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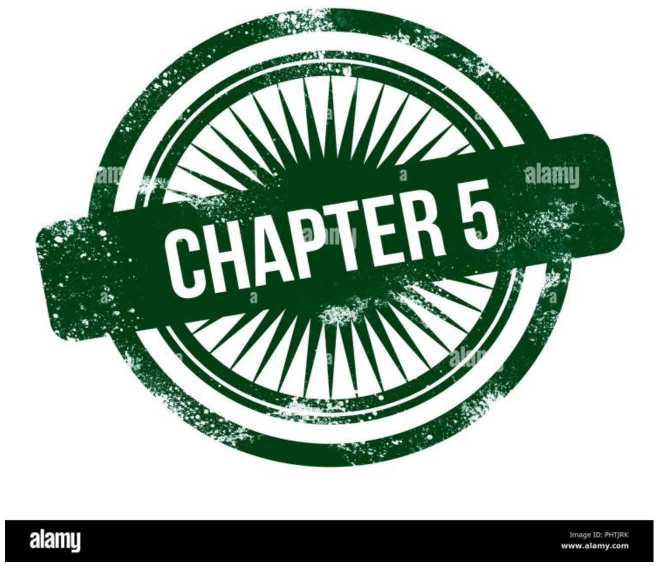

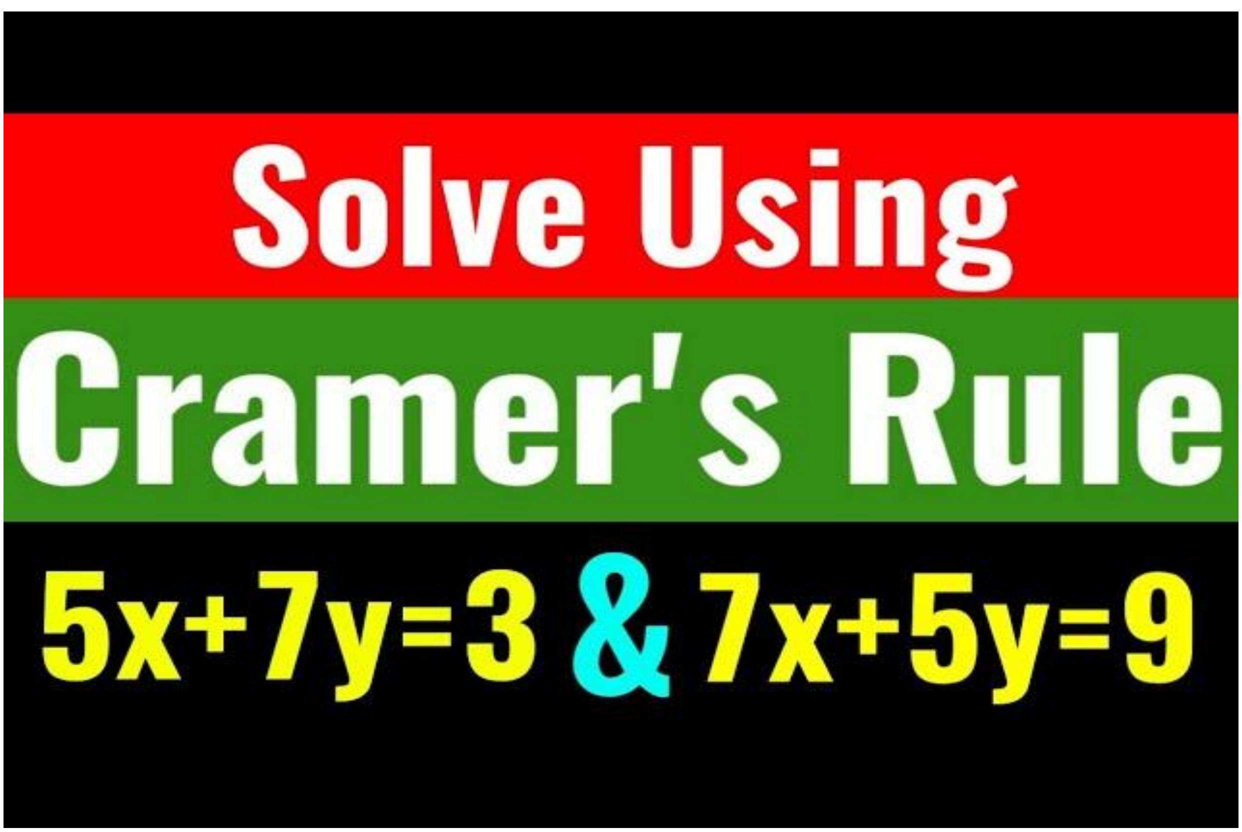

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Cramer's Rules for Systems of Linear Equations with Three Variables

x-column z-column

$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}$$

y-column constant-column

Coefficient matrix:

$$D = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad D_x = \begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix}$$

X – matrix

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$$D_y = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix}$$

Y – matrix

$$D_Z = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$$

Z – matrix

$$x = \frac{|D_x|}{|D|} = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \quad y = \frac{|D_y|}{|D|} = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}} \quad z = \frac{|D_Z|}{|D|} = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

How to Solve Systems of Linear Equations with Three Variables using Cramer's Rule

Example 1: **Solve the system with three variables by Cramer's Rule.**

$$\begin{cases} x+2y+3z=-5 \\ 3x+y-3z=4 \\ -3x+4y+7z=-7 \end{cases}$$

From the given system of linear equations, I will construct the four matrices that will be used to solve for the values of **x**, **y**, and **z**.

$$D = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & -3 \\ -3 & 4 & 7 \end{bmatrix} \qquad D_x = \begin{bmatrix} -5 & 2 & 3 \\ 4 & 1 & -3 \\ -7 & 4 & 7 \end{bmatrix}$$

$$D_y = \begin{bmatrix} 1 & -5 & 3 \\ 3 & 4 & -3 \\ -3 & -7 & 7 \end{bmatrix} \qquad D_z = \begin{bmatrix} 1 & 2 & -5 \\ 3 & 1 & 4 \\ -3 & 4 & -7 \end{bmatrix}$$

Next, I will solve for the determinant of each matrix.

The values of the determinants are listed below.

Determinants of each matrix:

$$|D| = 40$$
$$|D_x| = -40$$
$$|D_y| = 40$$
$$|D_z| = -80$$

The final answers or solutions are easily computed or calculated once all the required determinants are found.

Solved values for x, y, and z.

$$x = \frac{|D_x|}{|D|} = \frac{-40}{40} = -1$$
$$y = \frac{|D_y|}{|D|} = \frac{40}{40} = 1$$
$$z = \frac{|D_z|}{|D|} = \frac{-80}{40} = -2$$

The final answer written in point notation is : $(x,y,z)=(-1,1,-2)$.

Example 2: **Solve the system with three variables by using Cramer's Rule.**

$$\begin{cases} -2x - y - 3z = 3 \\ 2x - 3y + z = -13 \\ 2x \quad - 3z = -11 \end{cases}$$

$$D = \begin{bmatrix} -2 & -1 & -3 \\ 2 & -3 & 1 \\ 2 & 0 & -3 \end{bmatrix} \quad D_x = \begin{bmatrix} 3 & -1 & -3 \\ -13 & -3 & 1 \\ -11 & 0 & -3 \end{bmatrix}$$

$$D_y = \begin{bmatrix} -2 & 3 & -3 \\ 2 & -13 & 1 \\ 2 & -11 & -3 \end{bmatrix} \qquad D_z = \begin{bmatrix} -2 & -1 & 3 \\ 2 & -3 & -13 \\ 2 & 0 & -11 \end{bmatrix}$$

After solving the determinant of each matrix, I have them all written down.

Determinants of each matrix:

$$|D| = -44$$
$$|D_x| = 176$$
$$|D_y| = -88$$
$$|D_z| = -44$$

Solved values for *x*, *y*, and *z*.

$$x = \frac{|D_x|}{|D|} = \frac{176}{-44} = -4$$

$$y = \frac{|D_y|}{|D|} = \frac{-88}{-44} = 2$$

$$z = \frac{|D_z|}{|D|} = \frac{-44}{-44} = 1$$

Our final answer is : $(x, y, z) = (-4, 2, 1)$.

Example 3: Solve the system with three variables by Cramer's Rule.

$$\begin{cases} \quad -y - 2z = -8 \\ x \quad\quad + 3z = 2 \\ 7x + y + z = 0 \end{cases}$$

- **coefficient matrix**

$$D = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 3 \\ 7 & 1 & 1 \end{bmatrix}$$

$$D_x = \begin{bmatrix} -8 & -1 & -2 \\ 2 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

X – matrix

$$D_y = \begin{bmatrix} 0 & -8 & -2 \\ 1 & 2 & 3 \\ 7 & 0 & 1 \end{bmatrix}$$

$$D_z = \begin{bmatrix} 0 & -1 & -8 \\ 1 & 0 & 2 \\ 7 & 1 & 0 \end{bmatrix}$$

Solving for their determinants, I got the following values.

Determinants of each matrix:

$$|D| = -22$$

$$|D_x| = 22$$

$$|D_y| = -132$$

$$|D_z| = -22$$

This leads us to easily set up and calculate the final answers.

Solved values for **X**, **y**, and **Z**.

$$x = \frac{|D_x|}{|D|} = \frac{22}{-22} = -1$$

$$y = \frac{|D_y|}{|D|} = \frac{-132}{-22} = 6$$

$$z = \frac{|D_z|}{|D|} = \frac{-22}{-22} = 1$$

The final answer is: $(x, y, z) = (-1, 6, 1)$.

Example 4: Solve the system with three variables by Cramer's Rule

$$\begin{cases} -2x + y + z = 4 \\ -4x + 2y - z = 8 \\ -6x - 3y + z = 0 \end{cases}$$

Write down the four special matrices.

- **coefficient matrix**

$$D = \begin{bmatrix} -2 & 1 & 1 \\ -4 & 2 & -1 \\ -6 & -3 & 1 \end{bmatrix}$$

$$D_x = \begin{bmatrix} 4 & 1 & 1 \\ 8 & 2 & -1 \\ 0 & -3 & 1 \end{bmatrix}$$

x – matrix

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$$D_y = \begin{bmatrix} -2 & 4 & 1 \\ -4 & 8 & -1 \\ -6 & 0 & 1 \end{bmatrix}$$

y– matrix

$$D_z = \begin{bmatrix} -2 & 1 & 4 \\ -4 & 2 & 8 \\ -6 & -3 & 0 \end{bmatrix}$$

z– matrix

Evaluate each matrix to find its determinant.

$$|D| = 36$$
$$|D_x| = -36$$
$$|D_y| = 72$$
$$|D_z| = 0$$

Use the Cramer's Rule to get the following solutions.

Solved values for x, y, and z.

$$x = \frac{|D_x|}{|D|} = \frac{-36}{36} = -1$$

$$y = \frac{|D_y|}{|D|} = \frac{72}{36} = 2$$

$$z = \frac{|D_z|}{|D|} = \frac{0}{36} = 0$$

The final answer is $(x,y,z)=(-1,2,0)$.

Example 5: Solve the system with three variables by Cramer's Rule

$$\begin{cases} x - 8y + z = 4 \\ -x + 2y + z = 2 \\ x - y + 2z = -1 \end{cases}$$

Construct the four special matrices.

- **coefficient matrix**

$$D = \begin{bmatrix} 1 & -8 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$D_x = \begin{bmatrix} 4 & -8 & 1 \\ 2 & 2 & 1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$D_y = \begin{bmatrix} 1 & 4 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$D_z = \begin{bmatrix} 1 & -8 & 4 \\ -1 & 2 & 2 \\ 1 & -1 & -1 \end{bmatrix}$$

Find the determinant of each square matrix.

Determinants of each matrix

$$|D| = -20$$
$$|D_x| = 60$$
$$|D_y| = 16$$
$$|D_z| = -12$$

Solved values for x, y, and z.

$$x = \frac{|D_x|}{|D|} = \frac{60}{-20} = -3$$
$$y = \frac{|D_y|}{|D|} = \frac{16}{-20} = \frac{-4}{5}$$
$$z = \frac{|D_z|}{|D|} = \frac{-12}{-20} = \frac{3}{5}$$

The final answer in point form is $(-3,-45,35)$.

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Summary

Find the solution to the given 3 × 3 system using Cramer's Rule.

$$a_1x+b_1y+c_1z=d_1$$
$$a_2x+b_2y+c_2z=d_2$$
$$a_3x+b_3y+c_3z=d_3$$

$$D=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix},\ D_x=\begin{vmatrix}d_1 & b_1 & c_1\\ d_2 & b_2 & c_2\\ d_3 & b_3 & c_3\end{vmatrix},\ D_y=\begin{vmatrix}a_1 & d_1 & c_1\\ a_2 & d_2 & c_2\\ a_3 & d_3 & c_3\end{vmatrix},\ D_z=\begin{vmatrix}a_1 & b_1 & d_1\\ a_2 & b_2 & d_2\\ a_3 & b_3 & d_3\end{vmatrix}$$

$X = D_X / D$, $y = D_Y / D$, $z = D_Z / D$ $\quad D \neq 0$

Cramer's Rule

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$\text{Let } D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\text{If } D \neq 0 \text{ then}$$

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{D} \qquad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{D} \qquad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{D}$$

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Home Work 2

Problem 1: Solve the given system of equations using Cramer's Rule.

$$x + 3y + 3z = 5$$
$$3x + y - 3z = 4$$
$$-3x + 4y + 7z = -7$$

The final answer can be given in point notation: *(x , y, z) = (-1, 1, 2).*

Problem 2:

Solve the given system of equations using Cramer's Rule.

$$-y - 2z = -8$$
$$x + 3z = 2$$
$$7x + y + z = 0$$

The answer is: *(x, y, z) = (-1, 6, 1)*

Problem 3:

Solve the given system of equations using Cramer's Rule.

$$-2x - 5y + 4z = 21$$
$$-5x - 5y + z = 21$$
$$-4y - 4z = 8$$

The answer is: *(x,y,z)* = (−1, −3, 1)

Problem 4:

Solve the given system of equations using Cramer's Rule.

$$5x + y - 4z = -4$$

$$-3y - 6z = -21$$

$$-x - y - z = -6$$

The answer is: $(x, y, z) = (5, -5, 6)$

Problem 5:

Solve the given system of equations using Cramer's Rule.

$$4x - 4y + 2z = -14$$

$$4x + 2y = 14$$

$$-3y + z = -10$$

No solution