Ministry of Higher Education and Scientific Research

Al-Ma'moun University College

كليــة المــأمون الجامعــة

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Physics of Computed Tomography

Lecture (8)

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Physics of Computed Tomography

Data Processing

CT Image Reconstruction

CT perfusion imaging requires the reconstruction of a series of time-dependent volumetric datasets. The sequence of CT volumes measures the dynamics of contrast agent both in the vasculature and in the parenchyma.
CT image reconstruction refers to the computational process of determining tomographic images from X-ray projection images of the irradiated patient. Image reconstruction is a compute-intensive task and one of the most crucial steps in the CT imaging process.
As the basics of X-ray physics, we assume that the result of the X-ray image formation is an attenuation image. Each individual pixel on the detector therefore represents a line integral, that is, the accumulation of all X-ray attenuation coefficients along the projection line. Here, the projection line is the connecting line of the X-ray focal spot with the center of the respective detector pixel.

Computed Tomography

The relation between the removed part of the X-ray intensity dI and the attenuation coefficient μ is found as

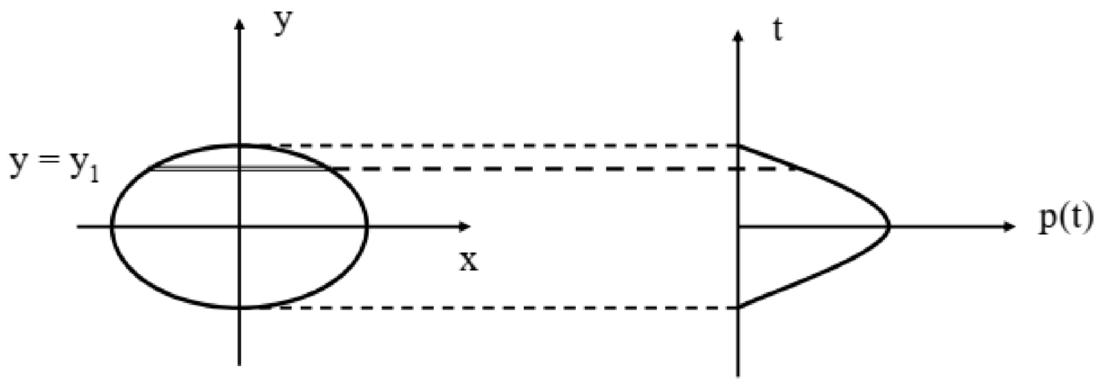
$$dI = -\mu I dx \qquad (1)$$

Here dI represents differential change in X-ray intensity along x-direction and μ is homogeneous. In general, 1) the attenuation coefficient is inhomogenous

$$\mu = \mu(x, y, z) \qquad (2)$$

, i.e., the x-ray beam can be in any direction

Assume a two-dimensional (2D) body with $\mu = \mu(x, y)$,



then at any differential distance dx, the removed X-ray intensity along $y = y_1$, is

$$dI = -\mu(x, y_1)\, I\, dx$$

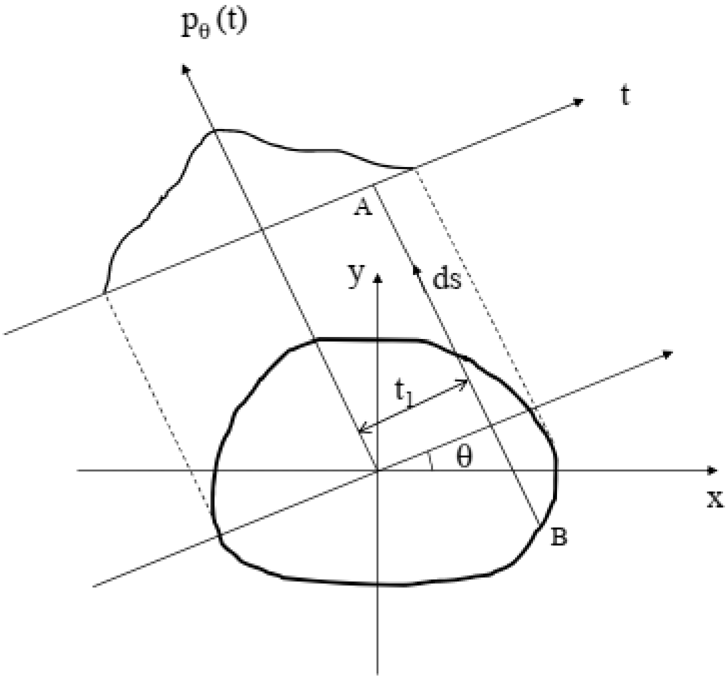
$$dI = -\mu(x, y_1)\, I\, dx$$

$$\int \frac{dI}{I} = -\int_{y=y_1} \mu(x, y_1)\, dx$$

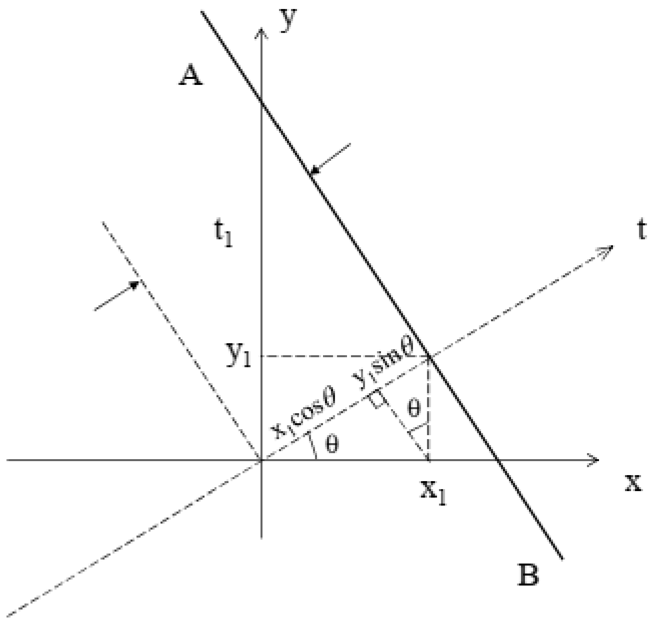
$$\Rightarrow -\ln\frac{I}{I_0} = \int_{y=y_1} \mu(x, y_1)\, dx$$

$$Define: \quad p(t) = p(y_1) = -\ln\frac{I}{I_0} = \int_{y=y_1} \mu(x, y_1)\, dx$$

Where $p(t)$ is the projection function



The projection function is also determined by the angle of view θ



At the AB line:

$$x\,cos\boldsymbol{\theta} + y\,sin\boldsymbol{\theta} = t$$

The projection function can be written as

$$p_\theta(t) = \int_{(\theta,t)\ line} \mu(x,y)\, ds$$

$$\boxed{p_\theta(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu(x,y)\,\delta(x\cos\theta + y\sin\theta - t)\, dxdy}$$

$p_\theta(t)$ is known as the *Radon Transform* of the function $\mu\,(x,y)$.

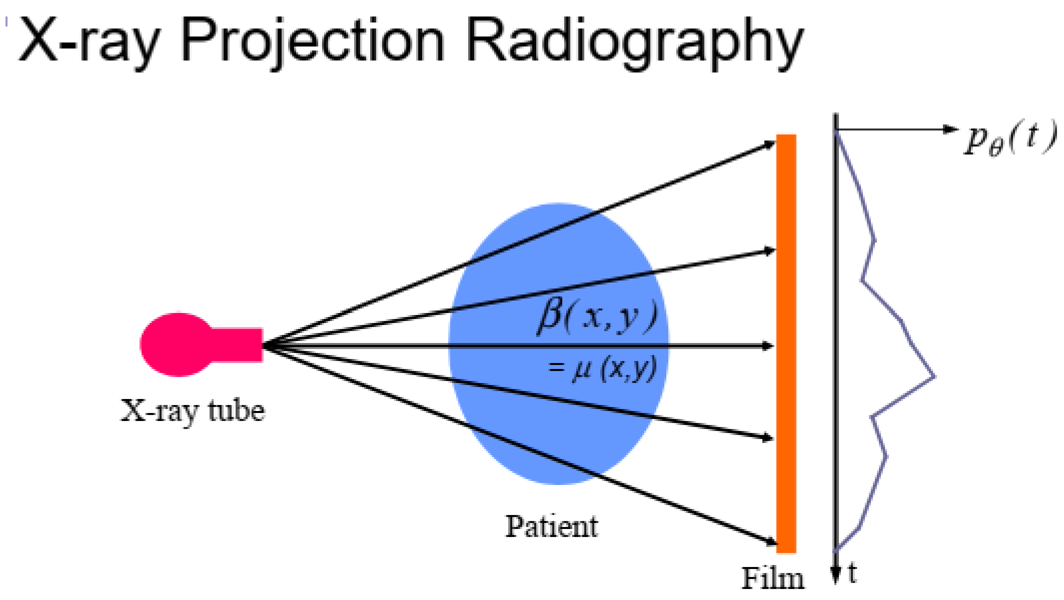
A projection is formed by combining a set of line integrals.

The simplest projection is a collection of parallel ray integrals as given by $p_\theta(t)$ for a constant θ. This is known as *parallel projection*.

Image reconstruction algorithms are derived to construct $\propto (x,y)$ from $p\backslash(t)$.

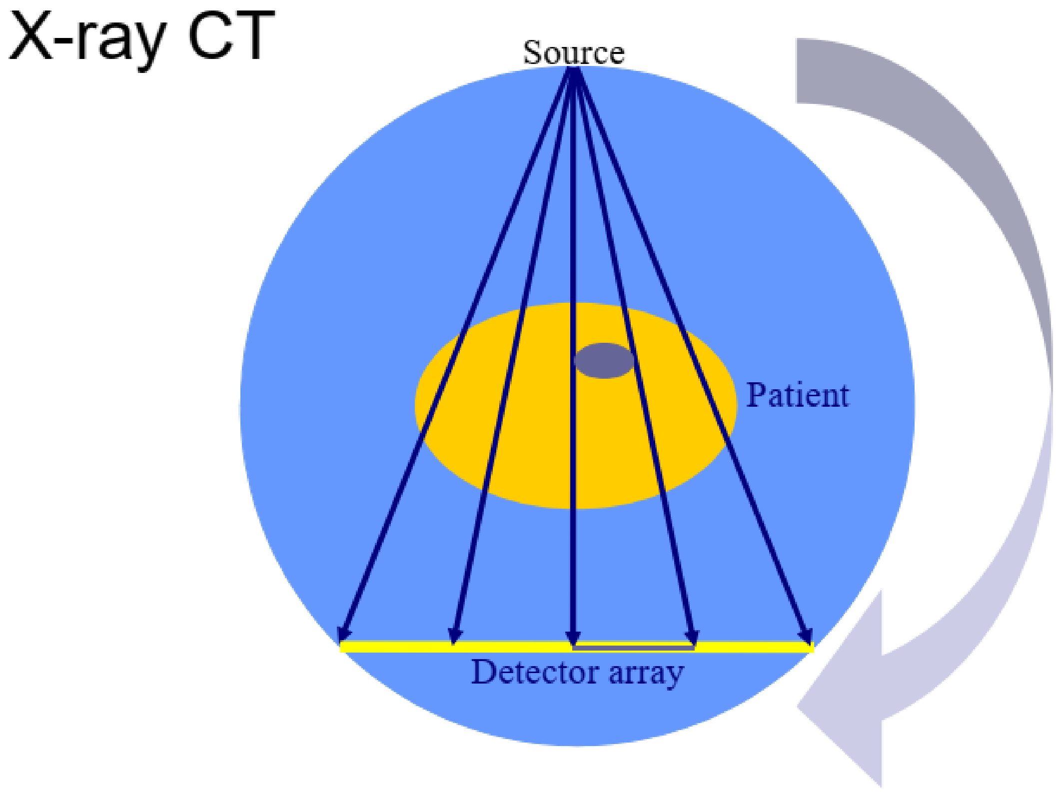
Algorithms

1- Backprojection
2- Fourier Domain Approach
3- Filtered Backprojection
4- Iterative Methods
5- Algebraic Reconstruction
6- Technique (ART) Iterative Least Squares
7- Simultaneous Iterative
8- Reconstruction Technique (SIRT)

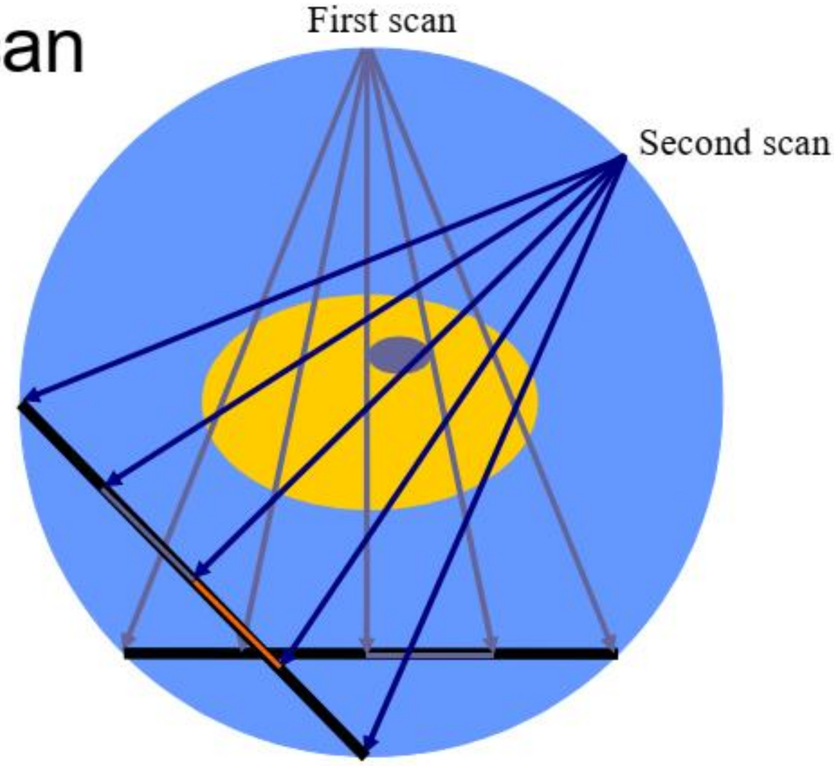


$$p_{\boldsymbol{\theta}(t)} = \int \beta(x,y)\ \mathbf{ds} = \iint_{-\infty}^{\infty} \beta(x,y)\delta(x\cos\theta + y\sin\theta - t)dxdy$$

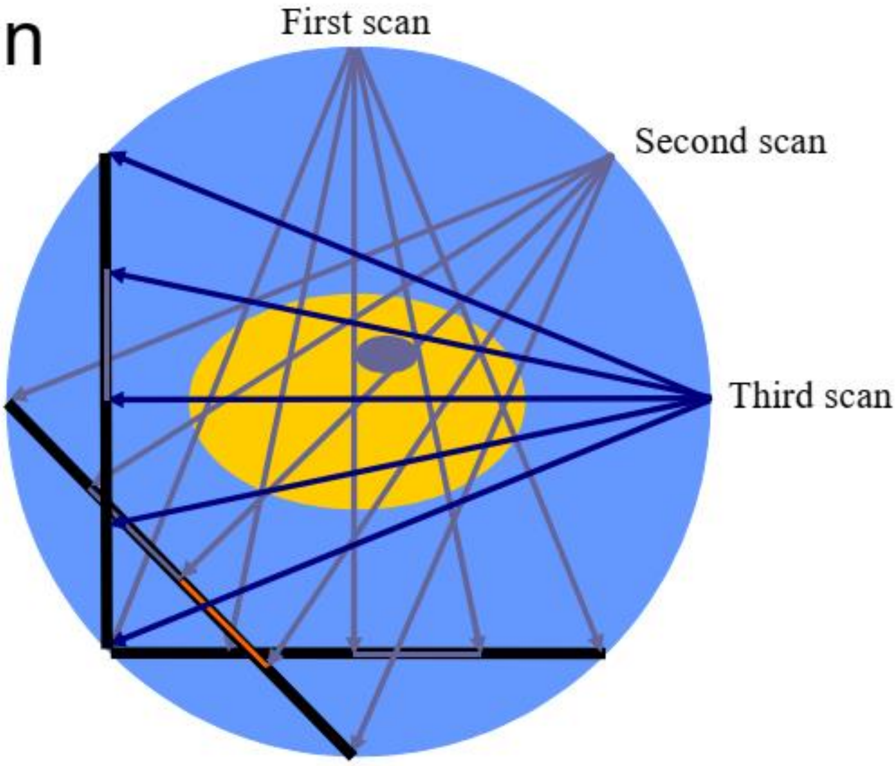
Radon Transform



CT Scan



CT Scan



CT Scan