$

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Example 8: Find the determinant of the 3×3 matrix below.

$$\begin{pmatrix} \mathbf{-1} & \mathbf{-1} & \mathbf{-1} \\ 4 & 5 & -3 \\ -1 & -6 & 3 \end{pmatrix}$$

Answer

$$\begin{bmatrix}-1&-1&-1\\4&5&-3\\-1&-6&3\end{bmatrix} = \begin{bmatrix}\boxed{-1}&-1&-1\\4&5&-3\\-1&-6&3\end{bmatrix} - \begin{bmatrix}-1&\boxed{-1}&-1\\4&5&-3\\-1&-6&3\end{bmatrix} + \begin{bmatrix}-1&-1&\boxed{-1}\\4&5&-3\\-1&-6&3\end{bmatrix}$$

$$= \boxed{-1} \times \begin{vmatrix}5&-3\\-6&3\end{vmatrix} - \boxed{-1} \times \begin{vmatrix}4&-3\\-1&3\end{vmatrix} + \boxed{-1} \times \begin{vmatrix}4&5\\-1&-6\end{vmatrix}$$

$$= -1(-3) + 1(9) - 1(-19)$$

$$= 31$$

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Example 9: Calculate the determinant of the 3×3 matrix below.

$$\begin{pmatrix} \mathbf{7} & \mathbf{5} & \mathbf{7} \\ 6 & -5 & -5 \\ 6 & 2 & 3 \end{pmatrix}$$

Answer:

$$\begin{bmatrix} 7 & 5 & 7 \\ 6 & -5 & -5 \\ 6 & 2 & 3 \end{bmatrix} = \begin{bmatrix} \boxed{7} & 5 & 7 \\ 6 & -5 & -5 \\ 6 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 7 & \boxed{5} & 7 \\ 6 & -5 & -5 \\ 6 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 7 & 5 & \boxed{7} \\ 6 & -5 & -5 \\ 6 & 2 & 3 \end{bmatrix}$$

$$= \boxed{7} \times \begin{vmatrix} -5 & -5 \\ 2 & 3 \end{vmatrix} - \boxed{5} \times \begin{vmatrix} 6 & -5 \\ 6 & 3 \end{vmatrix} + \boxed{7} \times \begin{vmatrix} 6 & -5 \\ 6 & 2 \end{vmatrix}$$

$$= 7(-5) - 5(48) + 7(42)$$

$$= 19$$

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Example 10: Calculate the determinant of the 3×3 matrix below.

$$\begin{pmatrix} \mathbf{4} & \mathbf{-6} & \mathbf{4} \\ -4 & -7 & -5 \\ 2 & 7 & 3 \end{pmatrix}$$

Answer:

$$\begin{bmatrix} 4 & -6 & 4 \\ -4 & -7 & -5 \\ 2 & 7 & 3 \end{bmatrix} = \begin{bmatrix} \boxed{4} & -6 & 4 \\ -4 & -7 & -5 \\ 2 & 7 & 3 \end{bmatrix} - \begin{bmatrix} 4 & \boxed{-6} & 4 \\ -4 & -7 & -5 \\ 2 & 7 & 3 \end{bmatrix} + \begin{bmatrix} 4 & -6 & \boxed{4} \\ -4 & -7 & -5 \\ 2 & 7 & 3 \end{bmatrix}$$

$$= \boxed{4} \times \begin{vmatrix} -7 & -5 \\ 7 & 3 \end{vmatrix} - \boxed{-6} \times \begin{vmatrix} -4 & -5 \\ 2 & 3 \end{vmatrix} + \boxed{4} \times \begin{vmatrix} -4 & -7 \\ 2 & 7 \end{vmatrix}$$

$$= 4(14) + 6(-2) + 4(-14)$$

$$= -12$$

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Example 11: Calculate the determinant of the 3×3 matrix below.

$$\begin{pmatrix} \mathbf{-1} & \mathbf{-3} & \mathbf{4} \\ -1 & 2 & 6 \\ -3 & -7 & 2 \end{pmatrix}$$

Answer:

$$\begin{bmatrix} -1 & -3 & 4 \\ -1 & 2 & 6 \\ -3 & -7 & 2 \end{bmatrix} = \begin{bmatrix} \boxed{-1} & -3 & 4 \\ -1 & 2 & 6 \\ -3 & -7 & 2 \end{bmatrix} - \begin{bmatrix} -1 & \boxed{-3} & 4 \\ -1 & 2 & 6 \\ -3 & -7 & 2 \end{bmatrix} + \begin{bmatrix} -1 & -3 & \boxed{4} \\ -1 & 2 & 6 \\ -3 & -7 & 2 \end{bmatrix}$$

$$= \boxed{-1} \times \begin{vmatrix} 2 & 6 \\ -7 & 2 \end{vmatrix} - \boxed{-3} \times \begin{vmatrix} -1 & 6 \\ -3 & 2 \end{vmatrix} + \boxed{4} \times \begin{vmatrix} -1 & 2 \\ -3 & -7 \end{vmatrix}$$

$$= -1(46) + 3(16) + 4(13)$$

$$= 54$$

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Example 12: Determine the determinant of the 3×3 matrix below.

$$\begin{pmatrix} 2 & 7 & 5 \\ 1 & 2 & 5 \\ 0 & 0 & -4 \end{pmatrix}$$

Answer:

$$\begin{bmatrix} 2 & 7 & 5 \\ 1 & 2 & 5 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} \boxed{2} & 7 & 5 \\ 1 & 2 & 5 \\ 0 & 0 & -4 \end{bmatrix} - \begin{bmatrix} 2 & \boxed{7} & 5 \\ 1 & 2 & 5 \\ 0 & 0 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 7 & \boxed{5} \\ 1 & 2 & 5 \\ 0 & 0 & -4 \end{bmatrix}$$

$$= \boxed{2} \times \begin{vmatrix} 2 & 5 \\ 0 & -4 \end{vmatrix} - \boxed{7} \times \begin{vmatrix} 1 & 5 \\ 0 & -4 \end{vmatrix} + \boxed{5} \times \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix}$$

$$= 2(-8) - 7(-4) + 5(0)$$

$$= 12$$

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Example 13: Determine the determinant of the 3×3 matrix below.

$$\begin{pmatrix} \mathbf{1} & \mathbf{-2} & \mathbf{2} \\ -5 & -1 & 5 \\ 4 & -1 & 0 \end{pmatrix}$$

Answer:

$$\begin{bmatrix} 1 & -2 & 2 \\ -5 & -1 & 5 \\ 4 & -1 & 0 \end{bmatrix} = \begin{bmatrix} \boxed{1} & -2 & 2 \\ -5 & -1 & 5 \\ 4 & -1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & \boxed{-2} & 2 \\ -5 & -1 & 5 \\ 4 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -2 & \boxed{2} \\ -5 & -1 & 5 \\ 4 & -1 & 0 \end{bmatrix}$$

$$= \boxed{1} \times \begin{vmatrix} -1 & 5 \\ -1 & 0 \end{vmatrix} - \boxed{-2} \times \begin{vmatrix} -5 & 5 \\ 4 & 0 \end{vmatrix} + \boxed{2} \times \begin{vmatrix} -5 & -1 \\ 4 & -1 \end{vmatrix}$$

$$= 1(5) + 2(-20) + 2(9)$$

$$= -17$$

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Example 14: Compute the determinant of the 3×3 matrix below.

$$\begin{pmatrix} -5 & 2 & -3 \\ -2 & -1 & -5 \\ 5 & -5 & -3 \end{pmatrix}$$

Answer

$$\begin{bmatrix} -5 & 2 & -3 \\ -2 & -1 & -5 \\ 5 & -5 & -3 \end{bmatrix} = \begin{bmatrix} \boxed{-5} & 2 & -3 \\ -2 & -1 & -5 \\ 5 & -5 & -3 \end{bmatrix} - \begin{bmatrix} -5 & \boxed{2} & -3 \\ -2 & -1 & -5 \\ 5 & -5 & -3 \end{bmatrix} + \begin{bmatrix} -5 & 2 & \boxed{-3} \\ -2 & -1 & -5 \\ 5 & -5 & -3 \end{bmatrix}$$

$$= \boxed{-5} \times \begin{vmatrix} -1 & -5 \\ -5 & -3 \end{vmatrix} - \boxed{2} \times \begin{vmatrix} -2 & -5 \\ 5 & -3 \end{vmatrix} + \boxed{-3} \times \begin{vmatrix} -2 & -1 \\ 5 & -5 \end{vmatrix}$$

$$= -5(-22) - 2(31) - 3(15)$$

$$= 3$$

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Example 15: Compute the determinant of the 3×3 matrix below.

$$\begin{pmatrix} \mathbf{-4} & \mathbf{1} & \mathbf{2} \\ 1 & -3 & 2 \\ -4 & 4 & 0 \end{pmatrix}$$

Answer:

$$\begin{bmatrix} -4 & 1 & 2 \\ 1 & -3 & 2 \\ -4 & 4 & 0 \end{bmatrix} = \begin{bmatrix} \boxed{-4} & 1 & 2 \\ 1 & -3 & 2 \\ -4 & 4 & 0 \end{bmatrix} - \begin{bmatrix} -4 & \boxed{1} & 2 \\ 1 & -3 & 2 \\ -4 & 4 & 0 \end{bmatrix} + \begin{bmatrix} -4 & 1 & \boxed{2} \\ 1 & -3 & 2 \\ -4 & 4 & 0 \end{bmatrix}$$

$$= \boxed{-4} \times \begin{vmatrix} -3 & 2 \\ 4 & 0 \end{vmatrix} - \boxed{1} \times \begin{vmatrix} 1 & 2 \\ -4 & 0 \end{vmatrix} + \boxed{2} \times \begin{vmatrix} 1 & -3 \\ -4 & 4 \end{vmatrix}$$

$$= -4(-8) - 1(8) + 2(-8)$$

$$= 8$$

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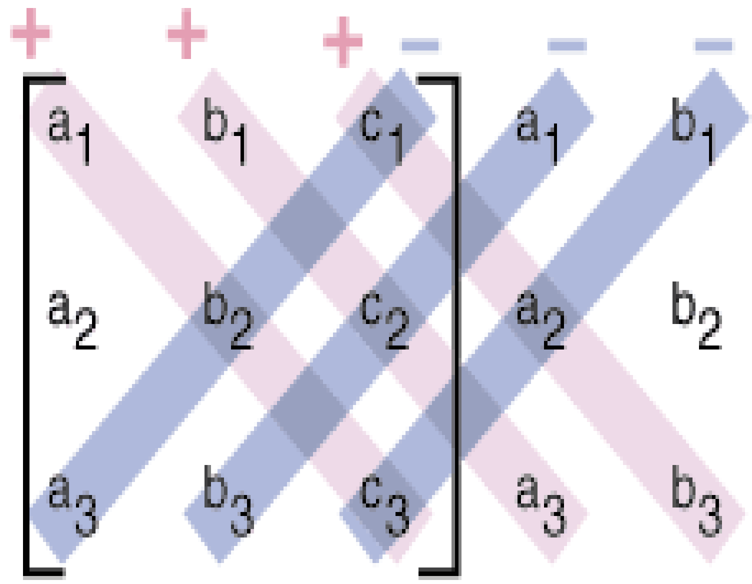
Determinant of 3×3 Matrix

(Second Method)

To find the determinant of a 3×3 matrix, copy the first two columns of the matrix to the right of the original matrix. Next, multiply the numbers on the three downward diagonals, and add these products together. Multiply the numbers on the upward diagonals, and add *these* products together. Then subtract the sum of the products of the upward diagonals from the sum of the product of the downward diagonals (subtract the second number from the first number):

$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \begin{matrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{matrix}$$



$$\det A = (a_1 b_2 c_3 + b_1 c_2 a_3 + c_1 a_2 b_3) - (a_3 b_2 c_1 + b_3 c_2 a_1 + c_3 a_2 b_1)$$

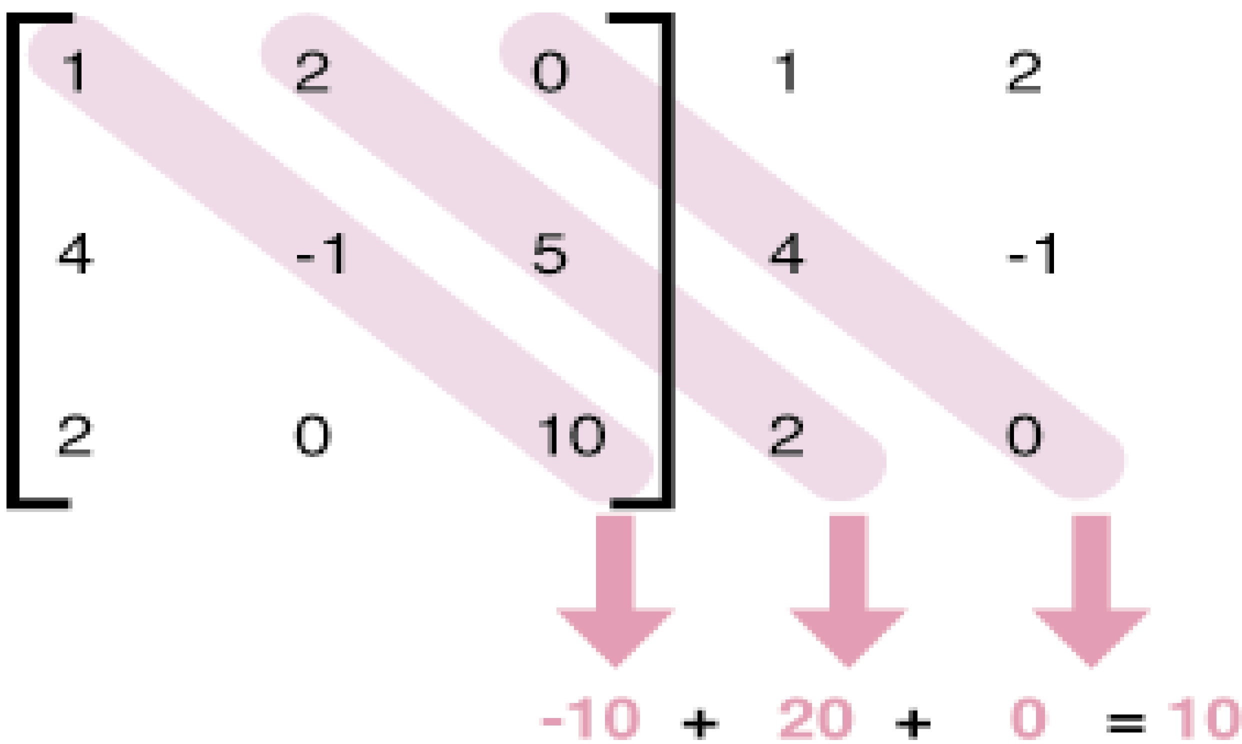
Example: Find the determinant of:

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & -1 & 5 \\ 2 & 0 & 10 \end{bmatrix}$$

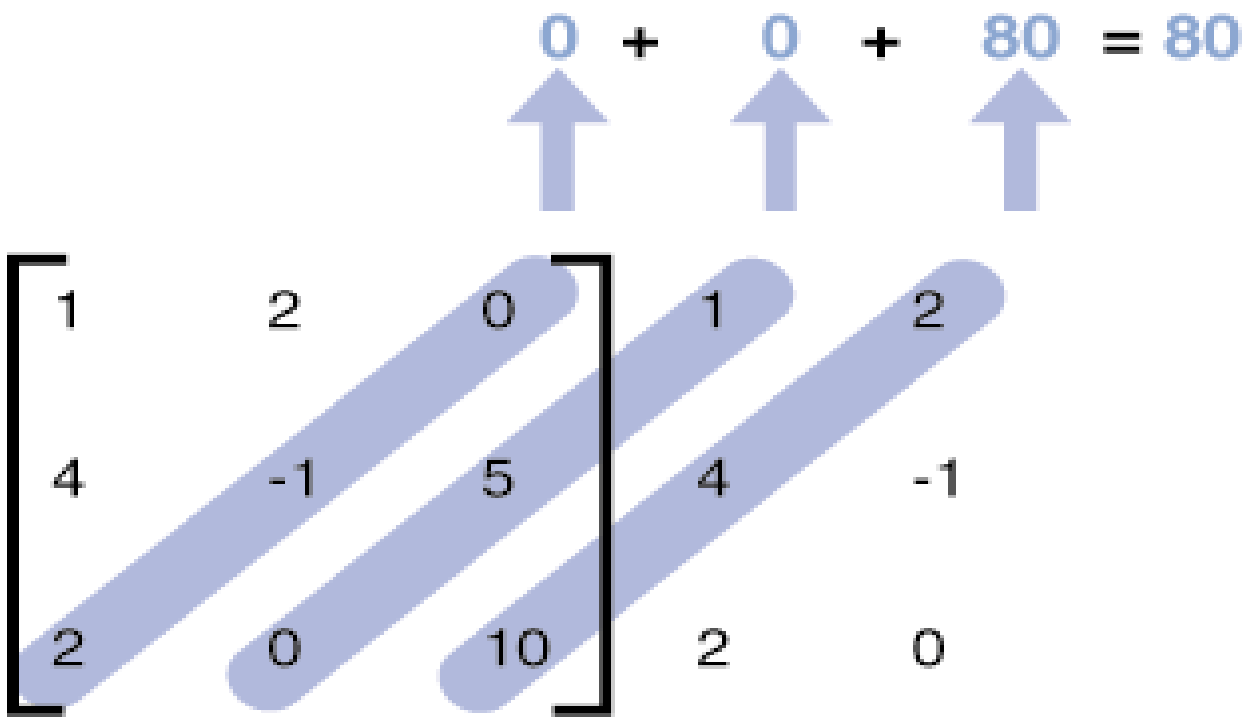
Step 1:

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & -1 & 5 \\ 2 & 0 & 10 \end{bmatrix} \begin{matrix} 1 & 2 \\ 4 & -1 \\ 2 & 0 \end{matrix}$$

Step 2:



Step 3:

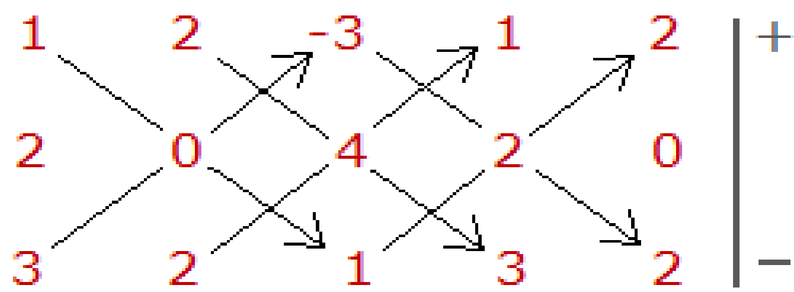


Step 4:

10 - 80 = -70. *Det A* **= - 70.**

For example: Find the determinant of $|A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 0 & 4 \\ 3 & 2 & 1 \end{vmatrix}$ by the rule of Sarrus.

Solution: $|A| =$

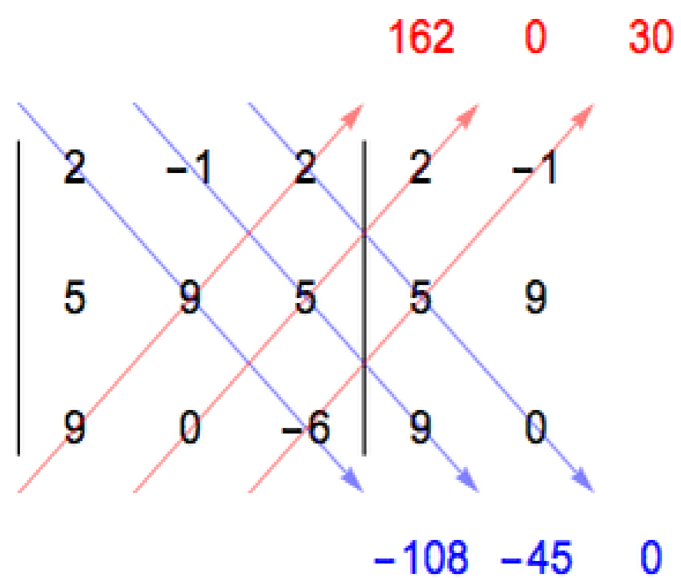


$= (0 + 24 - 12) - (0 + 8 + 4)$

$= 12 - 12$

$= 0$

new numbers



$$\therefore \begin{vmatrix} 2 & -1 & 2 \\ 5 & 9 & 5 \\ 9 & 0 & -6 \end{vmatrix} = -108 + -45 + 0 - (162 + 0 + 30) = -345$$

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Find the solution to the given 3 × 3 system using Cramer's Rule.

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \; D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \; D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \; D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

$X = D_X / D$, $y = D_Y / D$, $z = D_Z / D$ $D \neq 0$

Cramer's Rule

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$\text{Let } D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

If $D \neq 0$ then

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{D} \qquad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{D} \qquad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{D}$$

Solve the linear equations with 3 variables using Cramer's rule.

$$x + y = 2$$

$$x - y + z = 4$$

$$x + y - z = 6$$

Solution:

By Cramer's rule for 3 variables, $\mathbf{x = D_x / D}$, $\mathbf{y = D_y / D}$, $\mathbf{z = D_z / D}$, where D, D_x, D_y and D_z are determinants.

$$D = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$D = 1(1-1) -1 (-1-1) + 0$

$D = 0 + 2 + 0$

$D = 2$

$$D_x = \begin{vmatrix} 2 & 1 & 0 \\ 4 & -1 & 1 \\ 6 & 1 & -1 \end{vmatrix}$$

$D_x = 2(1-1) -1 (-4-6) + 0$

$D_x = 0 -1(-10) + 0$

$D_x = 10$

$$D_y = \begin{vmatrix} 1 & 2 & 0 \\ 1 & 4 & 1 \\ 1 & 6 & -1 \end{vmatrix}$$

$D_y = 1(-4-6) -2 (-1-1) + 0$

$D_y = 1 (-10) + 4 + 0$

$D_y = -6$

$$D_z = \begin{vmatrix} 1 & 1 & 2 \\ 1 & -1 & 4 \\ 1 & 1 & 6 \end{vmatrix}$$

$D_z = 1 (-6-4) -1 (6-4) + 2(1+1)$

$D_z = -10) -2 + 4$

$D_z = -8$

$x = D_x \ / D = 10/2 = 5$

$y = D_y \ / D = -6/2 = -3$

$z = D_z \ / D = -8/2 = -4$

Homework 1

Solve the following 3 x 3 system of equations by using Cramer Rule:

Q1: $x + y - z = 6$

$3x - 2y + z = -5$

$X + 3y - 2z = 14$

The solution is x=1, y=3, and z=-2

Q2: $2x + y - z = 3$

$X + y + z = 1$

$X - 2y - 3z = 4$

The solution is x=2, y=-1, and z=0

Q3: $x + y + z = 6$

$2x + 3y - z = 5$

$6x - 2y - 3z = -7$

The solution is x=1, y=2, and z=3

Q4: $X + Y = 2$

$X - Y + Z = 4$

$X + Y - Z = 6$

The solution is x=5, y=-3, and z=-4

Q5: $3x - 4y + 8z = 34$

$4x + y - 2z = 1$

$-6x - 13y + 20z = 61$

The solution is: x=2, y=-1, and z=3

Q6: $X + 4Y + 3Z = 2$

$2X - 6Y + 6Z = -3$

$5X - 2Y + 3Z = -5$

The solution is x=-1, y=1/2, and z=1/3