Ministry of Higher Education and Scientific Research

Al-Ma’moun University College

كليــة المــأمون الجامعــة

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General Physics

Lecture (3)

Mechanics

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Lecture (3)

Motion in a Straight line

Motion

The world, and everything in it, moves. Even stationary things, such as a roadway, move with Earth's rotation, Earth's orbit around the Sun, the Sun's orbit around the center of the Milky Way galaxy, and that galaxy's migration relative to other galaxies. The classification and comparison of motions (called **kinematics**) is often challenging. What exactly do you measure, and how do you compare?

Before we attempt an answer, we shall examine some general properties of motion that is restricted in three ways.

1. The motion is along a straight line only. The line may be vertical, horizontal, or diagonal, but it must be straight.

2. Forces (pushes and pulls), Does the moving object speed up, slow down, stop, or reverse direction If the motion does change, how is time involved in the change?

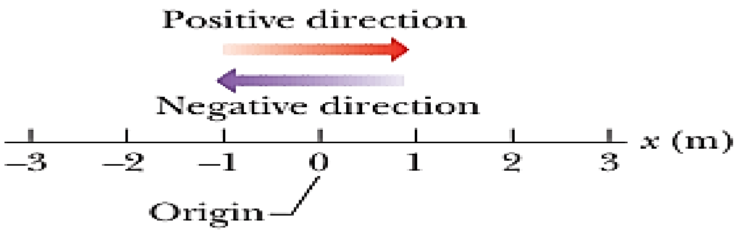
3. The moving object is either a **particle** (by which we mean a point-like object such as an electron) or an object that moves like a particle (that every portion moves in the same direction and at the same rate). A stiff pig slipping down a straight playground slide might be considered to be moving like a particle; however, a tumbling tumbleweed would not.

Position and Displacement

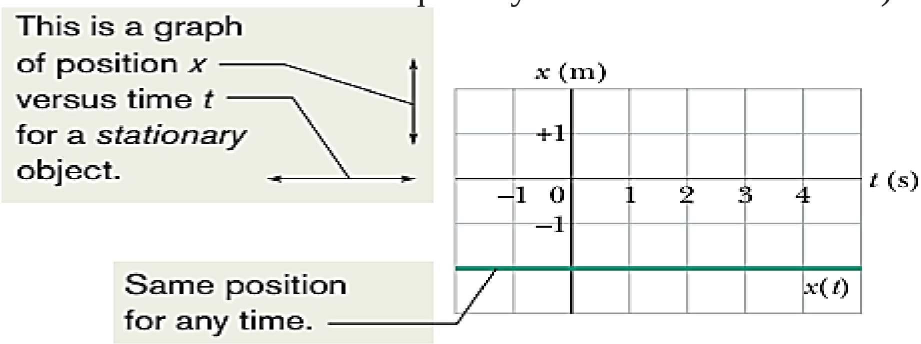
To locate an object means to find its position relative to some reference point, often the **origin** (or zero point) of an axis such as the *x-axis*. The **positive direction** of the axis is in the direction of increasing numbers (coordinates), which is to the right. The opposite is the **negative direction.** For example, a particle might be located at x=5 m, which means it is 5 m in the positive direction from the origin. If it were at x =5 m, it would be just as far from the origin but in the opposite direction. On the axis, a coordinate of 5 m is less than a coordinate of 1 m, and both coordinates are less than a coordinate of 5 m. A plus sign for a coordinate need not be shown, but a minus sign must always be shown.

A change from position x_1 to position x_2 is called a **displacement** Δx, where

$$\mathbf{\Delta x = x_2 - x_1} \text{--------------- } \mathbf{(1)}$$



(The symbol Δ, the Greek uppercase delta, represents a change in a quantity, and it means the final value of that quantity minus the initial value.)



Displacement is an example of a **vector quantity**, which is a quantity that has both a direction and a magnitude.

Average Velocity and Average Speed

A compact way to describe position is with a graph of position *x* plotted as a function of time *t*—a graph of *x*(*t*). (The notation *x*(*t*) represents a function *x* of *t*, not the product *x* times *t*.)

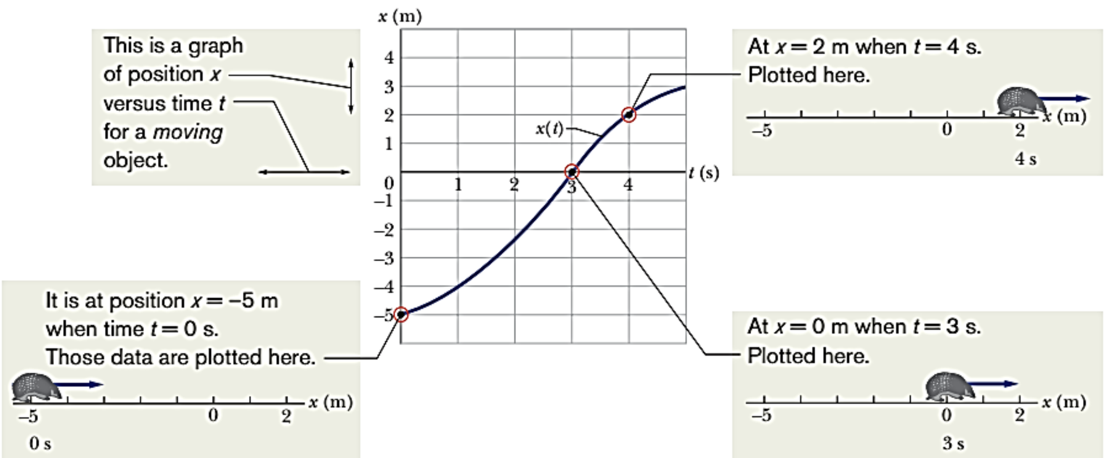


Figure (2)

Actually, several quantities are associated with the phrase "how fast." One of them is the **average velocity** v_{avg}, which is the ratio of the displacement Δx that occurs during a particular time interval Δt to that interval:

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}. \quad \text{---------------(2)}$$

The notation means that the position is $\boldsymbol{x}_1$at time $\boldsymbol{t}_1$ and then $\boldsymbol{x}_2$at time $\boldsymbol{t}_2$. A common unit for v_{avg} is the meter per second (m/s). You may see other units in the problems, but they are always in the form of length/time.

Graphs. On a graph of *x* versus *t*, v_{avg}, is the **slope** of the straight line that connects two particular points on the *x*(*t*) curve: one is the point that corresponds to $x_2$and t_2, and the other is the point that corresponds to x_1 andt_1. Like displacement, v_{avg}, has both magnitude and direction (it is another vector quantity). Its magnitude is the magnitude of the line's slope. A positive v_{avg}, (and slope) tells us that the line slants upward to the right; a negative v_{avg}, (and slope) tells us that the line slants downward to the right. The average velocity v_{avg}, always has the same sign as the displacement Δx because Δt in the above equation is always positive. Figure (3) shows how to find v_{avg}, in Fig. 2 for the time interval $t = 1$ s to $t = 4$ s. We draw the straight line that connects the point on the position curve at the beginning of the interval and the point on the curve at the end of the interval. Then we find the slope $\Delta x/\Delta t$ of the straight line. ,For the given time interval, the average velocity is:

$$v_{ave} = \frac{6m}{3s} = 2\ m/s$$

Average speed s_{avg} is a different way of describing "how fast" a particle moves. Whereas the average velocity involves the particle's displacement Δ*x*, the average speed involves the total distance covered (for example, the number of meters moved), independent of direction; that is,

$$S_{ave} = \frac{total\ distance}{\Delta t} \quad \text{------------------------ (3)}$$

Because average speed does *not* include direction, it lacks any algebraic sign. Sometimes S_{ave} is the same (except for the absence of a sign) as v_{ave}. However, the two can be quite different.

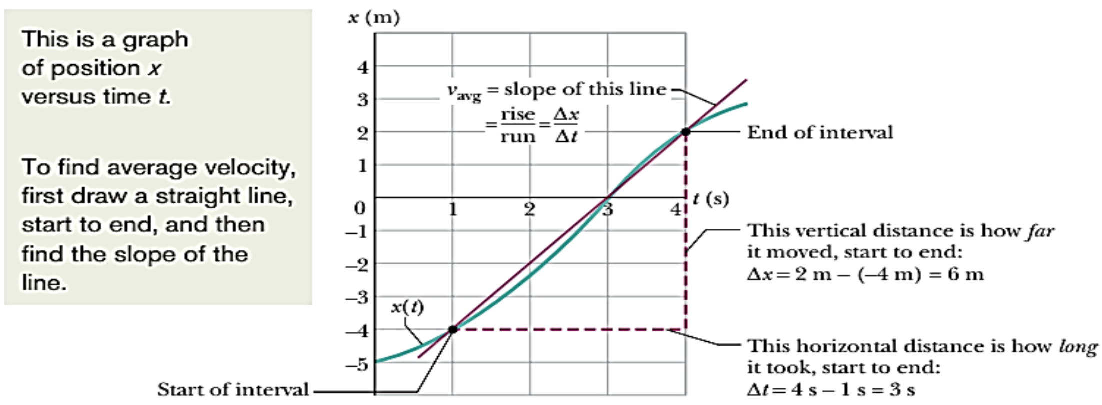


Figure (3)

Instantaneous Velocity and Speed

You have now seen two ways to describe how fast something moves: average velocity and average speed, both of which are measured over a time interval Δt. However, the phrase "how fast" more commonly refers to how fast a particle is moving at a given instant—its **instantaneous velocity** (or simply **velocity**) v. The velocity at any instant is obtained from the average velocity by shrinking the time interval Δt closer and closer to 0. As Δt dwindles, the average velocity approaches a limiting value, which is the velocity at that instant:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}.$$ ---------------- (4)

Note that v is the rate at which position x is changing with time at a given instant; that is, v is the derivative of x with respect to t. Also, note that v at any instant is the slope of the position–time curve at the point representing that instant. Velocity is another vector quantity and thus has an associated direction.

Speed is the magnitude of velocity; that is, speed is velocity that has been stripped of any indication of direction, either in words or via an algebraic sign.

ACCELERATION

When a particle's velocity changes, the particle is said to undergo **acceleration** (or to accelerate). For motion along an axis, the **average acceleration** a_{avg} over a time interval Δt is:

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t},$$ ------------------------ (5)

Where the particle has velocity v_1 at time t_1 and then velocity v_2 at time t_2. The **instantaneous acceleration** (or simply **acceleration**) is:

$$a = \frac{dv}{dt}.$$ ----------------------- (6)

In words, the acceleration of a particle at any instant is the rate at which its velocity is changing at that instant. Graphically, the acceleration at any point is the slope of the curve of $v(t)$ at that point. We can combine the last two equations to write:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}.$$ ----------- (7)

In words, the acceleration of a particle at any instant is the second derivative of its position $x(t)$ with respect to time.

A common unit of acceleration is the meter per second : m/(s s) or m/s2. Other units are in the form of length/(time time) or $length/time$2.

Acceleration has both magnitude and direction (it is yet another vector quantity). Its algebraic sign represents its direction on an axis just as for displacement and velocity; that is, acceleration with a positive value is in the positive direction of an axis, and acceleration with a negative value is in the negative direction.

CONSTANT ACCELERATION

- The following five equations describe the motion of a particle with constant acceleration:

$$v = v_0 + at, \qquad x - x_0 = v_0 t + \frac{1}{2}at^2,$$

$$v^2 = v_0^2 + 2a(x - x_0), \qquad x - x_0 = \frac{1}{2}(v_0 + v)t, \qquad x - x_0 = vt - \frac{1}{2}at^2.$$

These are *not* valid when the acceleration is not constant.

Constant Acceleration: A Special Case

In many types of motion, the acceleration is either constant or approximately so. For example, you might accelerate a car at an approximately constant rate when a traffic light turns from red to green. Then graphs of your position, velocity, and acceleration would resemble those in Fig. (4) (Note that a (t) in Fig. 4C is constant, which requires that $v(t)$ in Fig. 4b have a constant slope.) Later when you brake the car to a stop, the acceleration (or deceleration in common language) might also be approximately constant.

Such cases are so common that a special set of equations has been derived for dealing with them.

First Basic Equation. When the acceleration is constant, the average acceleration and instantaneous acceleration are equal and we can write Eq. 6, with some changes in notation, as

$$a = a_{avg} = \frac{v - v_0}{t - 0}.$$ ---------------- (8)

Here $\boldsymbol{v0}$ is the velocity at time $\boldsymbol{t} = \boldsymbol{0}$ and $\boldsymbol{v}$ is the velocity at any later time $\boldsymbol{t}$. We can recast this equation as

$$v = v_0 + at.$$ --------------------(9)

The last equation reduces to $\boldsymbol{v} = \boldsymbol{v_0}$ for $\boldsymbol{t} = \boldsymbol{0}$, as it must. As a further check, take the derivative of Eq. (9). Doing so yields $\frac{\boldsymbol{dv}}{\boldsymbol{dt}} = \boldsymbol{a}$, which is the definition of a. Figure 4 b shows a plot of Eq. 9, the $\boldsymbol{v(t)}$ function; the function is linear and thus the plot is a straight line

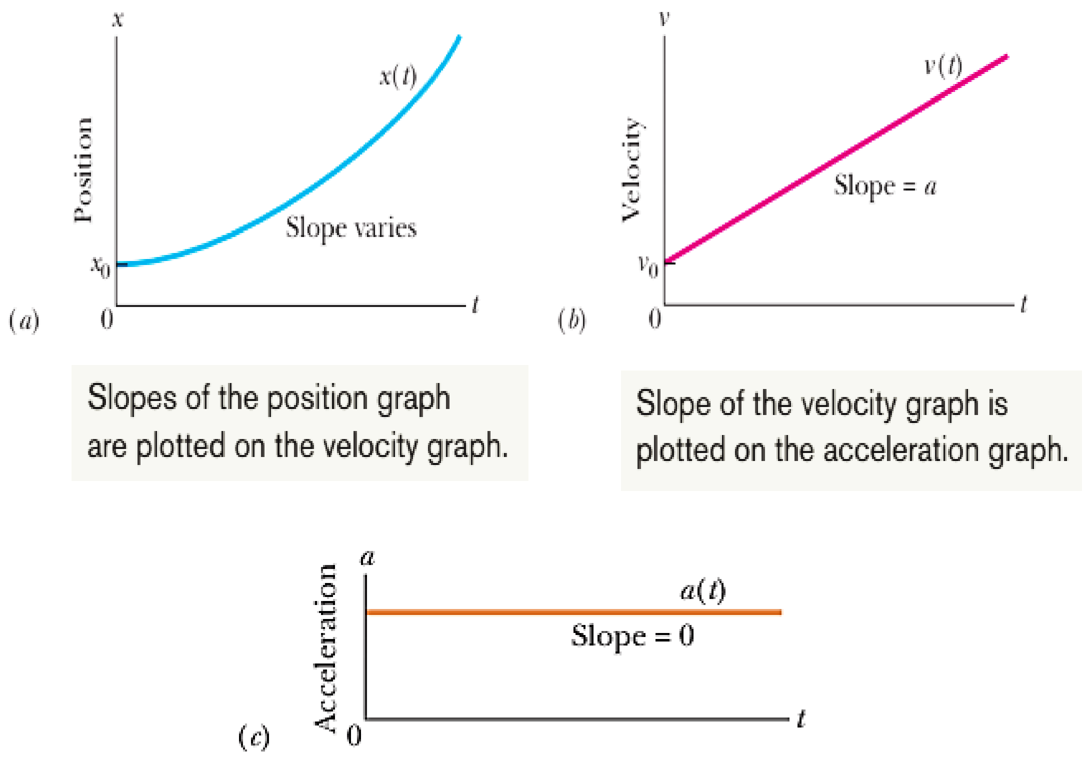


Figure (4) (*a*) The position *x*(*t*) of a particle moving with constant acceleration. (*b*) Its velocity *v*(*t*), given at each point by the slope of the curve of *x*(*t*). (*c*) Its (constant) acceleration, equal to the (constant) slope of the curve of *v*(*t*).

Second Basic Equation. In a similar manner, we can rewrite Eq. 2 (with a few changes in notation) as

$$v_{avg} = \frac{x - x_0}{t - 0}$$ ------------------ (10)

And

$$x = x_0 + v_{avg}t, \quad \text{---------------- (11)}$$

In which x_0 is the position of the particle at $\boldsymbol{t} = \mathbf{0}$ and $\boldsymbol{v_{avg}}$ is the average velocity between $\boldsymbol{t} = \mathbf{0}$ and a later time t.
For the linear velocity function in Eq. 10, the *average* velocity over any time interval (say, from $\boldsymbol{t} = \mathbf{0}$ to a later time t) is the average of the velocity at the beginning of the interval (v_0) and the velocity at the end of the interval (v). For the interval from $\boldsymbol{t} = \mathbf{0}$ to the later time t then, the average velocity is:

$$v_{avg} = \tfrac{1}{2}(v_0 + v). \quad \text{-------(12)}$$

Substituting the right side of Eq. 10 for v yields, after a little rearrangement,

$$v_{avg} = v_0 + \tfrac{1}{2}at. \quad \text{-------------- (13)}$$

Finally, substituting Eq. 12 into Eq. 10 yields

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2. \quad \text{-------------- (14)}$$