AL-MAMON UNIVERSITY COLLAGE

DEPARTMENT OF ELECTRICAL POWER
ENGINEERING TECHNIQUES

Lecture notes 1

Mechanical Vibrations

Part I

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Basic Concepts of Mechanical Vibrations

This chapter introduces the subject of mechanical vibrations including the following topics:

- Vibration and simple harmonic motion;
- Importance of the study of vibration;
- Characteristics of vibration;
- Main elements of a vibrating system;
- Classification of vibration;
- Harmonic analysis;
- Degrees of freedom;
- Modelling of vibrating systems.

1.1 Vibration and simple harmonic motion:

1.1.1 What is vibration?

Vibration can be defined as any motion that repeats itself in a certain interval of time around a certain equilibrium position. The study of vibration is concerned with the oscillatory motion of bodies and the forces associated with them. Most engineering machines and structures experience vibration to some degree and their design generally requires consideration of their oscillatory behavior

يمكن تعريف الاهتزاز على أنه أي حركة تكرر نفسها في فترة زمنية معينة حول موضع توازن معين. تهتم دراسة الاهتزاز بالحركة التذبذبية للأجسام والقوى المرتبطة بها. تتعرض معظم الآلات والهياكل الهندسية للاهتزاز إلى حد ما ويتطلب تصميمها عمومًا مراعاة سلوكها التذبذب.

1.1.2 Simple harmonic motion:

If the motion is repeated after equal intervals of time, it is called **periodic motion**. The simplest type of periodic motion is harmonic motion. Harmonic motion can be represented as shown in **Fig. 1.1**

إذا تكررت الحركة بعد فترات زمنية متساوية ، فإنها تسمى الحركة الدورية. أبسط أنواع الحركة الدورية هي الحركة التوافقية. يمكن تمثيل الحركة التوافقية كما هو موضح في الشكل ١,١

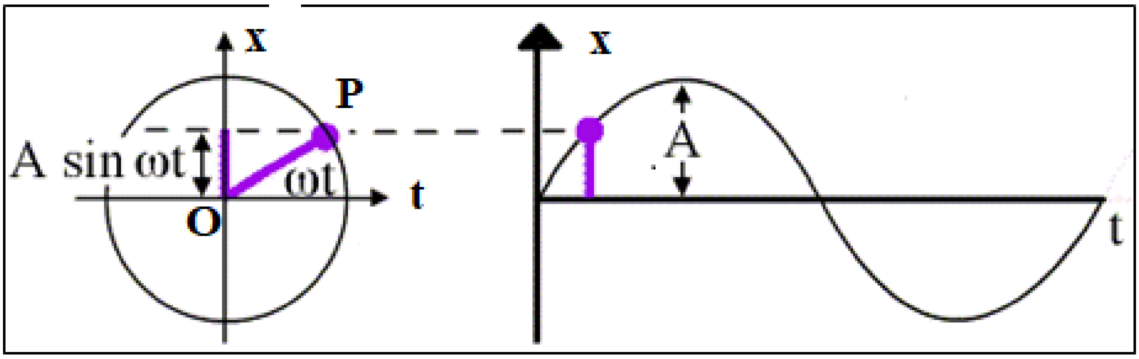

Fig. 1.1 Harmonic motion as the projection of the end of a rotating vector

by means of vector ***OP*** of magnitude ***A*** rotating at a constant angular velocity **ω**. The projection of the end of the vector ***OP*** is given by:

Where: -

X = displacement (m) المسافة

A = Amplitude السعة

ω = Angular velocity السرعة الزاوية

$$x(t) = A\sin(\omega t) \quad (1)$$

This is considered the harmonic displacement. Velocity and acceleration can be obtained by differentiating Eq. (1) with respect to time once and twice respectively as follows:

Where: -

X′ = Velocity (m/S) السرعة الخطية

X′′ = Acceleration (m/S²) التعجيل

$$\dot{x}(t) = A\omega\cos(\omega t) \quad (2)$$

$$\ddot{x}(t) = -A\omega^2\sin(\omega t) \quad (3)$$

ملاحظة : مشتقة المسافة بالنسبة للزمن X(t) هي السرعة X′ . اما مشتقة السرعة فهي التعجيل X″ او المشتقة الثانية للمسافة

Figure 1.2 shows the three measures of vibration; displacement, velocity and acceleration waveforms and vectors;

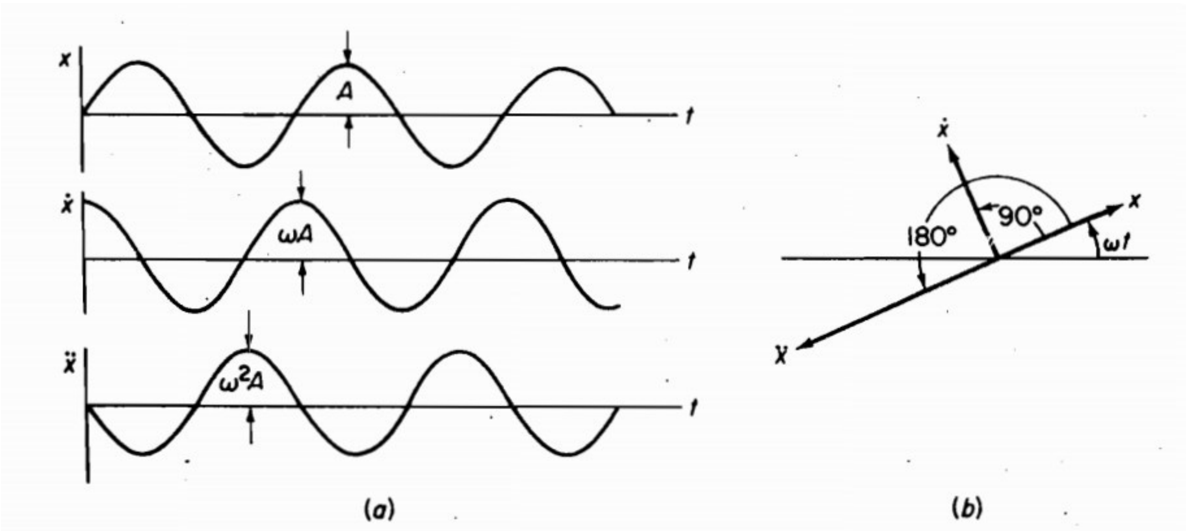

Fig. 1.2 Displacement, velocity and acceleration (a) waveforms, (b) vectors

1.2 Importance of the study of vibrations:

We experience many examples of vibration in our daily lives. A pendulum set in motion vibrates. A plucked guitar string vibrates. Vehicles driven on rough roads vibrate, and geological activity can cause massive vibrations in the form of earthquakes. All machines and structures are subject to vibrations which arise due to internal and external forces applied to the machine. There are good and bad effects of mechanical vibrations. The following are examples of the bad effects of vibration:

1- Increased wear of machine components (Ex. Bearings, couplings, etc.)

2- Looseness of fasteners (may cause catastrophic accidents in vehicles and aircrafts).

3- Failure of machine components due to fatigue.

4- Poor surface finish due to tool chatter in metal cutting processes.

5- Excessive noise.

6- Earthquakes.

7- Resonance.

8- Instability.

9- Discomfort in operating machines or vehicles.

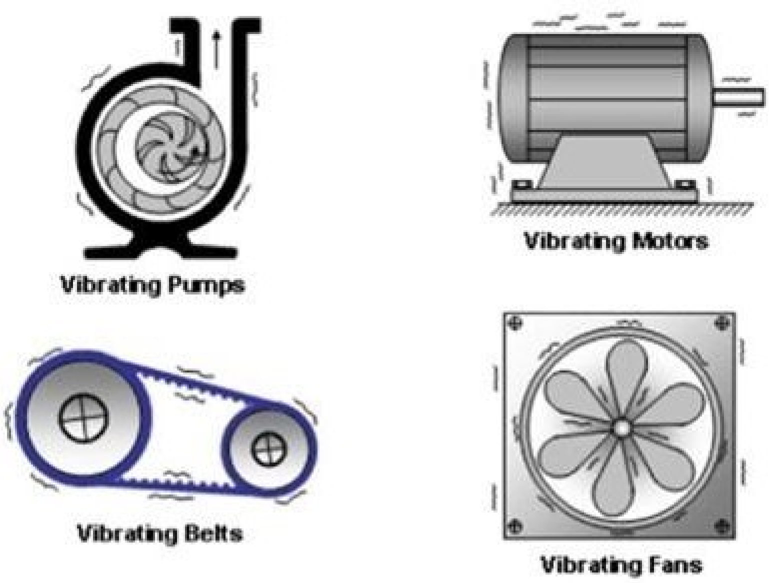

Fig. 1.3 Examples of bad machine vibrations

Figure 1.3 shows some examples of bad machine vibration. There are some applications where vibrations are intentionally generated to obtain useful functions

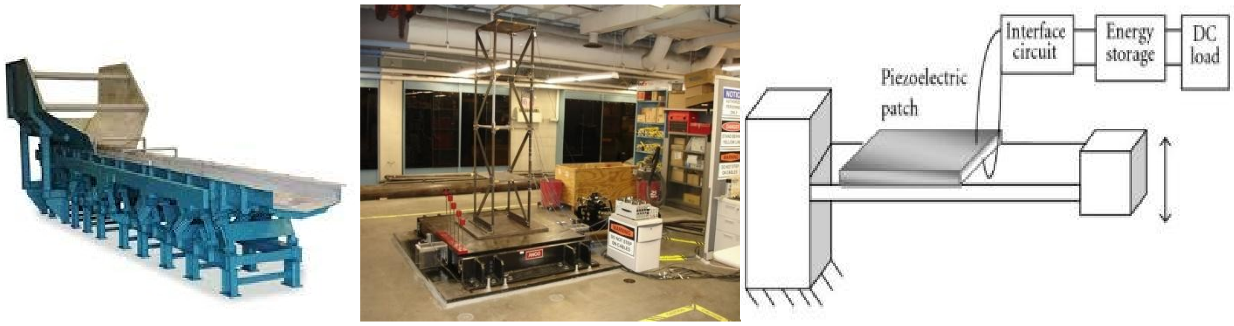

Fig. 1.4 Examples of good vibration applications, (a) Vibratory conveyor, (b) Shaking table, (c) Piezoelectric energy harvesting device

Some applications of good vibrations include (Fig. 1.4):

• Vibratory conveyors;

• Shaking table for earthquake testing of buildings;

• Vibratory sieves;

• Grinding machines;

• Energy harvesting using piezoelectric elements.

Therefore, it is important to study vibration in order to reduce its bad effects through proper design of machines and their mountings (vibration isolation and vibration absorption) and also to design new systems that use the vibration phenomenon to perform useful functions.

1.3 Characteristics of vibration:

1- Cycle of vibration:

This is the motion of a vibrating body as shown in Fig. 1.5: from its neutral position (Position O) to the maximum position in one direction (Position A), then passing through the neutral position to the maximum position in the other direction (Position B) and finally to the neutral position again.

دورة الاهتزاز: -هي حركة الجسم المهتز كما هو موضح في الشكل ١,٥: من موقعه الساكن (النقطة O) إلى أقصى موقع في (نقطة A) ، ثم المرور عبر الموضع السكون إلى أقصى موقع في الاتجاه الآخر (النقطة B) وأخيراً إلى موقع السكون مرة اخرى (النقطة O).

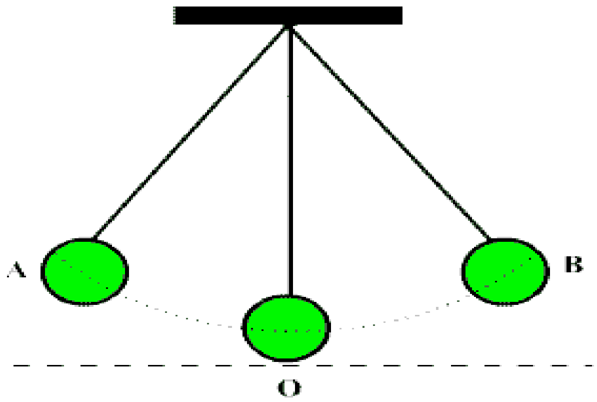

Fig. 1.5 One cycle of vibration

2- Period (*T*):

It is the time taken to complete one cycle of motion and is denoted by ***T*** and measured in seconds.

الفترة: - هو الوقت اللازم لإكمال دورة واحدة من الحركة، ويُرمَز بواسطة T ويقاس بالثواني.

3- Frequency (*f*):

It is the number of cycles per unit time and is denoted by ***f*** and measured in **Hz** or **cps**.

التردد: - هو عدد الدورات لكل وحدة زمنية ويُشار إليه بالرمز f ويقاس بالهرتز أو cps.

$$f = \frac{1}{T} \qquad (4)$$

4- Angular (Circular) frequency (ω):

This is the angular velocity of the cyclic motion measured in rad/s.

التردد الزاوي :- هي السرعة الزاوية للحركة الدورية المقاسة بوحدة rad/ s.

$$\omega = 2\pi f = \frac{2\pi}{T} \qquad (5)$$

5- Amplitude of vibration (A):-

It is the maximum displacement of a vibrating body from its equilibrium position.

سعة الاهتزاز:- إنها أقصى إزاحة لجسم مهتز من موضع توازنه وتقاس بوحدات الطول مثل المتر.

6- Phase angle (φ):

It is the angular difference between the occurrence of similar points of two harmonic motions. If we consider the two waves shown in Fig. 1.6 denoted by:

زاوية فرق الطور :- إنه الفرق الزاوي بين نقاط متشابهة لحركتين متناسقين.

$$x_1 = A_1 \sin \omega t \qquad (6)$$

And

$$x_2 = A_2 \sin(\omega t + \varphi) \qquad (7)$$

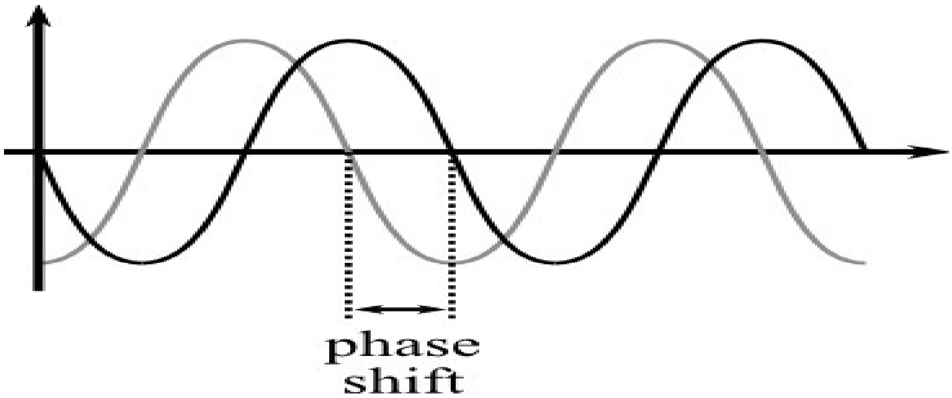

Fig. 1.6 Phase difference between two waves

It can be noticed that there is a lag (difference) between the peak of x_1 and the peak of x_2. The difference is called phase lag and is measured by the phase angle φ. The two waves are described to be out of phase by φ degrees.

Example 1: - A harmonic motion has a frequency of 15 cycle/second and its maximum velocity is 5 m/s. Determine its Amplitude, Period and maximum acceleration.

Solution: -

$\omega = 2\pi f$

$\omega = 94.248 \quad rad.sec$

$T = \frac{1}{f}$

$T = 0.0106 \quad sec$

At maximum velocity

$x^{*}_{max} = A\omega$

$5 = A\, 94.248$

$A = 0.05305\, m$

$x^{**}_{max} = A\omega^2$

$x^{**}_{max} = 471.24\, m/s^2$

Example 2: - A harmonic motion is given as:

$X(t) = (4\, m)\ \cos(\, 3\pi\, t + \pi)$, x in meter, t in second

Find:

a) Frequency and Period
b) Amplitude
c) Phase constant
d) The Position body at t= 0.25 sec.

Solution: -

Compaction between $X(t) = (4\,m)\ \cos(\,3\pi\,t + \,\pi)$

And equation $X(t) = A\ \cos(\,\omega\,t + \,\varphi)$

We get: -

a) $\omega = 2\pi f = 3\pi$ then $f = 1.5\ H_z$

And

$$T = \frac{1}{f} = 0.667\ second$$

b) $A = 4\ m$

c) $\phi = \pi$ rad or $\phi = 3.142$ rad

d) $x(0.25) = (4\,m)\ \cos(\,3\pi\,(0.25) + \,\pi) = 2.83m$

Example 3: - A simple harmonic motion takes 12 second to complete 5 vibration cycle. Find

a) Time Period.
b) Frequency in $H_{z.}$
c) Angular Frequency in rad/s

Solution: -

a) $T = \frac{12}{5} = 2.4\ s$

b) $f = \frac{1}{T} = \frac{1}{2.4} = 0.417\ H_z$

c) $\omega = 2\pi f = 2\pi\,(0.417) = 2.62\ \frac{rad}{s}$