Ohm's law

Ohm's law states that the current through a conductor between two points is directly proportional to the potential difference across the two points. Introducing the constant of proportionality, the resistance,

$$I=V/R$$

where *I* is the current through the conductor in units of amperes, *V* is the potential difference measured *across* the conductor in units of volts, and *R* is the resistance of the conductor in units of ohms. More specifically, Ohm's law states that the *R* in this relation is constant, independent of the current.

Electric current: • Net charge flowing through given cross-sectional area per unit time. •

$$I = dQ\ dt\ .$$

SI unit: 1C/s = 1A (one Ampere)

Types of Current

The current can be divided into two types.

Direct Current:

- Direct current travels in the same direction at all points, although the instantaneous magnitude can differ.
- An example of DC is the current generated by an electrochemical cell.

Alternating Current:

- The flow of charge carriers is towards the opposite direction periodically in an alternating current.

- The number of AC cycles per second is known as frequency and calculated in Hertz.

- **What Is Current Density?**
- The amount of electric current traveling per unit cross-section area is called as current density and expressed in amperes per square meter. The more the current in a conductor, the higher will be the current density.

The formula for Current Density is given as,

$$\mathbf{J = I / A}$$

Where,

I = current flowing through the conductor in Amperes

A = cross-sectional area in m^2.

Current density is expressed in A/m^2.

Solved Problem on Current Density

Example 1: **Determine the current density when 40 amperes of current is flowing through the battery in a given area of 10 m^2.**

Solution:

The current density formula is given by,

$J = I / A = 40 / 10 = 4\ A/m^2$.

Example 2: If the current density is 4 A/m^2 and the current through the conductor is 24 A. Find the cross-sectional area.

Solution:

Since, $J = I/A$

$A = I/J = 24/4 = 6\ m^2$

Asked Questions –

Q1: What is current?

We can define current as the flow of electrically charged particles travelling. Electric current is represented as I.

Q2: What are the types of current?

The following are the types of electric current:

- Direct Current
- Alternating Current

Q3: What is meant by current density?

The amount of current travelling per unit cross-section area is known as current density and expressed in amperes per square metre.

Q4 :Give the formula for the current density.

The formula for current density is J = I/A.

Q5: Which type of current is generated by an electrochemical cell?

Direct current is generated by an electrochemical cell.

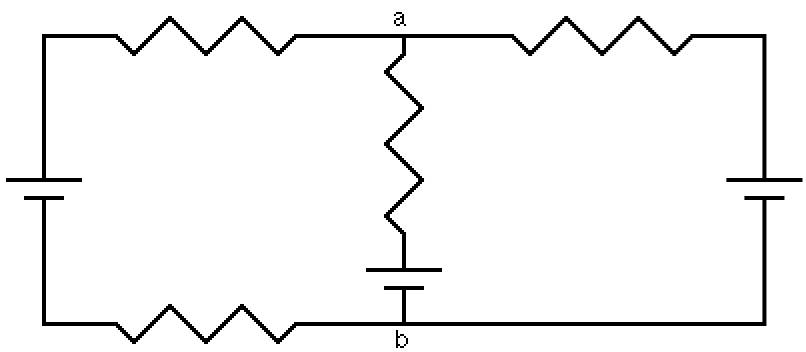
Multi-loop Circuits and Kirchoff's Rules

It is helpful to define two terms, junction and branch.

A junction is a point where at least three circuit paths meet.

A branch is a path connecting two junctions.

In the circuit below, there are two junctions, labeled a and b. There are three branches: these are the three paths from a to b.



Multi-loop circuits

In a circuit involving one battery and a number of resistors in series and/or parallel, the resistors can generally be reduced to a single equivalent resistor. With more than one battery, the situation is trickier. If all the batteries are part of one branch they can be combined into a single equivalent battery. Generally, the batteries will be part of different branches, and another method has to be used to analyze the circuit to find the current in each branch. Circuits like this are known as multi-loop circuits.

Finding the current in all branches of a multi-loop circuit (or the emf of a battery or the value of a resistor) is done by following guidelines known as Kirchoff's rules. These guidelines also apply to very simple circuits.

K**irchoff's first rule** : the junction rule. The sum of the currents coming in to a junction is equal to the sum leaving the junction. (Basically this is conservation of charge)

Kirchoff's second rule : the loop rule. The sum of all the potential differences around a complete loop is equal to zero. (Conservation of energy)

The branch current method

To analyze a circuit using the branch-current method involves three steps:

1. Label the current and the current direction in each branch. Sometimes it's hard to tell which is the correct direction for the current in a particular loop. That does NOT matter. Simply pick a direction. If you guess wrong, youšll get a negative value. The value is correct, and the negative sign means that the current direction is opposite to the way you guessed. You should use the negative sign in your calculations, however.

2. Use Kirchoff's first rule to write down current equations for each junction that gives you a different equation. For a circuit with two inner loops and two junctions, one current equation is enough because both junctions give you the same equation.

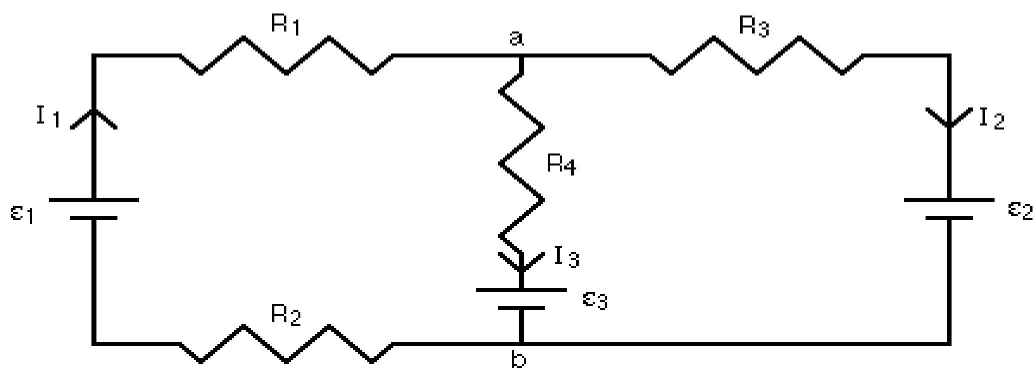
3. Use Kirchoff's second rule to write down loop equations for as many loops as it takes to include each branch at least once. To write down a loop equation, you choose a starting point, and then walk around the loop in one direction until you get back to the starting point. As you cross batteries and resistors, write down each voltage change. Add these voltage gains and losses up and set them equal to zero.

When you cross a battery from the - side to the + side, that's a positive change. Going the other way gives you a drop in potential, so that's a negative change.

When you cross a resistor in the same direction as the current, that's also a drop in potential so it's a negative change in potential. Crossing a resistor in the opposite direction as the current gives you a positive change in potential.

An example

Running through an example should help clarify how Kirchoff's rules are used. Consider the circuit below:



The battery emf's are : $\varepsilon_1 = 19$ V $\quad \varepsilon_2 = 6$ V $\quad \varepsilon_3 = 2$ V

The resistors are : $R_1 = 6\ \Omega \quad R_2 = 4\ \Omega \quad R_3 = 4\ \Omega \quad R_4 = 1\ \Omega$

Step 1 of the branch current method has already been done. The currents have been labeled in each branch of the circuit, and the directions are shown with arrows. Again, you don't have to be sure of these directions at this point. Simply choose directions, and if any of the currents come out to have negative signs, all it means is that the direction of that current is opposite to the way you've shown on your diagram.

Applying step 2 of the branch current method means looking at the junctions, and writing down a current equation. At junction a, the total current coming in to the junction equals the total current flowing away. This gives:

$$\text{at junction a}: I_1 = I_2 + I_3$$

If we applied the junction rule at junction b, we'd get the same equation. So, applying the junction rule at one of the junctions is all we need to do. In some cases you will need to get equations from more than one junction, but you'll never need to get an equation for every junction.

There are three unknowns, the three currents, so we need to have three equations. One came from the junction rule; the other two come from going to step 3 and applying the loop rule.

There are three loops to use in this circuit: the inside loop on the left, the inside loop on the right, and the loop that goes all the way around the outside.

We just need to write down loop equations until each branch has been used at least once, though, so using any two of the three loops in this case is sufficient.

When applying the loop equation, the first step is to choose a starting point on one loop. Then walk around the loop, in either direction, and write down the change in potential when you go through a battery or resistor. When the potential increases, the change is positive; when the potential decreases, the change is negative. When you get back to your starting point, add up all the potential changes and set this sum equal to zero, because the net change should be zero when you get back to where you started.

When you pass through a battery from minus to plus, that's a positive change in potential, equal to the emf of the battery. If you go through from plus to minus, the change in potential is equal to minus the emf of the battery.

Current flows from high to low potential through a resistor. If you pass through a resistor in the same direction as the current, the potential, given by IR, will decrease, so it will have a minus sign. If you go through a resistor opposite to the direction of the current, you're going from lower to higher potential, and the IR change in potential has a plus sign.

Keeping all this in mind, let's write down the loop equation for the inside loop on the left side. Picking a starting point as the bottom left corner, and moving clockwise around the loop gives:

$$+\varepsilon_1 - I_1R_1 - I_3R_4 - \varepsilon_3 - I_1R_2 = 0$$

Make sure you match the current to the resistor; there is one current for each branch, and a loop has at least two branches in it.

The inner loop on the right side can be used to get the second loop equation. Starting in the bottom right corner and going counter-clockwise gives:

$$+\varepsilon_2 + I_2R_3 - I_3R_4 - \varepsilon_3 = 0$$

Plugging in the values for the resistances and battery emf's gives, for the three equations:

$I_1 = I_2 + I_3$ (equation 1)

$+19 - 6I_1 - I_3 - 2 - 4I_1 = 0$ so $10I_1 + I_3 = 17$ (equation 2)

$+6 + 4I_2 - I_3 - 2 = 0$ so $4I_2 - I_3 = -4$ (equation 3)

The simplest way to solve this is to look at which variable shows up in both loop equations (equations 2 and 3), solve for that variable in equation 1, and substitute it in in equations 2 and 3.

Rearranging equation 1 gives:

$$I_3 = I_1 - I_2$$

Substituting this into equation 2 gives:

$10I_1 + (I_1 - I_2) = 17$ so $11I_1 - I_2 = 17$ (equation 4)

Making the same substitution into equation 3 gives:

$4I_2 - (I_1 - I_2) = -4$ so $-I_1 + 5I_2 = -4$ (equation 5)

This set of two equations in two unknowns can be reduced to one equation in one unknown by multiplying equation 4 by 5 and adding the result to equation 5.

5 x equation 4: $55I_1 - 5I_2 = 85$

+ equation 5: $-I_1 + 5I_2 = -4$

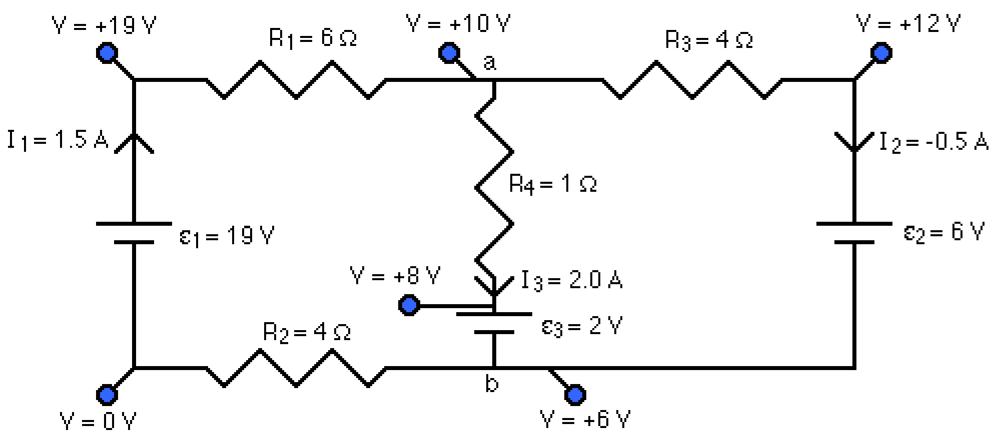
$54I_1 = 81$ so $I_1 = 1.5$ A

Substituting this into equation 5 gives:

$$I_2 = (-4 + 1.5) / 5 = -0.5 \text{ A}$$

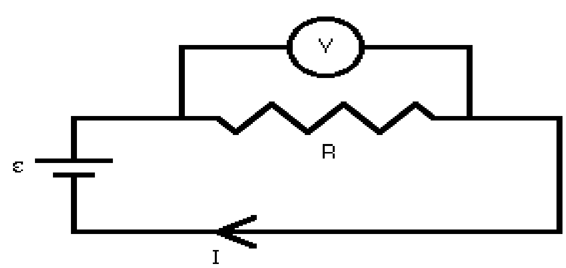
The negative sign means that the current is 0.5 A in the direction opposite to that shown on the diagram. Solving for the current in the middle branch from equation 1 gives:

$$I_3 = 1.5 - (-0.5) = 2.0 \text{ A}$$



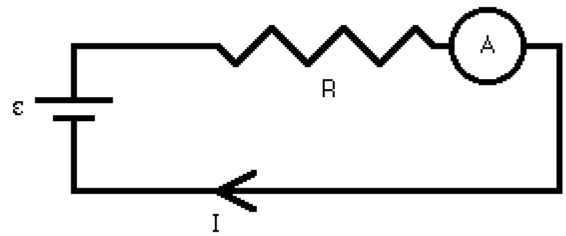
One final note: you can use this method of circuit analysis to solve for more things than just the current. If one or more of the currents was known (maybe the circuit has an ammeter or two, measuring the current magnitude and direction in one or two branches) then an unknown battery emf or an unknown resistance could be found instead.

Voltmeters



Resistors in parallel have the same voltage across them, so if you want to measure the voltage across a circuit element like a resistor, you place the voltmeter in parallel with the resistor. To prevent the voltmeter from changing the current in the circuit (and therefore the voltage across the resistor), the voltmeter must have a resistance much larger than the resistor's. With a large voltmeter resistance, hardly any of the current in the circuit makes a detour through the meter.

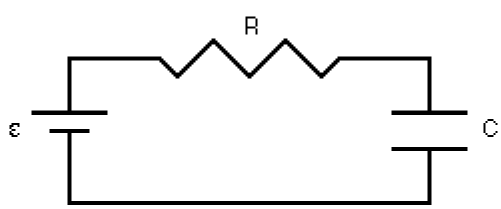
Ammeters



Remember that resistors in series have the same current flowing through them. An ammeter, then, must be placed in series with a resistor to measure the current through the resistor. On a circuit diagram, an ammeter is shown as an A in a circle. Again, the ammeter acts as a resistor, so to minimize its impact on the circuit it must have a small resistance relative to the resistance of the resistor whose current is being measured.

RC Circuits

Resistors are relatively simple circuit elements. When a resistor or a set of resistors is connected to a voltage source, the current is constant. If a capacitor is added to the circuit, the situation changes. In a simple series circuit, with a battery, resistor, and capacitor in series, the current will follow an exponential decay. The time it takes to decay is determined by the resistance (R) and capacitance (C) in the circuit.



A capacitor is a device for storing charge. In some sense, a capacitor acts like a temporary battery. When a capacitor is connected through a resistor to a battery, charge from the battery is stored in the capacitor. This causes a potential difference to build up across the capacitor, which opposes the potential difference of the battery. As this potential difference builds, the current in the circuit decreases.

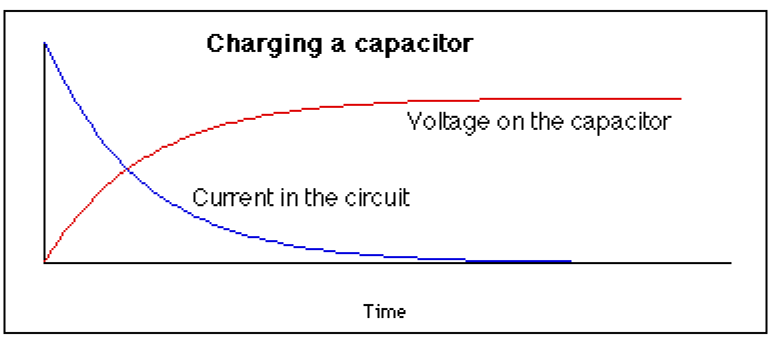
If the capacitor is connected to a battery with a voltage of Vo, the voltage across the capacitor varies with time according to the equation:

$$V = V_0 [1 - e^{-t/RC}]$$

The current in the circuit varies with time according to the equation:

$$I = I_0 e^{-t/RC}$$

Graphs of voltage and current as a function of time while the capacitor charges are shown below.

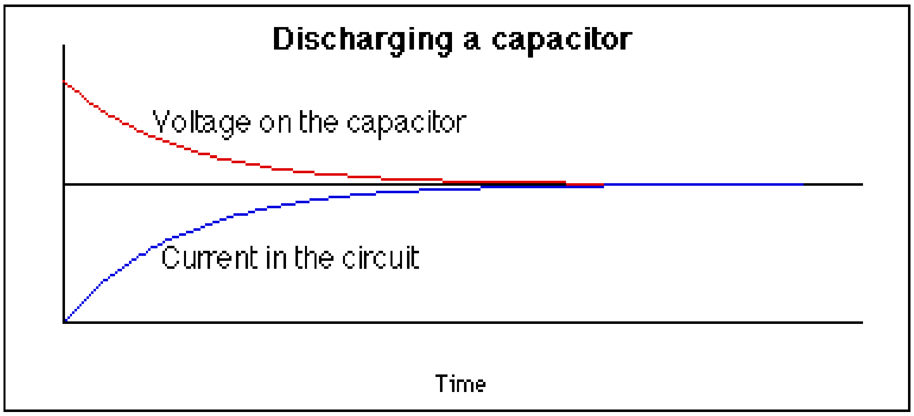


The product of the resistance and capacitance, RC, in the circuit is known as the time constant. This is a measure of how fast the capacitor will charge or discharge.

After charging a capacitor with a battery, the battery can be removed and the capacitor can be used to supply current to the circuit. In this case, the current obeys the same equation as above, decaying away exponentially, and the voltage across the capacitor will vary as:

$$V = V_0 e^{-t/RC}$$

Graphs of the voltage and current while the capacitor discharges are shown here. The current is shown negative because it is opposite in direction to the current when the capacitor charges.



What Is Electromotive Force?

Electromotive force is defined as *the electric potential produced by either an electrochemical cell or by changing the magnetic field.* **Symbol for Electromotive Force ,** is ε.

What Is Electromotive Force Formula?

Following is the formula for electromotive force:

$$\varepsilon = V + Ir$$

Where,

- V is the voltage of the cell, I is the current across the circuit, r is the internal resistance of the cell, ε is the electromotive force

What Is the Unit of EMF?

The **unit for electromotive force is Volt.**

Dimension of Electromotive Force :

EMF is given as the ratio of work done on a unit charge

Difference between Electromotive Force and Potential Difference

Electromotive Force	**Potential Difference**
EMF is defined as the work done on a unit charge	Potential difference is defined as the energy which is dissipated as the unit charge pass through the components
EMF remains constant	Potential difference is not constant
EMF is independent of circuit resistance	The potential difference depends on the resistance between the two points during the measurement
Due to EMF, electric, magnetic, and the gravitational field is caused	Due to the potential difference, the only electric field is induced
It is represented by E	It is represented by V

Definition of Capacitance

The **capacitance**, *C*, of a capacitor is defined the capacitance of a capacitor is the amount of charge the capacitor can store per unit of potential difference.

The SI unit of capacitance is the **farad** (F).

The farad is a large unit, typically you will see microfarads (mF) and picofarads (pF).

Capacitance will always be a positive quantity .The capacitance of a given capacitor is constant.

The capacitance is a measure of the capacitor's ability to store charge.

Makeup of a Capacitor

A capacitor consists of two conductors.

- These conductors are called plates.
- When the conductor is charged, the plates carry charges of equal magnitude and opposite directions.

A potential difference exists between the plates due to the charge.

Parallel Plate Capacitor

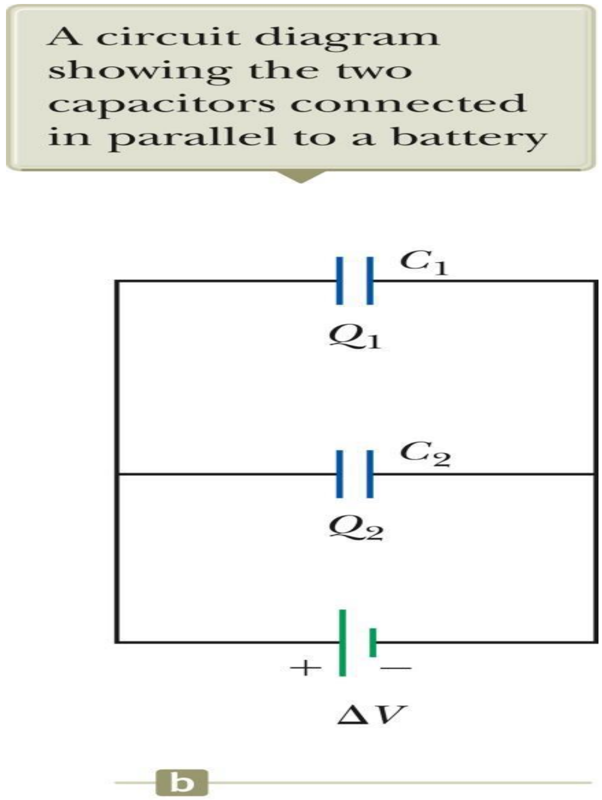
Each plate is connected to a terminal of the battery.

- The battery is a source of potential difference.

If the capacitor is initially uncharged, the battery establishes an electric field in the connecting wires.

The capacitors can be replaced with one capacitor with a capacitance of *equivalent capacitor* (C_{eq}.) .

- The C_{eq}. must have exactly the same external effect on the circuit as the original capacitors

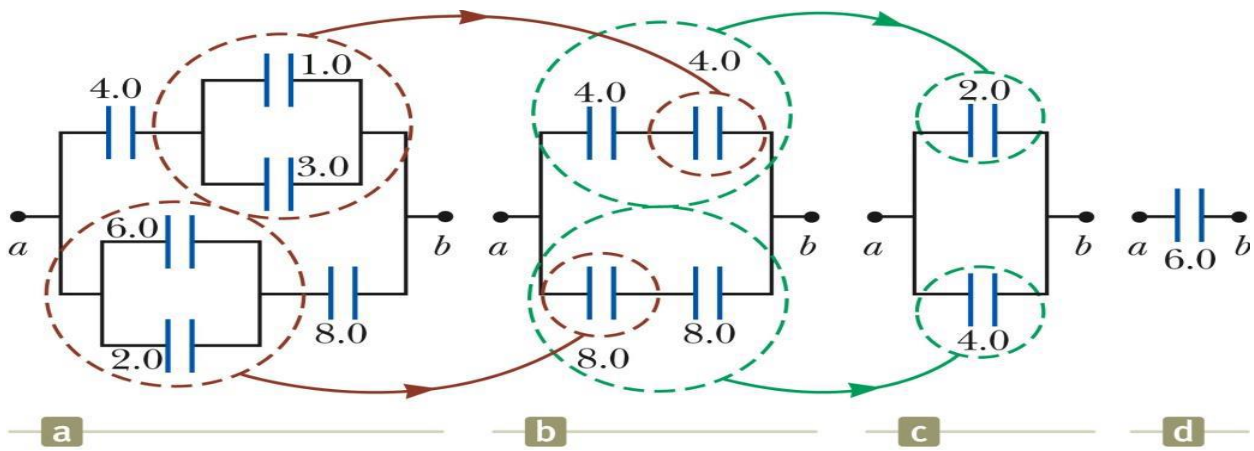


$$C_{eq} = C_1 + C_2 + C_3 + \ldots$$

The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors.

Equivalent Capacitance, Example

The 1.0-mF and 3.0-mF capacitors are in parallel as are the 6.0-mF and 2.0-mF capacitors. These parallel combinations are in series with the capacitors next to them. The series combinations are in parallel and the final equivalent capacitance can be found.



Capacitance –Parallel Plates

The charge density on the plates is $\sigma = Q/A$.

□ A is the area of each plate, the area of each plate is equal

□ Q is the charge on each plate, equal with opposite signs

The electric field is uniform between the plates and zero elsewhere.

$$C = Q/V = Q/Ed = Q/\ Qd/\varepsilon_o\ A = \varepsilon_o\ A/d$$

$$C = \varepsilon_o\ A/d$$

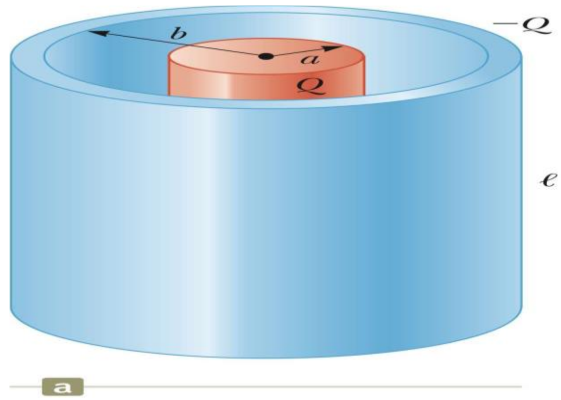
The capacitance is proportional to the area of its plates and inversely proportional to the distance between the plates

Capacitance of a Cylindrical Capacitor

$$V = -2k\ \lambda \ln (b/a)$$

Where □λ= Q/ℓ

The capacitance is□ C=Q/V= □ ℓ /2 k ln (b /a)



Capacitance of a Spherical Capacitor

The potential difference will be kQ(1/b-1/a)

The capacitance will be C=Q/V = ab/k(b-a)

Capacitors in Parallel

The capacitors can be replaced with one capacitor with a capacitance of *C*eq.

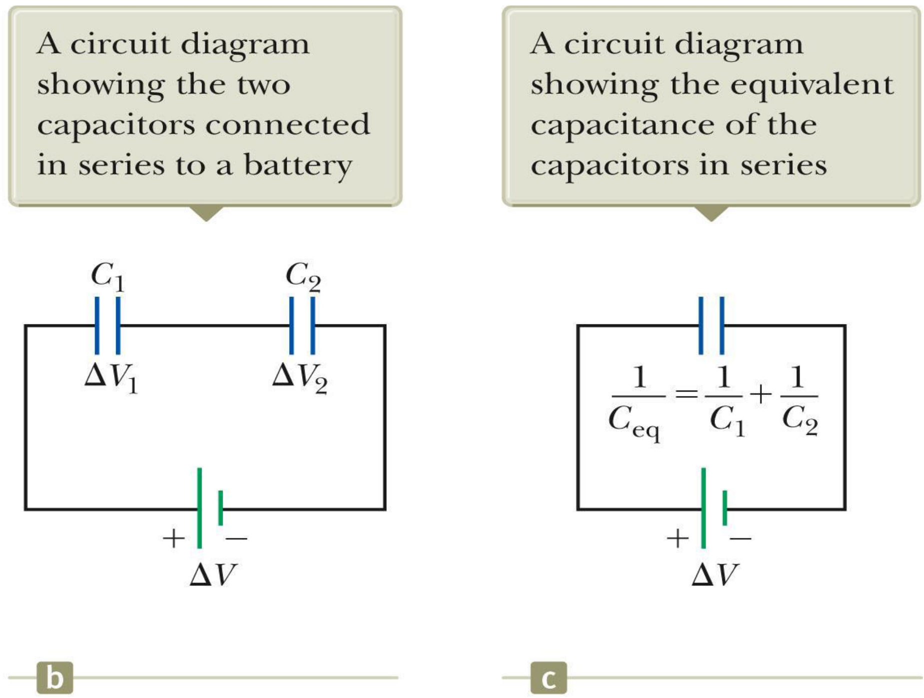
The *equivalent capacitor*must have exactly the same external effect on the circuit as the original capacitors.

Capacitors in Series

An equivalent capacitor can be found that performs the same function as the series combination.

The charges are all the same.

$$Q_1 = Q_2 = Q$$



Energy Stored in a Capacitor

Assume the capacitor is being charged, at some point, has a charge(q*)* on it.

The work needed to transfer a charge from one plate to the others

$$W=V\,dq = q/c\,dq$$

The total work required is $W = \int (q/C)\,dq = Q\,/2\,C$

The work done in charging the capacitor appears as electric potential energy

$$U=Q^2/2C = \tfrac{1}{2} QV = 1/2CV^2$$

The energy stored increases as the charge increases and as the potential difference increases.

The energy can be considered to be stored in the electric field .

For a parallel-plate capacitor, the energy can be expressed in terms of the field as

$$U= \tfrac{1}{2} (\varepsilon_o A d) E^2.$$

It can also be expressed in terms of the energy density (energy per unit volume)

The energy density $(U/Ad) = ½\, \varepsilon_o E^2$.

Capacitors with Dielectrics

A dielectric is a nonconducting material that, when placed between the plates of a capacitor, increases the capacitance. Dielectrics include rubber, glass, and waxed paper

With a dielectric, the capacitance becomes $C = \kappa C_o$.
κ is the dielectric constant of the material.

The capacitance increases by the factor κ when the dielectric completely fills the region between the plates.

If the capacitor remains connected to a battery, the voltage across the capacitor necessarily remains the same.
If the capacitor is disconnected from the battery, the capacitor is an isolated system and the charge remains the same.

For a parallel-plate capacitor,

$$C = \kappa\, (\varepsilon_o A) / d$$

d is limited by the electric discharge that could occur though the dielectric medium separating the plates.

In theory, *c*ould be made very small to create a very large capacitance.
In practice, there is a limit to *d*.

For a given *d*, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** of the material.