Problems on Gauss Law

Problem 1: A uniform electric field of magnitude E = 100 N/C exists in the space in the X-direction. Using the Gauss theorem, calculate the flux of this field through a plane, square area of edge 10 cm placed in the Y-Z plane. Take the normal along the positive X-axis to be positive.

Solution:

The flux $\Phi = \int E.\cos\theta\, dA$

As the normal to the area points along the electric field, $\theta = 0$

Also, E is uniform so, $\mathbf{\Phi = E.\Delta A = (100\ N/C)\ (0.10m)^2 = 1\ \ N\text{-}m^2}$

Problem 2: A large plane charge sheet having surface charge density $\sigma = 2.0 \times 10^{-6}$ C-m^{-2} lies in the X-Y plane. Find the flux of the electric field through a circular area of radius 1 cm lying completely in the region where x, y and z are all positive and with its normal, making an angle of 60^0 with the Z-axis.

Solution:

The electric field near the plane charge sheet is $E = \sigma/2\varepsilon_0$ in the direction away from the sheet.

The flux $= E.\Delta A \cos\theta = \sigma/2\varepsilon_0 \times \pi r^2 \cos 60^o$

$=2.0\times10^{-6} / (2\times8.85\times10^{-12}) \times 3.14\times(0.01)^2 x\ 0.5$

$= 17.5$ N-m^{2}C^{-1}.

Problem 3: A charge of 4×10^{-8} C is distributed uniformly on the surface of a sphere of radius 1 cm. It is covered by a concentric, hollow conducting sphere of radius 5 cm.

- Find the electric field at a point 2 cm away from the Centre.

Solution:

The flux through this surface $=\oint E.dA\ \ = E\oint dA\ \ === 4\pi\ x^2\ E$

$$\Rightarrow 4\pi x^2 E = q/\varepsilon_0$$

$$\text{or, } E = q/4\pi\varepsilon_0 x^2 = 1/4\pi\varepsilon_0 \; {}_q x^2$$

$$= (9 \times 10^9) \times [(4 \times 10^{-8})/(4 \times 10^{-4})] = 9 \times 10^5 \text{ N C}^{-1}.$$

Frequently Asked Questions on Gauss Law

Q1:**Can the Gauss law be applied to all surfaces?**

For any closed surface and for any distribution of charges, the Gauss law is valid.

Q2:**Can the Gauss law be applied to the non-uniform electric field?**

The Gauss law can be applied to uniform and non-uniform electric fields.

Q3:**State the Gauss law.**

According to the Gauss law, the net flux of an electric field in a closed surface is directly proportional to the charge enclosed.

Q4:**What is the factor on which the electric field lines depend?**

The Gauss law is interpreted in terms of the electric flux through the surface. The electric flux through the surface is the number of lines of force passing normally through the surface. The electric flux depends on the charge enclosed by the surface.

Q5:**When is the flux through the surface taken as positive or negative?**

The flux through the surface is taken as positive if the flux lines are directed outwards and negative if the flux is directed inwards.

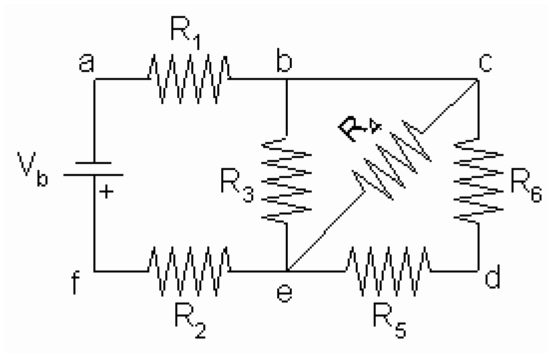
Q6 :**What is called a Gaussian surface?**

A Gaussian surface is a surface through which the electric flux is calculated.

Q7 :**Define surface charge density.**

When the charge is uniformly distributed over the surface of the conductor, it is called surface charge density.

Problem: Find the potential difference across and the current through each resistor of the circuit shown below. Given values are listed to the

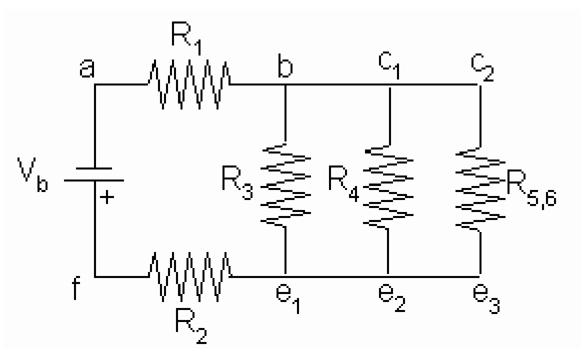


Circuit diagram 1

V_b	12 V
R_1	1 Ω
R_2	5 Ω
R_3	2 Ω
R_4	3 Ω
R_5	4 Ω
R_6	2 Ω

<u>Finding the equivalent resistance</u>

The next step is to find the equivalent resistance of the circuit. You can then use that result to find the total current.

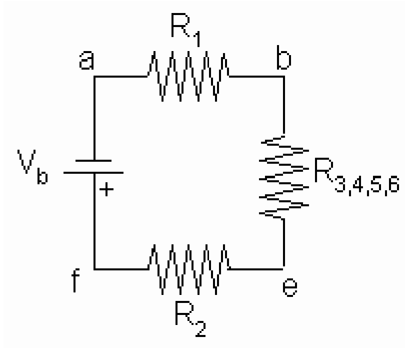


Circuit diagram 2

By the formula for resistors in series.

$$R_{5,6} = R_5 + R_6.$$

Now we can combine R_3, R_4, and $R_{5,6}$ as shown below.



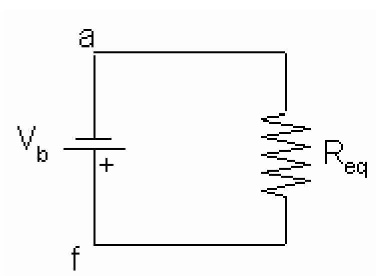
Circuit diagram 3

By the formula for resistors in parallel,

$$\frac{1}{R_{3,4,5,6}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_{5,6}}$$

$$R_{3,4,5,6} = \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5 + R_6}\right)^{-1}$$

We now see that R_1, $R_{3,4,5,6}$, and R_2 are in series, because the current has only one path: feba. They can be combined into a single resistance which is the equivalent resistance, R_{eq}, of the entire circuit. We calculate that value to the right of the circuit.



$$\begin{aligned} R_{eq} &= R_1 + R_2 + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5 + R_6}\right)^{-1} \\ &= 1 + 5 + \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{6}\right)^{-1} \Omega \\ &= 6 + \left(\frac{6}{6}\right)^{-1} \Omega \\ &= 7\ \Omega \end{aligned}$$

Circuit diagram 4

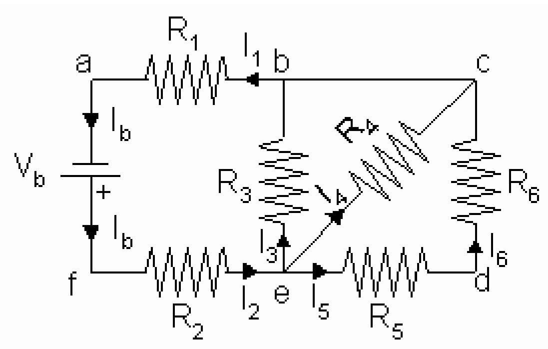
Finally, we note the other combined resistances, because they'll come in handy in finding currents and potential differences.

$R_{5,6} = 6\ \Omega$
$R_{3,4,5,6} = 1\ \Omega$

<u>Finding currents and potential differences</u>

Once R_{eq} is known, find the total current, I_b, provided by the battery. All the currents are shown in Circuit diagram 5. First we see that the currents I_1 and I_2 must be the same as I_b, because R_1 and R_2 are in series with the battery. We also see that currents I_5 and I_6 are the same, because R_5 and R_6 are in series.

However, by the junction rule, we know that $I_2 = I_3 + I_4 + I_5$.



Circuit diagram 5

$I_b = V_b/R_{eq}$ = (12 V)/(7 Ω) = 12/7 A.

Now let's move on to Circuit diagram 3. We see that R_1, R_2, $R_{3,4,5,6}$ and the battery are in series and must all have the same current.
Therefore, $I_b = I_1 = I_2 = I_{3,4,5,6}$ = 12/7 A. We can now calculate the potential differences across the resistors.

$V_1 = I_1R_1$ = (12/7 A)(1 Ω) = 12/7 V
$V_2 = I_2R_2$ = (12/7 A)(5 Ω) = 60/7 V
$V_{3,4,5,6} = I_{3,4,5,6}R_{,3,4,5,6}$ = (12/7 A)(1 Ω) = 12/7 V

As a check on our work so far, the loop rule tells us that the potential rise V_b across the battery should equal the sum of the potential drops across the resistors.

$V_1 + V_2 + V_{3,4,5,6}$ = (12/7 + 60/7 + 12/7) V = 84/7 V = 12 V = V_b.

We have left to deal with resistors R_3 to R_6 individually. Let's back up to Circuit diagram 2. We see that R_3, R_4, and $R_{5,6}$ must have the same potential difference across them. We've already calculated this value above as $V_{3,4,5,6}$.

Thus, $V_3 = V_4 = V_{5,6}$ = 12/7 V. We can use this value to find the currents through the resistors.

$I_3 = V_3/R_3$ = (12/7 V)/(2 Ω) = 6/7 A
$I_4 = V_4/R_4$ = (12/7 V)/(3 Ω) = 4/7 A
$I_{5,6} = V_{5,6}/R_{5,6}$ = (12/7 V)/(6 Ω) = 2/7 A

Before going on, we can use the junction rule to provide another check on our work. The three currents above all branch off at the same point. The current going in to that point is I_2 = 12/7 A. The sum of 6/7, 4/7, and 2/7 is 12/7, so the junction rule checks.

The last thing to do is find the potential differences
across R_5 and R_6 individually. We already know the current in these resistors, so the potential differences are:

$V_5 = I_5R_5 = (2/7\ \text{A})(4\ \Omega) = 8/7\ \text{V}$
$V_6 = I_6R_6 = (2/7\ \text{A})(2\ \Omega) = 4/7\ \text{V}$