General Physics

Lecture (6)

(**Periodic Motion**)

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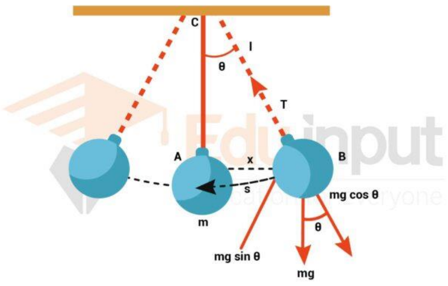
Periodic Motion

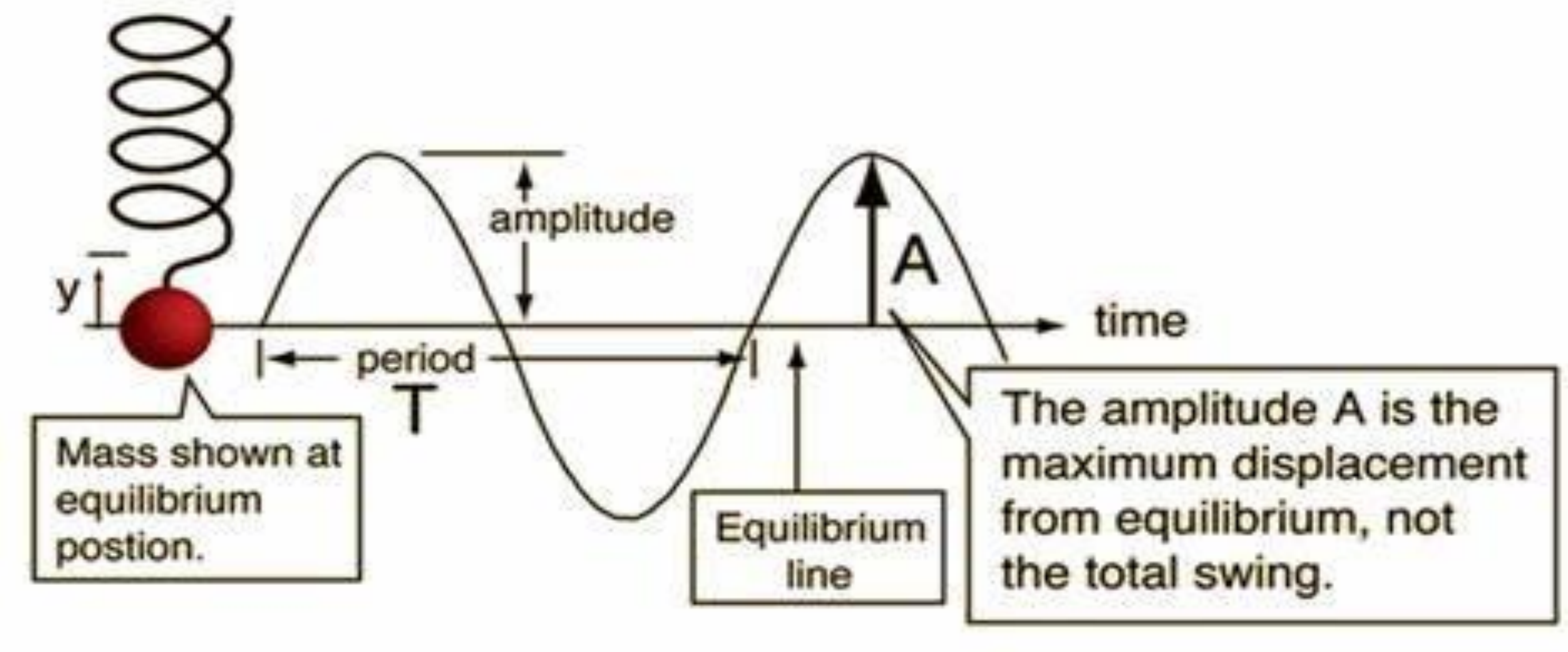


- Motion like:
  – Swinging pendulum
  – Sound vibrations
  – Vibrations of atoms
  - Earth rotating on its axis
  - Earth orbiting the sun.

- Any motion that is repeating
  (comes back to an original point and follows the same path again)
- Can be characterized by:
- – Amplitude of motion
- – Periodicity of motion (Or, the frequency)

One-dimensional Period Motion

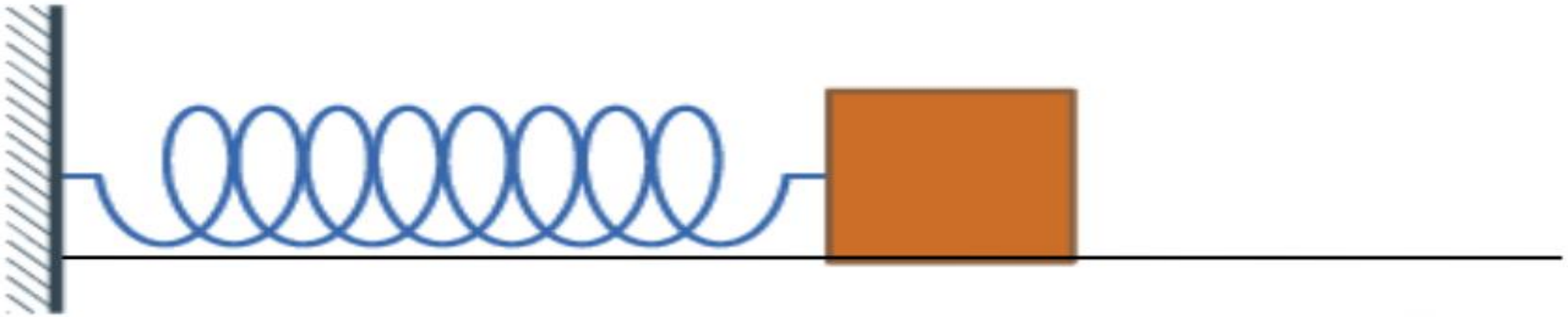
- There is a **repeating** aspect to the motion. Each repetition is referred to as a **cycle**. The same cycle repeats itself in consecutive fashion.
- The repetitions occur **regularly**. It takes the same amount of time to complete each consecutive cycle. The time to complete one cycle is known as the **period**.



Simple Harmonic Motion

- A type of periodic motion with a very explicit definition: • Motion about an equilibrium point with a restoring force proportional to the distance away from the equilibrium point.

$$F = -kx$$



Sinusoidal Relationships

- A motion detector placed below a vibrating mass can detect the position as a function of time. The mass vibrates about a fixed position known as the **resting position**. It oscillates back and forth from (for example) 20 cm above the
- detector to 120 cm above the detector. The position varies as a function of the sine of time.
  Period versus Frequency
- **Period** = time to complete a cycle of vibration
- **Frequency** = number of complete cycles of vibration per unit of time.

$$\text{Period} = \frac{\text{Time (s)}}{\text{\# of cycles}} \qquad \text{Frequency} = \frac{\text{\# of cycles}}{\text{Time (s)}}$$

- **Example**

An object undergoing periodic motion completes 60 cycles in 15 seconds. What is the period and what is the frequency?

$$T = \frac{15\ \text{s}}{60\ \text{cycles}} = 0.25\ \text{s/cycle} \qquad f = \frac{60\ \text{cycles}}{15\ \text{s}} = 4.0\ \text{cycles/s}$$

Simple Harmonic Motion

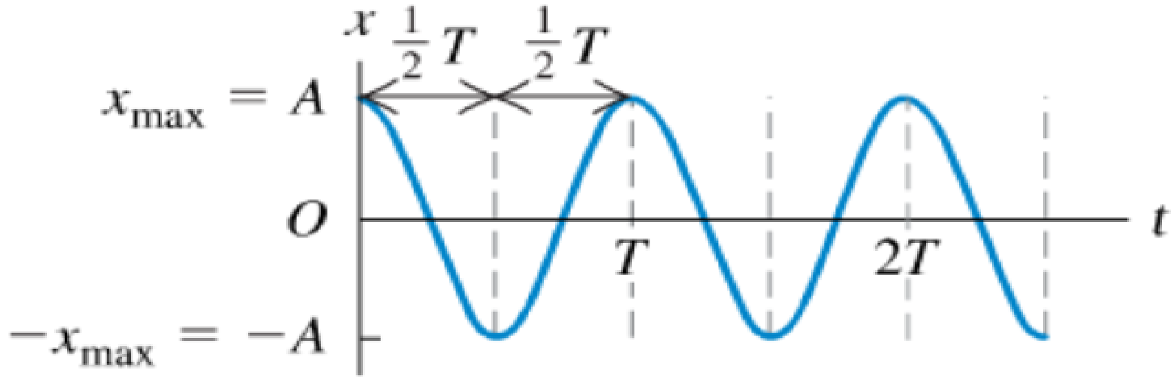
$$F = -kx = ma = m\frac{d^2x}{dt^2} \ggg \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$x(t) = \cos(ct) \Rightarrow \frac{dx}{dt} = -c\sin(ct) \Rightarrow \frac{d^2x}{dt^2} = -c^2\cos(ct) = -\frac{k}{m}\cos(ct)$$

$$\text{if } c^2 = \frac{k}{m}$$

- The general solution is:

$$x(t) = A\cos\left(\sqrt{\frac{k}{m}}\,t + \phi\right)$$



For $\varphi = 0$

Frequency, Angular Frequency and Period

$$x(t) = A\cos(\omega t + \phi), \quad \omega = \sqrt{\frac{k}{m}}$$

- ω is called the angular frequency.
- • x returns to its starting point when $\omega t = 2\pi$, so the
- period (amount of time to complete one cycle), is: T=$\frac{2\pi}{\omega}$
- • The frequency (number of cycles per second) is just:

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Homework

- **A mass m = 1.0kg is attached to a massless spring. The spring is stretched with a force of 6.0N to a distance of 0.03m and then released. What is the frequency of oscillations of the mass?**