AL-MAMON UNIVERSITY COLLAGE

DEPARTMENT OF ELECTRICAL POWER ENGINEERING TECHNIQUES

Lecture notes 2

Mechanical Vibrations

Part I

Prepared by

Eng. Ammar T. Ahmed

Eng. Hayder Baqer Mahdi

Prospective students: Second year

Semester: 2022/2023

Basic Concepts of Mechanical Vibrations

This lecture notes consist the following topics:

- Classification of vibration;
- Undamped and damped vibration;
- Free and forced vibration;
- Main elements of a vibrating system;
- Degrees of freedom;
- Undamped Free Vibration;
- Equation of motion;

2.1 Classification of vibration:

Vibration can be classified in several ways. The following are the most important classifications

2.1.1. <u>Undamped and damped vibration</u> (Fig. 2.1):

- Undamped vibration occurs when *no energy is lost or dissipated in friction or other resistance during oscillation* (No damping is present).

يحدث الاهتزاز غير المخمد عند عدم فقدان الطاقة أو تبديدها في الاحتكاك أو أي مقاومة أخرى أثناء التذبذب (لا يوجد تخميد).

- Damped vibration occurs when *some energy is lost due to damping*

يحدث الاهتزاز المخمد عند فقد بعض الطاقة بسبب التخميد

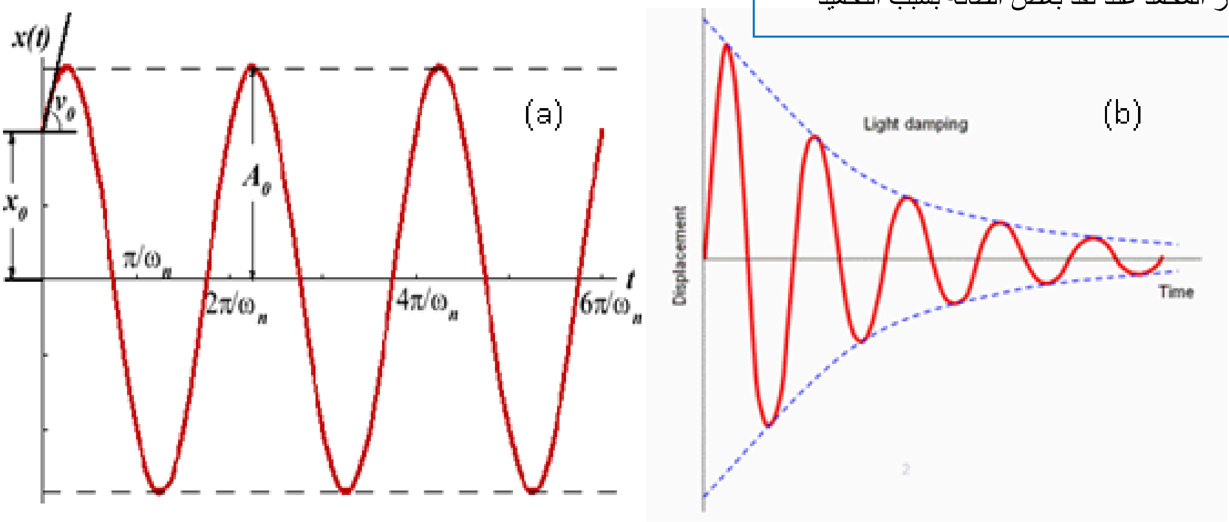

Fig. 2.1 Classification of vibration according to damping (a) Undamped, (b) Damped vibration

2.1.2 <u>Free and forced vibration</u>:

- Free vibration *is the condition when the system is given an initial disturbance (displacement, velocity or force) and then released to vibrate on its own.*

الاهتزاز الحر هو الحالة التي يتم فيها إعطاء النظام اضطرابًا أوليًا (إزاحة أو سرعة أو قوة) ثم إطلاقه ليهتز من تلقاء نفسه.

- Forced vibration *is the condition when the system is subjected to an external force.*

الاهتزاز القسري هو الحالة التي يتعرض فيها النظام لقوة خارجية و مستمره مع الزمن.

2.2 Main elements of a vibrating system:

Figure 2.2 shows the mass-spring-damper system (MSD) which demonstrates the main elements of a vibrating system. All machines and structures have these fundamental properties (mass, stiffness, damping) that combine to determine how the machine will react to the forces that cause vibrations just like the MSD system. Table 2.1 shows the function, forces and energies associated with each element. It may be noted that there are other forms of damping such as fluid resistance, friction, etc. however, in this course focus will be given to viscous damping shown in Fig. 2.2 which resembles shock absorbers used in vehicles.

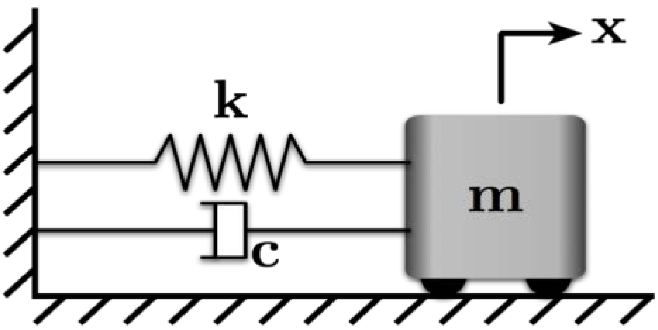

Fig. 2.2 Mass-spring-damper system

Table 2.1 Main elements of a vibrating system details

	Mass	Spring	Damper
Notation	m in Kg	***K*** (Spring constant or stiffness in N/m)	***C*** (Damping coefficient in N.s/m)
Function	Moving element	Restoring element	Energy dissipating element
Force	Inertia force $F = m\,a = m\ddot{x}$	Spring force $F = k\,x$	Damping force $F = C\,v = C\dot{x}$
Energy	Kinetic energy $K.E = \frac{1}{2} m\,V^2 = \frac{1}{2} m\dot{x}^2$	Potential energy $P.E = \frac{1}{2} k\,x^2$	Dissipated energy $D.E = \frac{1}{2} C\,\dot{x}^2$

2.3 Degrees of freedom:

Degrees of freedom can be defined as the minimum number of independent coordinates that describe the motion of a system completely. To specify the DOF for any system, you need to observe the number of independent displacements for each mass

يمكن تعريف درجات الحرية (DOF)على أنها الحد الأدنى لعدد الإحداثيات المستقلة التي تصف حركة النظام تمامًا. لتحديد DOF لأي نظام ، نحتاج إلى ملاحظة المسافات المستقلة المقطوعة لكل كتلة وتحديد عددها.

$$\text{DOF} = \sum (\text{Independent displacements of masses})$$

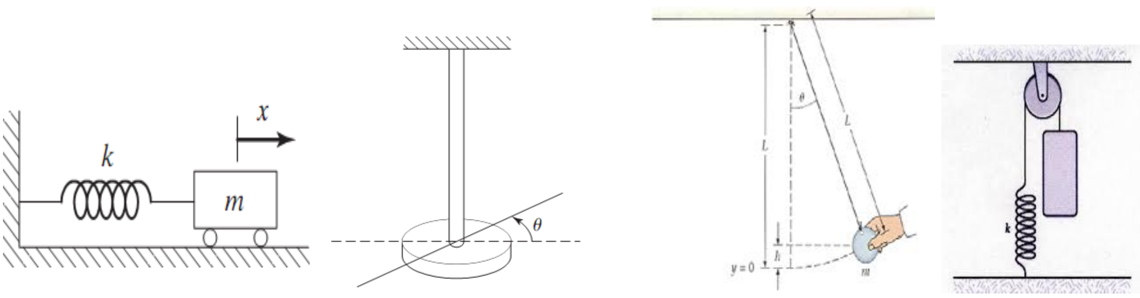

Fig. 2.3 Examples of 1-DOF systems

The systems shown in Fig. 2.3, represent single-degree-of-freedom systems. For example, the motion of the simple pendulum can be stated either in terms of the angle or in terms of the linear coordinates. In this example, we find that the choice of θ as the independent coordinate will be more convenient than the choice of x or y. For the torsional system (long bar with a heavy disk at the end) shown in Fig 2.3, the angular coordinate can be also used to describe the motion. The mass-pulley system motion can be represented by either the linear displacement of the mass or the angular displacement of the pulley because the two coordinates are dependent on each other through this equation ($x = r\ \theta$) assuming no slip condition where r is the radius of the pulley.

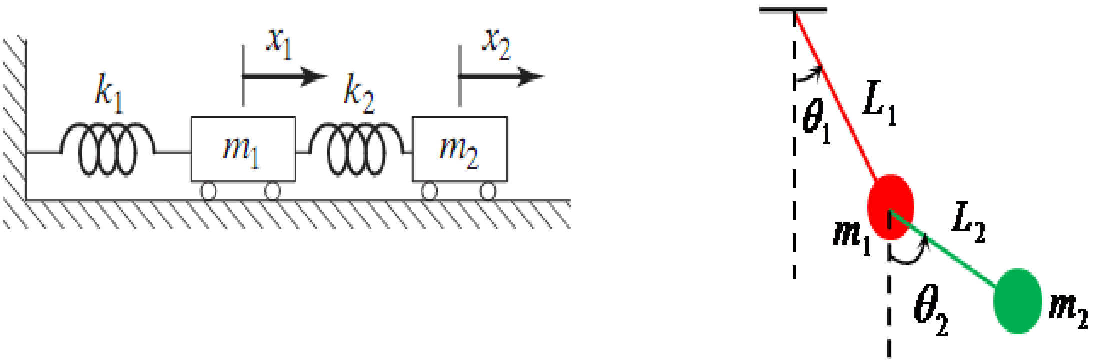

Fig. 2.4 Examples of 2-DOF systems

Fig. 2.4 and show examples of two-DOF, some systems especially those involving continuous elastic members, have an infinite number of degrees of freedom. As a simple example, consider the cantilever beam shown in Fig. 2.5. Since the beam has an infinite number of mass points, we need an infinite number of coordinates to specify its deflected configuration. Thus, the cantilever beam has an infinite number of degrees of freedom. Most structural and machine systems have deformable (elastic) members and therefore have an infinite number of degrees of freedom.

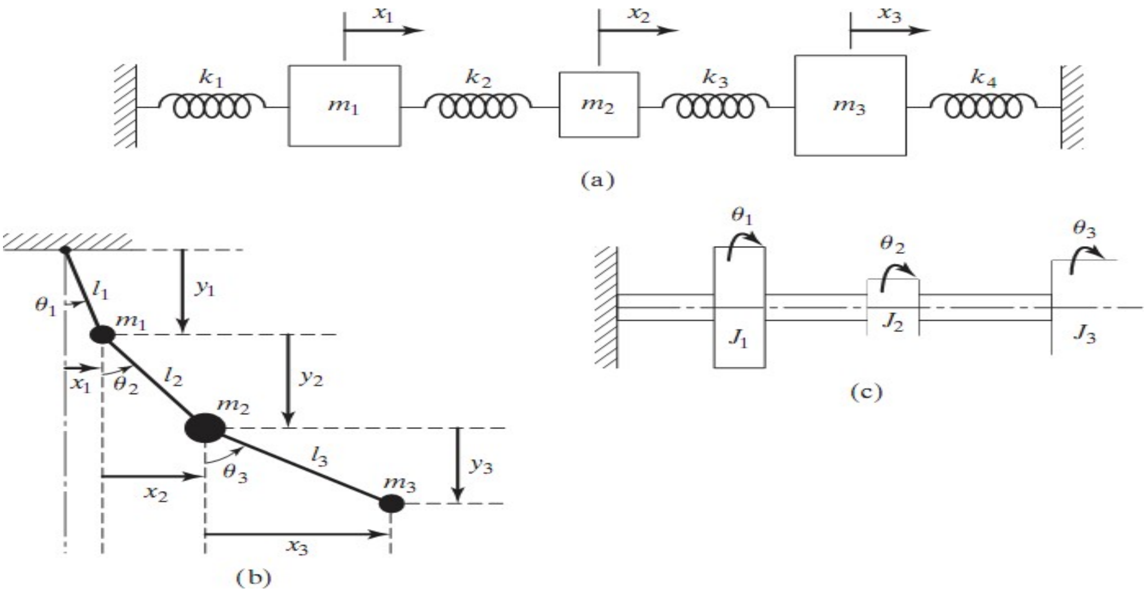

Fig. 2.5 Examples of 3-DOF systems

2.4 Undamped Free Vibration:

Figure 2.6 shows a spring-mass system that represents the simplest possible vibratory system. The motion is called free vibration because there is no external force applied to the mass. Motion is started by initial displacement and/or velocity at t = 0. The system is undamped because there is no element that causes dissipation of energy during the motion of the mass. Therefore, the amplitude of motion remains constant with time.

وضح الشكل ٢,١ نظام كتلة مع النابض (mass, spring) يمثل أبسط نظام اهتزازي ممكن. تسمى الحركة الاهتزاز الحر(Undamped Free Vibration) لأنه لا توجد قوة خارجية مسلطة على الكتلة. تبدأ الحركة بالإزاحة الأولية عند t = 0. النظام غير مخمد لأنه لا يوجد عنصر يسبب تبديد الطاقة أثناء حركة الكتلة لذلك ، فإن سعة الحركة تظل ثابتة مع مرور الوقت.

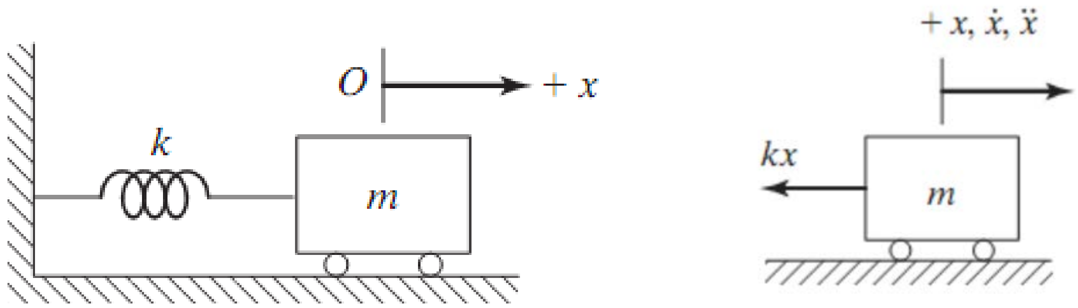

Fig. 2.6 Spring-mass system in horizontal direction

The governing equation of a vibrating system is called the equation of motion (EOM) and it is a second order ordinary differential equation whose solution gives the displacement of the system at any instant of time. There are different methods to determine the equation of motion of vibrating system. In this course, we will focus on two methods; Newton's law and energy method.

تسمى المعادلة الحاكمة لنظام الاهتزاز معادلة الحركة (EOM) وهي معادلة تفاضلية عادية من الدرجة الثانية يعطي حلها إزاحة النظام في أي لحظة من الزمن. توجد طرق مختلفة لتحديد معادلة حركة نظام الاهتزاز. في هذه المحاضرة ، سنركز على طريقتين: **(قانون نيوتن و طريقة الطاقة)**

2.4.1 <u>Equation of motion using Newton's second law of motion</u>:

Newton's law states that:

"The rate of change of momentum of a mass is equal to the resultant force acting on it."

معادلة الحركة يمكن صياغتها : ناتج القوى باتجاه الحركة يساوي حاصل ضرب الكتلة في التعجيل.

Applying Newton's law to the mass shown in Fig. 2.6, we get:

$$-kx = m\ddot{x}$$

$$\therefore m\ddot{x} + kx = 0 \qquad (8)$$

Eq. (8) represents the equation of motion of the system (EOM).

From comparing equation (3) and (8) we get the following relationship:

$$\ddot{x}(t) = -A\omega^2 \sin(\omega t) \qquad (3)$$

ملاحظة :اعلى ازاحة تكون معادلة التعجيل (3)

$$\ddot{x}(t) = -A\omega^2$$

And eq (8) $$m\ddot{x} + kx = 0$$

Or $$\ddot{x} + \frac{k}{m}x = 0 \qquad (8)$$

<u>We get</u>: $$\omega_n = \sqrt{\frac{k}{m}} \qquad (9)$$

Where ω_n is **Natural frequency** of a given system can be defined as *<u>the frequency at which a system vibrates naturally without external forces.</u>*

التردد الطبيعي لنظام معين هو التردد الذي يهتز فيه النظام بشكل طبيعي بدون قوى خارجية.

Example 4: The mass (0.25 kg) is supported by spring has constant (150 N/m). Compute the **Natural Angular Frequency** and the **Periodic**.

Solution: -

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{150}{0.25}} = 24.5\ rad/s$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{24.5} = 0.26\ \ s$$

Example 5: A (0.5 kg) mass connected to spring has a stiffens (8 N/m) move in simple harmonic motion with (10 cm) amplitude. Find:

a) **max Velocity** and **max Acceleration**.
b) The **Velocity** and **Acceleration** at distance (**6 cm**) from equilibrium position.
c) The **time** required to move the body from (**X= 6 cm**) to (**X= 8 cm**).

Solution: -

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{8}{0.5}} = 4\ rad/s$$

ملاحظة : عند اعلى سرعة يصل اليها الجسم **max Velocity** يكون قانون السرعة

$$\dot{x}_{max}(t) = A\omega$$

$$\dot{x}_{max}(t) = 40\ cm/s$$

ملاحظة : وعند اعلى تعجيل للجسم **max Acceleration** يكون قانون التعجيل

$$\ddot{x} = A\omega^2$$

$$\ddot{x}_{max} = 160\ cm/s^2$$

b) at A= 6 cm we need to find t.

$$x\ (t) = A\sin(\omega t)$$

$$t = \frac{1}{\omega}\sin^{-1}\left(\frac{x}{A}\right)$$

t= 0.161 s

$$x^{\cdot}(t) = A\omega\cos(\omega t)$$

$$x^{\cdot}(t) = 10 * 4\cos(4 * 0.161)$$

$$x^{\cdot}(t) = 40\ cm/s$$

$$x^{\cdot\cdot}(t) = -A\omega^2\sin(\omega t)$$

$$x^{\cdot\cdot}(t) = -10 * 4^2\sin(4 * 0.161)$$

$$x^{\cdot\cdot}(t) = -96\ cm/s^2$$

c) we need to find the time (t) at x= 6 cm and at x= 8 cm.

$$t = \frac{1}{\omega}\sin^{-1}\left(\frac{x}{A}\right)$$

$$t_1 = \frac{1}{4}\sin^{-1}\left(\frac{6}{10}\right) = 0.161\ s$$

$$t_2 = \frac{1}{\omega}\sin^{-1}\left(\frac{8}{10}\right) = 0.232\ s$$

$$\Delta t = t_2 - t_1$$

=0.232 -0.161 = 0.071 s

2.4.2 *Equation of motion using Energy method (principle of conservation of energy)*:

For a conservative system, the sum of the kinetic energy and potential energy is a constant:

$$\text{K.E.} + \text{P.E.} = \text{Constant} \qquad (10)$$

$$\frac{d}{dt}(K \cdot E \cdot + P.E.) = 0 \qquad (11)$$

$$\frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2\right) = 0$$

Applying the differentiation and rearranging we get:

$$\dot{x}(m\ddot{x} + kx) = 0$$

$$\because \dot{x} \neq 0$$

$$\therefore m\ddot{x} + kx = 0$$

which represents the same equation of motion got using Newton's law.

Example 6: A mass- spring system oscillate with amplitude of (3.5 cm). If spring stiffness is (250 N/m), mass is (0.5 kg), Find.

a) Mechanical Energy of the system.
b) Maximum Velocity
c) Maximum Acceleration.

Solution: -

$$Energy = \frac{1}{2} k x^2$$

$$Energy = \frac{1}{2} 250\ (3.5 * 10^{-2})^2 = 0.153\ \text{J}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{250}{0.5}} = 22.4\ rad/s$$

$$\dot{x}_{max}(t) = A\omega$$

$$\dot{x}_{max}(t) = 0.784\ m/s$$

$$\ddot{x}(t) = -A\omega^2$$

$$\ddot{x}_{max}(t) = 17.56\ m/s^2$$