## The AC equivalent circuit of the BJT amplifier

Is obtained according the following steps as shown in Figs. (10-5a, 10-5b, 10-5c):
a) Setting all dc sources to zero and replacing then by short cet. equivalent.
b) Replacing all capacitors by a short cct.
c) Remove all elements by pass by the short cct. equivalents introduced by steps $a$ and $b$.
d) Redrawing the circuit.


Fig.(10-5a) transistor circuit under examination.


Fig.(10-5b) the network of Fig.(10-5a) following the removal of the dc
supply and inserting short cct. for capacitor.


Fig.(10-5c) circuit of fig(10-1b) redrawn for small ac signal.

## NOTES

1- The hybrid parameter equivalent cet continuous to be very popular.
2- Manufacturers continue to specify the hybrid parameters for particular operating region on their specification sheets.
3- The parameter of the re-model can be derived directly from the hybrid parameters the hybrid equivalent cet suffer from being limited to a particular set of operating condition.
4- The hybrid equivalent cet suffer from being limited to a particular set of operating conditions.
5- The parameter of the other equivalent cct (re-model can be determined for any region of operation within the active region and are not limited by the single set of parameters provided by the specification sheet.

## Transistor hybrid equivalent cct (h-parameter)

- For the basic three terminal electronic device or system. It is obvious, from fig(10-6), that there are two ports(pair of terminal) interest . For our purpose the set at the left will represent the input terminal and the set at the right, the output terminal.
-For each set of terminals, there are two variables of interest.
-For the general hybrid two-port system of fig (10-6):-


Fig. (10-6).
$\mathbf{V}_{i}=h_{11} I_{i}+h_{12} \mathbf{V}_{0}$ (10-12a)
$\mathbf{I}_{\mathbf{o}}=\mathbf{h}_{21} \mathbf{I}_{\mathbf{i}}+\mathbf{h}_{22} \mathbf{V}_{\mathbf{o}}$
(10-12b)

The parameters relating the four variables are called $h$-parameters from the word hybrid. The term hybrid was chosen because the mixture of variables (v and i)
$\mathbf{h}_{11}=\left.\frac{V_{i}}{I_{i}}\right|_{\mathrm{Vo}=0}=\mathbf{h}_{\mathbf{i}}(\mathbf{\Omega})$, short cet input impedance parameter (10-13a)
$\mathbf{h}_{12}=\left.\frac{V_{i}}{V_{o}}\right|_{\mathrm{I}_{\mathrm{i}}=0}=\mathbf{h}_{\mathbf{r}}$ (unitless), open cet reverse transfer voltage ratio parameter
$\mathbf{h}_{21}=\left.\frac{I_{o}}{I_{i}}\right|_{\mathrm{v}_{0}=0}=\mathbf{h}_{\mathbf{f}}$ (unitless), short cct. forward transfer current ratio parameter ...(10-13c)
$\mathbf{h}_{22}=\left.\frac{I_{o}}{V_{o}}\right|_{\mathrm{I}_{\mathrm{i}}=0}=\mathbf{h}_{\mathbf{0}}$ (S), open cct. admittance parameter $\ldots(10-13 \mathrm{~d})$
-Since each term of eq(10-12a) has the units of volt, let us now apply Kirchhoff's voltage law in reverse to obtain the circuit of fig (10-7).
-Since each term of eq(10-12b) has the units of volt, let us now apply Kirchhoff's voltage law in reverse to obtain the circuit of fig (10-8).


Fig(10-7) hybrid input equivalent cet


Fig (10-8) hybrid output equivalent cet
-The complete ac equivalent cct for the basic three terminal linear device is shown in fig (10-9).


Fig(10-9) complete hybrid equivalent cct.
From the BJT hybrid equivalent cet of fig (10-9) Eqs.(10-12a) and (10-12b) becomes

$$
\begin{align*}
& \mathbf{V}_{i}=\mathbf{h}_{i} \mathbf{I}_{\mathbf{i}}+\mathbf{h}_{\mathbf{r}} \mathbf{V}_{o}  \tag{10-14a}\\
& \mathbf{I}_{\mathbf{o}}=\mathbf{h}_{\mathbf{f}} \mathbf{I}_{\mathbf{i}}+\mathbf{h}_{\mathbf{o}} \mathbf{V}_{o} \tag{10-14b}
\end{align*}
$$

- The circuit of fig (10-9) is applicable to any three- terminal electronic device or system with no internal independent sources.
- For the transistor, it has three basic configurations they are all three terminal configurations, so that the resulting equivalent cet will have the same format as shown in Fig. (10-9).
- In each case the bottom of the input and output section of the network of fig (10-9) can be connected as shown in Fig. (10-10), since the potential level is the same. The hybrid equivalent network for the C.E is shown in fig (1010).Note that:


Fig. (10-10) C.E configuration.
For C.B hybrid equivalent network is shown in $\mathbf{f i g}(\mathbf{1 0 - 1 1})$.Note that:
$\mathbf{I}_{\mathrm{i}}=\mathbf{I}_{\mathrm{e}} \quad, \quad \mathbf{I}_{0}=\mathbf{I}_{\mathrm{c}} \quad, \quad \mathbf{V}_{\mathrm{i}}=\mathbf{V}_{\mathrm{eb}}, \quad \mathbf{V}_{\mathbf{o}}=\mathbf{V}_{\mathrm{cb}}$


Fig. (10-11) C.B configuration.

## Types of hybrid parameters

Since there are three possible configurations for $\mathbf{B J} \mid \mathbf{T}$, there are three different sets of $\mathbf{h}$-parameters.
A second subscript has been added to the $h$-parameters
$h_{i e}, \quad h_{f e}, \quad h_{\text {re }}, \quad h_{\text {ie }}:$ C.E h-parameters
$h_{i c}, h_{f c}, h_{\text {rc }}, h_{\text {ic }}: C . C h$-parameters
$h_{i b}, h_{f b}, h_{r b}, h_{\text {ib }}: C . B h-p a r a m e t e r s$

- If all of $h$-parameter values in one configuration are known, then the values corresponding to any other configuration can be determined
- The C.E values are the ones most often given.
- Table below lists typical parameter values in each of the three-transistor configuration.

| h-parameter | C.E | C.B | C.C |
| :---: | :---: | :---: | :---: |
| $\mathbf{h}_{\mathbf{i}}$ | $1400 \Omega$ | $14 \Omega$ | $1400 \Omega$ |
| $\mathrm{h}_{\mathrm{r}}$ | $2 * 10^{-4}$ | 4*10-5 | 1 |
| $\mathrm{h}_{\mathrm{f}}$ | 100 | -0.99 | -101 |
| $\mathrm{h}_{0}$ | $2 * 10^{-5} \mathrm{~S}$ | $2 * 10{ }^{-7} \mathrm{~S}$ | $2 * 10{ }^{-5} \mathrm{~S}$ |

## Approximate C.E and C.B hybrid equivalent cct

Since $h_{\text {re }}, h_{\text {rb }}$ are normally a relatively small quantity ,their removal are approximate by $\mathbf{h}_{\mathrm{re}}=\mathbf{0}, \mathbf{h}_{\mathrm{rb}}=\mathbf{0}$ and $\mathbf{h}_{\mathrm{re}} \mathbf{V}_{\mathbf{0}}=\mathbf{0}, \mathbf{h}_{\mathrm{rb}} \mathbf{V}_{\mathbf{o}}=\mathbf{0}$ resulting a short cet for the feedback element as shown in fig(10-12a).The resistance determined by $\mathbf{1 /} h_{\text {oe }}$ and $1 / h_{o b}$ are large enough to be ignored ,in comparison to a parallel load ,which can be replaced by an open cct. as shown in fig(10-12b).


Fig.(10-12b)


Fig.(10-12a)

For the C.E configuration the approximate equivalent model will appear as shown in $\operatorname{fig}(10-13)$, it is obvious that


Fig. (10-13) approximate C.E hybrid equivalent model

$$
I_{i}=I_{b}
$$

$I_{o}=I_{c}=I_{f c} I_{b}$
$Z_{i}=h_{i e}$
$A_{i}=\frac{I_{c}}{I_{b}}=h_{f e}$
$A_{v}=\frac{V_{o}}{V_{i}}=\frac{V_{o}}{I_{b} h_{i e}}$

## The re model

The re model derived below will permit the determination of an equivalent cct. using the dc operating point condition of the network (but the hybrid parameters specified at a particular operating point).

## 1- C.B transistor configuration

The derivation of the alternate equivalent cct. Begin with approximation of the input and output of the C.B transistor configuration, as redrawn in Fig. (10-14).
Note that straight line segments are used to represent the collector characteristics and a single diode characteristic for the emitter cet. resulting in equivalent cct.



O/P characteristics
Fig. (10-14) approximate C.B characteristics


Fig. (10-15a) C.B configuration

$\operatorname{Fig}(10-15 b)$ approximate C.B equivalent cct as defined by Fig. (10-14)

For ac condition, the input at the emitter of the C.B transistor can be determined using the dynamic resistance of the diode, which can be obtained by using the following equation
$r e=\frac{26 m V}{I_{e}} \quad$ ohms $\qquad$
Substituting re will result in the re model of the C.B configuration as shown in Fig. (10-16).


Fig. (10-16)
Note the similarities of the re model of fig (10-16) with approximate hybrid equivalent model of fig (10-12) for C.B.
A comparison of the two models (h model and re model) shows that:

$$
\begin{gather*}
h_{i b}=r e \ldots \ldots \ldots \ldots \ldots . .(10-16 a) \\
h_{f b}=-1 \ldots \ldots \ldots \ldots . . .(10-16 b)
\end{gather*}
$$

For the C.B model of Fig. (10-16) the following parameters are defined:

$$
\mathbf{I}_{\mathrm{i}}=\mathbf{I}_{\mathrm{e}} \quad, \mathbf{Z}_{\mathrm{i}}=\operatorname{re} \boldsymbol{\Omega} \quad, \mathbf{Z}_{0}=\infty \Omega \quad, \mathbf{I}_{0}=\mathbf{I}_{\mathrm{c}}=\mathbf{I}_{\mathrm{e}}
$$

## 2- C.E transistor configuraion

For C.E configuration appearing in Fig. (10-17a) the input and output characteristics have been approximated by the set appearing in Fig. (10-17b) and Fig. (1017c).

[a]

$\begin{array}{lllll}0.2 & 0.4 & 0.6 & 0.8 & 1\end{array}$
[b]

[c]

Fig. (10-17): [a] C.E configuration [b] I/P characteristic [c] O/P characteristic
The base characteristics are approximated to be those of a diode and
$\mathbf{r}_{\mathrm{ac}}=\frac{26 \mathrm{mV}}{I B}$. (10-17)

But $\mathbf{I}_{\mathbf{E}}=\mathbf{I}_{\mathrm{C}}=\boldsymbol{\beta} \mathbf{I}_{\mathrm{B}} \quad$ and $\quad \mathbf{I}_{\mathrm{B}}=\frac{I E}{\beta}$
$\mathbf{r a c}_{\mathrm{ac}}=\frac{26 m V}{I B}=\frac{26 m V}{I E / \beta}=\beta \frac{26 m V}{I E}$

$$
\begin{equation*}
\mathbf{r}_{\mathrm{ac}}=\boldsymbol{\beta} \mathbf{r}_{\mathrm{e}} \tag{10-18}
\end{equation*}
$$

The common-emitter $r_{e}$ model is shown in fig(10-18)


Fig. (10-18) CE $\mathrm{r}_{\mathrm{e}}$ model
A comparison of the two models (hybrid and $r_{e}$ ) shows that:
$\beta=h_{f_{e}} \quad . . . . . . . . . . . . . . . .(10-19 a)$
$\beta r_{e}=h_{\text {ie }} . . . . . . . . . . . . . . .(10-19 b)$

## From fig(10-18)

$\mathbf{I}_{\mathbf{i}}=\mathbf{I}_{\mathrm{b}}$
$\mathbf{I}_{\mathbf{o}}=\mathbf{I}_{\mathbf{c}}=\boldsymbol{\beta} \mathbf{I}_{\mathrm{b}}$
$\mathbf{Z}_{i}=\boldsymbol{\beta} \mathbf{r}_{\mathrm{e}} \quad, \quad \mathbf{Z}_{\mathbf{o}}=\infty \Omega$

## BJT small signal analysis

## 1-Common-emitter fixed-bias confisuration

The CE fixed bias amplifier is shown in Fig. (11-8). The small signal analysis begins by removing the dc effects of $V_{c c}$ and replacing the dc blocking capacitors C 1 and C 2 by short cct. equivalent, resulting in the network of Fig. (11-9).

Note in Fig. (11-9) that the common ground of the dc supply and the emitter the relocation of $R_{B}$ and $R_{C}$ in parallel with the input and output section of the transistor.

Substituting the approximate hybrid small-signal equivalent cet. for the transistor of Fig. (11-9) will result in network of Fig. (11-10)


Fig. (11-8)
Fig. (11-9)


Fig. (11-10)
a- $Z_{i}$ : Fig. (11-10) clearly shows that
$\mathbf{Z}_{\mathbf{i}} \mathbf{R}_{\mathbf{B}} / / \mathbf{h}_{\mathrm{ie}} \quad \boldsymbol{\Omega}$

For situations where $\mathbf{R}_{\mathrm{B}}$ is greater than hie by more than a factor 10 :
$Z_{i=} h_{i e} \quad$ if $R_{B} \geq 10 h_{i e}$ $\qquad$
Using the $\mathbf{r}_{\mathbf{e}}$ model equivalence will result in the following equation for $\mathbf{Z}_{\mathbf{i}}$

$$
\mathbf{Z}_{i=\beta} \beta \mathbf{r}_{e} \quad \Omega \quad \ldots \ldots \ldots \ldots . .(11-5) \quad \text { Where } \mathbf{r}_{\mathrm{e}}=\frac{26 m V}{I_{E}}
$$

CE $\mathbf{r}_{\mathrm{e}}$ model

$b-Z_{0}$ : The output impedance of any network is defined when $\mathbf{V}_{i}=\mathbf{0}$.For Fig. (11-10) when $V_{i}=0, I_{i}$ and therefore $I_{b}=0$ and $h_{f} f_{b}=0$
$Z_{o}=R_{c} \Omega \ldots$...(11-6) for both the hybrid and $r_{e}$ model.
c- $\mathbf{A v}_{\mathbf{v}}$ Voltage gain $=\frac{V o}{V i}$
If $\left.R_{B}\right\rangle{ }^{\prime} h_{i e}$ than $I_{b} \approx I_{i}$
$\mathbf{V}_{\mathbf{o}}=-\mathbf{I}_{0} \mathbf{R}_{\mathrm{c}}=-\mathbf{h}_{\mathrm{fe}} \mathbf{I}_{\mathrm{b}} \mathbf{R}_{\mathrm{c}=}=-\mathbf{h}_{\mathrm{fe}} \mathbf{I}_{\mathbf{i}} \mathbf{R}_{\mathrm{c}}$
But $\mathbf{I}_{\mathbf{i}}=\frac{V i}{h i e}$ than $\mathbf{V}_{\mathbf{o}}=-\mathbf{h}_{\text {fe }}\left(\frac{V i}{h i e}\right) \mathbf{R}_{\mathbf{c}}$
$\mathbf{A}_{\mathbf{v}}=\frac{V o}{V i}=\frac{h f e R c}{h i e}$
The negative sign in the resulting equation explains that an $180^{\circ}$ phase shift occurs between the input and output signals. Substituting
$\mathbf{h}_{\mathrm{he}_{\mathrm{e}}=\boldsymbol{\beta}}$
$\mathbf{h}_{\mathbf{i e}=}=\boldsymbol{\beta} \mathbf{r}_{\mathbf{e}}$ for $\mathbf{r}_{\mathbf{e}}$ model
$\mathbf{A}_{\mathbf{v}}=-\frac{h f e R c}{h i e}=-\frac{\beta R c}{\beta r e}$
$\mathbf{A} \mathbf{v}=-\frac{R c}{r e} \quad$ where $\mathbf{r}_{\mathrm{e}}=\frac{26 \mathrm{mV}}{I_{E}}$
d- $\mathrm{A}_{\mathrm{i}}: \quad$ Current gain $=\frac{I o}{I i}$
$\mathbf{I}_{0}=\mathbf{h}_{\mathrm{fe}} \mathbf{I}_{\mathrm{l}} \approx \mathbf{h}_{\mathrm{fe}} \mathbf{I}_{\mathbf{i}}$
$\mathbf{A}_{\mathbf{i}}=\frac{I o}{I i} \approx \mathbf{h}_{\mathrm{fe}}$
For the $\mathrm{r}_{\mathrm{e}}$ model $\mathrm{A}_{\mathrm{i}} \approx \boldsymbol{\beta}$
Note the relative simplicity of moving from one model to the other, simply recall that:

$$
\begin{aligned}
& \mathbf{h}_{\mathbf{h e}_{\mathrm{e}}=\boldsymbol{\beta}} \\
& \mathbf{h}_{\mathbf{i e}}=\boldsymbol{\beta} \mathbf{r}_{\boldsymbol{e}}
\end{aligned}
$$

EX11-2: Determine $Z_{i}, Z_{o}, A v$ and $A_{i}$ for the network of Fig. (11-1) using $h$ and $r_{e}$ models. When $\mathbf{h}_{\mathrm{fe}}=100, \mathrm{~h}_{\mathrm{ie}}=1300 \boldsymbol{\Omega}$ at $\mathrm{I}_{\mathrm{C}}=\mathbf{2 m A}$.


## Solution:

$\mathrm{Z}_{\mathrm{i}}: \mathrm{R}_{\mathrm{B}}=560 \mathrm{k} \Omega \gg \mathrm{h}_{\mathrm{ie}}=1300 \Omega$
$\mathbf{Z}_{\mathrm{i}}\left(\mathbf{E}_{\mathrm{q}} 11.4\right) \mathbf{Z}_{\mathrm{i}}=\mathbf{h}_{\mathrm{ie}}=\mathbf{1 3 0 0 \Omega}$
$\mathbf{Z}_{0}(\mathbf{E} 11.6) \mathbf{Z}_{\mathrm{o}}=\mathbf{R}_{\mathrm{c}}=3 \mathrm{k} \Omega$
$\mathbf{A}_{\mathbf{v}}\left(\mathbf{E}_{\mathbf{q}} 11.7\right) \mathbf{A}_{\mathbf{v}}=\frac{-h_{f_{c}} R_{c}}{h_{i e}}=\frac{-(100)(3 k \Omega)}{1.3 k \Omega}=-230.77$
$\mathbf{A}_{\mathbf{i}}\left(\mathbf{E}_{\mathrm{q}} 11.9\right) \mathbf{A}_{\mathrm{i}} \approx \mathrm{h}_{\mathrm{fe}}=\mathbf{1 0 0}$

## re-model:

DC analysis $\mathbf{I}_{\mathbf{B}}=\frac{V_{C C}-V_{B E}}{R_{B}}=\frac{12-0.7}{560 k \Omega}=20 \mu \mathrm{~A}$
$\mathbf{I}_{\mathrm{C}}=\mathrm{BI}_{\mathrm{B}}=(\mathbf{1 0 0})(\mathbf{2 0 \mu} \boldsymbol{\mu})=\mathbf{2} \boldsymbol{\mu} \mathrm{A}=\mathrm{I}_{\mathrm{E}}$
$\mathbf{r}_{\mathbf{e}}=\frac{26 m v}{I_{E}}=\frac{26 m v}{2 m A}=13 \Omega$
$\mathrm{Z}_{\mathrm{i}}\left[\mathrm{E}_{\mathrm{q}} 11.5\right] \mathrm{Z}_{\mathrm{i}}=\mathrm{Br}_{\mathrm{e}}=(100)(13)=1300 \Omega$
$\mathrm{Z}_{\mathrm{o}}\left[\mathrm{E}_{\mathrm{q}} 11.6\right] \mathrm{Z}_{\mathrm{o}}=\mathrm{R}_{\mathrm{c}}=3 \mathrm{k} \Omega$
$\mathbf{A}_{\mathbf{v}}\left[\mathbf{E}_{\mathbf{q}} 11.8\right] \mathbf{A}_{\mathbf{v}}=-\frac{R_{c}}{r_{e}}=\frac{3 \mathrm{~km}}{13}=-230.77$
$\mathbf{A}_{\mathbf{i}}\left[\mathrm{E}_{\mathrm{q}} \mathbf{1 1 . 1 0 ]} \mathrm{A}_{\mathrm{i}} \approx \mathbf{B}=\mathbf{1 0 0}\right.$

## 2-Common-emitter voltage divider bias configuration

The voltage divider bias configuration is show in Fig. (11.14). Substituting the approximate hybrid equivalent cct will result in the network of Fig. (11.15). Note the absence of $\mathrm{R}_{\mathrm{E}}$ due to the low-impedance shorting effect of $\mathrm{C}_{\mathrm{E}}$. That is at the
frequency or (frequencies) of operation, the reactance of the capacitor is so small compared to $R_{E}$ and the network it is treated as a short circuit across $R_{E}$.


Fig. (11.14)


Fig. (11.15)
The parallel combination of $R_{B 1}, R_{B 2}$ is defined by $\mathbf{R}_{\mathrm{BB}}=\mathbf{R}_{\mathrm{B} 1} \mathbf{R}_{\mathrm{B} 2} / \mathbf{R}_{\mathrm{B} 1}+\mathbf{R}_{\mathrm{B} 2}$ (11.14)
a) $\mathrm{Z}_{\mathrm{i}}$ : from Fig. (11.15) $\mathbf{Z}_{\mathrm{i}}=\mathbf{R}_{\mathrm{BB}} / / \mathrm{h}_{\mathrm{ie}}$ (11.15)


## For re model

$$
\mathbf{Z}_{\mathrm{i}=} \mathbf{R}_{\mathrm{BB}} / / \mathbf{B r}_{\mathrm{e}} . . . . . . . . . . . .(\mathbf{1 1 . 1 6})
$$

b) $\mathrm{Z}_{0}: \mathrm{Z}_{0}=\mathbf{R}_{\mathrm{C}}$ for both model
c) $A_{v}: V_{o}=-I_{0} R_{C}=-h_{h_{f}} I_{b} R_{C}$
$\mathbf{V}_{\mathbf{o}_{-}}=-\mathbf{h}_{\mathrm{fe}}\left(\mathbf{V}_{\mathrm{i}} / \mathbf{h}_{\mathrm{ie}}\right) \mathbf{R}_{\mathbf{C}}$

$$
\begin{equation*}
\mathbf{A}_{\mathbf{v}}=\mathbf{V}_{\mathbf{o}} / \mathbf{V}_{\mathbf{i}}=-\left(\mathbf{h}_{\mathrm{f} e} / \mathbf{h}_{\mathbf{i e}}\right) \mathbf{R}_{\mathrm{C}} \tag{11.18}
\end{equation*}
$$

For re model
$\mathbf{V}_{\mathbf{o}}=-\mathbf{I}_{\mathbf{0}} \mathbf{R}_{\mathrm{C}}=-\boldsymbol{\beta} \mathbf{I}_{\mathrm{b}} \mathbf{R}_{\mathrm{C}}=-\left(\boldsymbol{\beta} \mathbf{V}_{\mathrm{i}} \mathbf{R}_{\mathrm{C}} / \boldsymbol{\beta r e}\right)=-\mathbf{V}_{\mathrm{i}} \mathbf{R}_{\mathrm{C}} / \mathbf{r e}$
$A_{v}=V_{o} / V_{i}=-R_{C} / r e$ (11.19)

$\mathbf{A}_{\mathrm{i}}$ :

$$
\begin{aligned}
& I_{b}=\frac{R_{B B} I_{i}}{R_{B B}+h i e} \\
& \therefore \frac{I_{b}}{I_{i}}=\frac{R_{B B}}{R_{B B}+h i e}
\end{aligned}
$$

But $I_{o}=h f e I_{b}$

$$
\therefore \frac{I_{o}}{I_{b}}=h f e
$$

The current gain
$A_{i}=\frac{I_{o}}{I_{i}}=\frac{I_{b}}{I_{i}} \times \frac{I_{o}}{I_{b}}$
$\therefore A_{i}=h f e \frac{R_{B B}}{R_{B B}+h i e}$
$\therefore A_{i}=\frac{h f e R_{B B}}{R_{B B}+\text { hie }} .$.

If $R_{B B} \gg h i e$
$\therefore A_{i} \cong h f e$
For re-model
$I_{b}=\frac{R_{B B} I_{i}}{R_{B B}+\beta r e}$
$I_{o}=\beta I_{b}=\frac{\beta R_{B B} I_{i}}{R_{B B}+\beta r e}$

$$
\begin{aligned}
& h f e=\beta \\
& h i e=\beta r e
\end{aligned}
$$

$\therefore A_{i}=\frac{I_{o}}{I_{i}}=\frac{\beta R_{B B}}{R_{B B}+\beta r e}$
EX 11.4: Determine $Z_{i}, \mathbf{Z}_{\mathbf{o}}, \mathbf{A v}$ and $\mathbf{A}_{\mathbf{i}}$ for the network of Fig. (11-16) using remodel and h-model. if $\beta=h f e=90$ and $h i e=2.835 \mathrm{~K} \Omega$.


## Solution

DC:
$V_{B B}=\frac{R_{B 2}}{R_{B 1}+R_{B 2}} V_{C C}=\frac{(5.6 K \Omega)(22)}{56 K \Omega+5.6 K \Omega}=2 \mathrm{~V}$
$R_{B B}=R_{B 1} / / R_{B 2}=\frac{(56 K \Omega)(5.6 K \Omega)}{56 K \Omega+5.6 K \Omega}=5.09 \mathrm{~K} \Omega$
$I_{B}=\frac{V_{B B}-V_{B e}}{R_{B B}+(1+\beta) R_{E}}=\frac{2-0.7}{5.09 \mathrm{k} \Omega+(1+90) \times 1.5 \mathrm{~K} \Omega}=9.18 \mu \mathrm{~A}$
$I_{C}=\beta I_{B}=90 \times(9.18 \mu \mathrm{~A})=0.836 \mathrm{~mA}$
$r e=\frac{26 m V}{I_{E}}=31.5 \Omega$
AC:
$\mathrm{Z}_{\mathrm{i}}$ : from eq (11.16)
$Z_{i}=R_{B B} / / \beta r e=\frac{5.09 \mathrm{~K} \Omega(90 \times 31.5)}{5.09 \mathrm{~K} \Omega+90 \times 31.5}=1.82 \mathrm{~K} \Omega$
$\mathbf{Z}_{\mathbf{0}}$ : from eq (11.17)
$Z_{o}=R_{C}=10 \mathrm{~K} \Omega$
$A_{v}$ : from eq (11.19)
$A_{V}=-\frac{R_{C}}{r e}=-\frac{10 K \Omega}{31.5}=-317.5$
$A_{i}$ : from eq (11.21)

$$
A_{i}=\frac{\beta R_{B B}}{R_{B B}+\beta r e}=\frac{5.09 K \Omega \times(90)}{5.09 K \Omega+(90) \times(31.5)}=57.8
$$

## 3-CE unbypassed emitter - bias configuration

The network shown in Fig. (11.18) includes an emitter resistor unbypassed in the AC domain. substituting the approximate hybrid equivalent model will result in the configuration of Fig. (11.19).


Fig. (11.18)



Fig. (11.19)

Applying Kirchoff's voltage law to the input side of fig (11.19) will result in $V_{i}=I_{b} h i e+R_{E}(1+h f e) I_{b}$
$Z_{b}=\frac{V_{i}}{I_{b}}=h i e+R_{E}(1+h f e)$
The result as displayed in Fig. (11.19) described that the input impedance to a transistor with an unbypassed resistor $\mathrm{R}_{\mathrm{E}}$ is determined by
$Z_{b}=h i e+R_{E}(1+h f e)$ $\qquad$ .(11.25)
Since hfe is normally much greater then, the equation reduced to $Z_{b}=h i e+R_{E} h f e$
But hfe $\mathrm{R}_{\mathrm{E}}$ is much greater then hie
$\therefore Z_{b}=h f e R_{E}$
For the re-model, the following equation is normally applied
$Z_{b}=\beta\left(r e+R_{E}\right)$ (11.28)

If $\beta \gg 1 \therefore Z_{b}=\beta R_{E} \ldots \ldots$ (11.29)


Fig. (11.20)
a) $Z_{i}$ : returning to Fig. (11.19)
$Z_{i}=R_{B} / / Z_{b}$
For both model
b) $Z_{0}$ : with $V_{i}$ set to zero,$I_{b}=0$ and $h f e I_{b}$ can be replaced by an open circuit equivalent
$\therefore Z_{o}=R_{C} \ldots .$. (11.31)
For both model
c) $\mathrm{Av}_{\mathrm{v}}$ :

$$
\begin{align*}
I_{b} & =\frac{V_{i}}{Z_{b}} \\
V_{o} & =-I_{o} R_{C}=-h f e I_{b} R_{C} \\
V_{o} & =-h f e \frac{V_{i}}{Z_{b}} R_{C} \\
\therefore A_{V} & =\frac{V_{o}}{V_{i}}=-\frac{h f e R_{C}}{Z_{b}} \ldots \ldots \tag{11.32}
\end{align*}
$$

For the approximation $Z_{b} \cong h f e R_{E}$
$\therefore A_{V}=-\frac{h f e R_{C}}{h f e R_{E}}=-\frac{R_{C}}{R_{E}}$
Which also applied to both models, due to the absence of any device parameters.
d) $\mathbf{A}_{\mathbf{i}}$ : returning to Fig. (11.19)

$$
\begin{aligned}
& I_{b}=\frac{R_{B} I_{i}}{R_{B}+Z_{b}} \\
& \therefore \frac{I_{b}}{I_{i}}=\frac{R_{B}}{R_{B}+Z_{b}}
\end{aligned}
$$

Out $I_{o}=$ hfe $_{b}$

$$
\therefore \frac{I_{o}}{I_{b}}=h f e
$$

$$
\therefore A_{i}=\frac{I_{o}}{I_{i}}=\frac{I_{b}}{I_{i}} \times \frac{I_{o}}{I_{b}}=h f e \frac{R_{B}}{R_{B}+Z_{b}}
$$

$$
\begin{equation*}
\therefore A_{i}=\frac{I_{o}}{I_{i}}=\frac{h f e R_{B}}{R_{B}+Z_{b}} . \tag{11.34}
\end{equation*}
$$

## For re-model

$$
\begin{aligned}
& I_{b}=\frac{R_{B} I_{i}}{R_{B}+Z_{b}} \\
& \therefore \frac{I_{b}}{I_{i}}=\frac{R_{B}}{R_{B}+Z_{b}} \\
& I_{o}=\beta I_{b} \quad \therefore \beta=\frac{I_{o}}{I_{b}}
\end{aligned}
$$

$\therefore A_{i}=\frac{I_{b}}{I_{i}} \times \frac{I_{o}}{I_{b}}=\frac{\beta R_{B}}{R_{B}+Z_{b}} \ldots \ldots$ (11.35)


EX 11.5 Determine $Z_{i}, Z_{i}, A_{v}$ and $A_{i}$ for the network of Fig. (11.21). When $h f e=120, h_{i e}=560 \Omega$.


Solution
$\mathbf{Z}_{\mathrm{b}}$ from Eq. (11.25)
$Z_{b}=h i e+R_{E}(1+h f e)=0.56 K \Omega+1.2 K \Omega(1+120)=145.76 K \Omega$
$\mathbf{Z}_{\mathbf{i}}$ from Eq. (11.30)
$Z_{i}=Z_{b} / / R_{B}=145.76 \mathrm{~K} \Omega / / 270 \mathrm{~K} \Omega=94.66 \mathrm{~K} \Omega$
$Z_{\mathbf{0}}$ from Eq. (11.31)
$Z_{o}=R_{C}=5.6 \mathrm{~K} \Omega$
Av from Eq. (11.32)
$A_{V}=-\frac{h f e R_{C}}{Z_{b}}=-\frac{(120)(5.6 \mathrm{~K} \Omega)}{145.76 \mathrm{~K} \Omega}=-4.61$
$\mathrm{A}_{\mathbf{i}}$ from Eq. (11.34)
$A_{i}=\frac{R_{B} h f e}{R_{B}+Z_{b}}=\frac{(270 K \Omega)(120)}{(270 K \Omega)+(145.76 K \Omega)}=77.93$
NOTE
Applied the approximate equation for $\mathbf{Z}_{\mathbf{b}}$, we find
$Z_{b} \cong h f e R_{E}=(120)(1.2 K \Omega)=144 K \Omega$
$Z_{i}=R_{B} / / Z_{b}=270 \mathrm{~K} \Omega / / 144 \mathrm{~K} \Omega=93.91 \mathrm{~K} \Omega$
$A_{V} \cong-\frac{R_{C}}{R_{E}}=-\frac{5.6 K \Omega}{1.2 K \Omega}-4.67$
Which are very close to result $(94.66 \mathrm{~K} \Omega,-4.61)$ obtain above

## 4-EMITTER - FOLLOWER CONFIGURATION

When the output is taken from the emitter terminal of the transistor as shown in Fig. (11-22), the network is referred to as an emitter follower. The emitter follower output voltage is in phase with input voltage. The fact that $V_{o}$ follows the magnitude of $V_{i}$ with an in-phase relationship. The voltage gains for emitter follower $A_{V} \cong 1$ the most common emitter follower configuration as shown in Fig. (11.22), because the collector is grounded for ac analysis, it is actually a common collector configuration. The emitter follower configuration is used for impedance at the input and a low impedance at the output, which is direct opposite of the standard fixed bias configuration. The resulting effect is much the same as that obtained with a transformer, where a load is matched to the source impedance for maximum power transfer through the system.
Substituting the approximate equivalent circuit to the network of Fig. (11.22) will result in the network of Fig. (11.23).


Fig. (11.22)


Fig. (11.23)
a) $\mathbf{Z}_{\mathrm{i}}$ :
$Z_{i}=R_{B} / / Z_{b}$

With $\mathbf{Z}_{\mathbf{b}}$ defined by Eqs (11.25) through (11.29)
$Z_{b}=h i e+(1+h f e) R_{E}$
..(11.25)
If hfe $\gg 1$
$\therefore Z_{b}=h i e+h f e R_{E}$
If $\operatorname{hfeR}_{\mathrm{E}} \gg$ hie

$$
\begin{equation*}
\therefore Z_{b}=h f e R_{E} \tag{11.27}
\end{equation*}
$$

For the re-model
$Z_{b}=\beta\left(r e+R_{E}\right)$
If $\beta \gg 1$
$\therefore Z_{b}=\beta R_{E}$
b) $\mathbf{Z}_{\mathbf{0}}: \quad I_{b}=\frac{V_{i}}{Z_{b}}$

$$
\begin{aligned}
I_{e} & =I_{b}+h f e I_{b}=I_{b}(1+h f e) \\
I_{e} & =(1+h f e) \frac{V_{i}}{Z_{b}}
\end{aligned}
$$

Substituting for $\mathbf{Z}_{\mathbf{b}}$

$$
\begin{aligned}
& I_{e}=\frac{(1+h f e) V i}{h i e+(1+h f e) R_{E}} \\
& I_{e}=\frac{V i}{\frac{h i e}{1+h f e}+R_{E}}
\end{aligned}
$$

If now construct the network defined by Eq. (11.37) the configuration of Fig. (11.24) will result: -

To determine $\mathbf{Z}_{\mathbf{0}}, \mathbf{V}_{\mathbf{i}}$ is set to zero
$Z_{o}=R_{E} / / \frac{h i e}{1+h f e}$


Fig. (11.24)
The network seen by the emitter branch is the input voltage $\mathbf{V}_{\mathbf{i}}$ in series with a resistance determined by the hybrid parameter hie and hfe. The resistance ( $\frac{\mathrm{hie}}{1+h f e}$ ) is usually quite small, dropping $Z_{o}$ well below the $R_{E}$ level.
c) $A_{v}$ : Fig. (11.24) can be used to determine $A_{v}$

$$
V_{o}=I_{e} R_{E}=\frac{R_{E} V_{i}}{R_{E}+\left(\frac{h i e}{1+h f e}\right)}
$$

$\therefore A_{V}=\frac{V_{o}}{V_{i}}=\frac{R_{E}}{R_{E}+\left(\frac{h i e}{1+h f e}\right)}$
Note the absence of a negative sign to indicate that $V_{o}$ and $V_{i}$ are in phase and recognize that the factor $\left(\frac{h i e}{1+h f e}\right)$ is the only reason $V_{o}$ dose not equal $V_{i}$.
d) $\mathbf{A}_{\mathbf{i}}$ : from Fig. (11.23)

$$
\begin{align*}
& I_{b}=\frac{R_{B} I_{i}}{R_{B}+Z_{b}} \\
& \therefore \frac{I_{b}}{I_{i}}=\frac{R_{B}}{R_{B}+Z_{b}} \\
& I_{o}=I_{e}=(1+h f e) I_{b} \\
& \therefore \frac{I_{o}}{I_{b}}=1+h f e \\
& \therefore A_{i}=\frac{I_{o}}{I_{i}}=\frac{I_{b}}{I_{i}} \times \frac{I_{o}}{I_{b}}=(1+h f e) \frac{R_{B}}{R_{B}+Z_{b}} \\
& \therefore A_{i}=\frac{(1+h f e) R_{B}}{R_{B}+Z_{b}} \ldots \ldots . .(\mathbf{1 1 . 4 0 )} \tag{11.40}
\end{align*}
$$

## re-model

The equation for the re-model can be determine directly from the above simply by substituting $h i e=\beta r e$ and $h f e=\beta$

a) $\mathrm{Z}_{\mathrm{i}}$

$$
\begin{align*}
& Z_{i}=R_{B} / Z_{b} \\
& Z_{b}=\beta r e+\beta R_{E} \\
& Z_{b}=\beta\left(r e+R_{E}\right) \text { if } \beta \gg 1 \tag{11.41}
\end{align*}
$$

b) $\mathbf{Z}_{\mathbf{0}}$

$$
\begin{aligned}
& I_{b}=\frac{V_{i}}{Z_{b}} \\
& I_{e}=I_{b}+\beta I_{b}=I_{b}(1+\beta) \\
& I_{e}=(1+\beta) \frac{V_{i}}{Z_{b}}=\frac{V_{i}(1+\beta)}{\beta\left(r e+R_{E}\right)}
\end{aligned}
$$

$$
\begin{equation*}
I_{e} \frac{V_{i}}{\frac{\beta\left(r e+R_{E}\right)}{1+\beta}}=\frac{V_{i}}{\beta\left(r e+R_{E}\right)} \ldots \ldots \ldots . \tag{11.42}
\end{equation*}
$$

If now construct the network defined by Eq. (11.42) will result: -


To determine $\mathbf{Z}_{\mathbf{0}}, \mathbf{V}_{\mathbf{i}}$ set to zero
$\therefore Z_{o}=R_{E} / / r e$ $\qquad$ (11.43)
c) $\mathbf{A v}_{\mathrm{v}}$ :

$$
\begin{align*}
& V_{o}=I_{o} R_{E}=\frac{R_{E} V_{i}}{r e+R_{E}} \\
& \therefore A_{V}=\frac{V_{o}}{V_{i}}=\frac{R_{E}}{r e+R_{E}} . \tag{11.44}
\end{align*}
$$

d) $\mathbf{A}_{\mathbf{i}}$ :

$$
\begin{align*}
& I_{b}=\frac{R_{B} I_{i}}{R_{B}+Z_{b}} \\
& \therefore \frac{I_{b}}{I_{i}}=\frac{R_{B}}{R_{B}+Z_{b}} \\
& I_{o}=I_{e}=(1+\beta) I_{b} \\
& \therefore \frac{I_{o}}{I_{b}}=1+\beta \\
& \quad \therefore A_{i}=\frac{I_{b}}{I_{i}} \times \frac{I_{o}}{I_{b}}=\frac{R_{B}(1+\beta)}{R_{B}+Z_{b}} \\
& \therefore A_{i}=\frac{\beta R_{B}}{R_{B}+Z_{b}} \ldots \ldots \ldots(11.45) \tag{11.45}
\end{align*}
$$

EX 11.6: For the emitter follower network. When $h f e=98, h_{i e}=1275 \Omega$.
a) Find expressions for $Z_{i}, Z_{\mathbf{o}}, A_{v}$ and $A_{i} u s i n g h$ parameter.
b) Determine $\mathbf{Z}_{i}, \mathbf{Z}_{0}, A_{v}$ and $\mathbf{A}_{\mathbf{i}}$.

(11.25)

## Solution

$Z_{b}=h i e+(1+h f e) R_{E}=1.275 K \Omega+(1+98) \times 3.3 \mathrm{~K} \Omega=327.98 \mathrm{~K} \Omega$
$\mathrm{Z}_{\mathrm{i}}$ :
$Z_{i}=R_{B} / / Z_{b}=220 \mathrm{~K} \Omega / / 327.98 \mathrm{~K} \Omega=131.68 \mathrm{~K} \Omega$
$\mathrm{Z}_{\mathrm{o}}$ :
$Z_{o}=R_{E} / / \frac{\text { hie }}{1+\text { hfe }}=3.3 K \Omega / / \frac{1275 \Omega}{1+98}=12.9 \Omega$
Av:
$\therefore A_{V}=\frac{R_{E}}{R_{E}+\left(\frac{h i e}{1+h f e}\right)}=\frac{3300}{3300+12.9}=0.996 \cong 1$
$\mathbf{A}_{\mathbf{i}}$ :
$\therefore A_{i}=\frac{(1+h f e) R_{B}}{R_{B}+Z_{b}}=\frac{(1+98) \times 220 K \Omega}{220 K \Omega+327.98 K \Omega}=39.75$

