KIRCHHOFF’S VOLTAGE LAW
KVL

Introduction

OPPJECTIVES

After completing this lesson, the student should be able to :

- Explain the characteristics of a series circuit.
- Illustrate the concept of Kirchhoff's voltage law and how important it is to the analysis of electric circuits.
- Illustrate how applied voltage will divide among series components and how to properly apply the voltage divider rule.
- Learn how to use a voltmeter, ammeter, and ohmmeter to measure the important quantities of a network.
- Solve the problem that related to Kirchhoff's voltage law .

INTRODUCTION

The law to be described in this section is one of the most important in this field. It has application not only to dc circuits but also to any type of signal whether it be ac, digital, and so on. This law is far-reaching and can be very helpful in working out solutions to networks that sometimes leave us lost for a direction of investigation.

The law, called Kirchhoff's voltage law (KVL),was developed by Gustav Kirchhoff in the mid-1800s. It is a cornerstone of the entire field and, in fact, will never be outdated or replaced.

Kirchhoff's Voltage Law

KIRCHHOFF'S VOLTAGE LAW

The law, called **Kirchhoff's voltage law (KVL),** was developed by Gustav Kirchhoff in the mid-1800s.

The law specifies that ***the algebraic sum of the potential rises and drops around a closed path (or closed loop) is zero.***

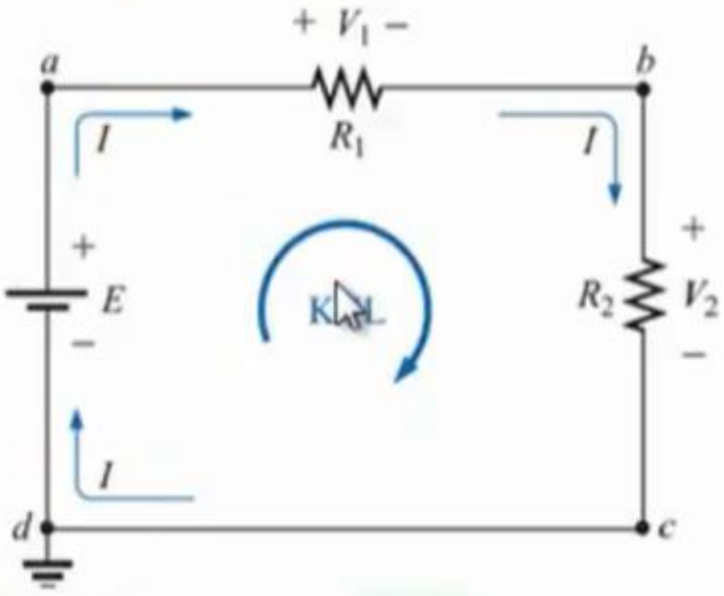


FIG. 6.1 *Applying Kirchhoff 's voltage law to a series dc circuit.*

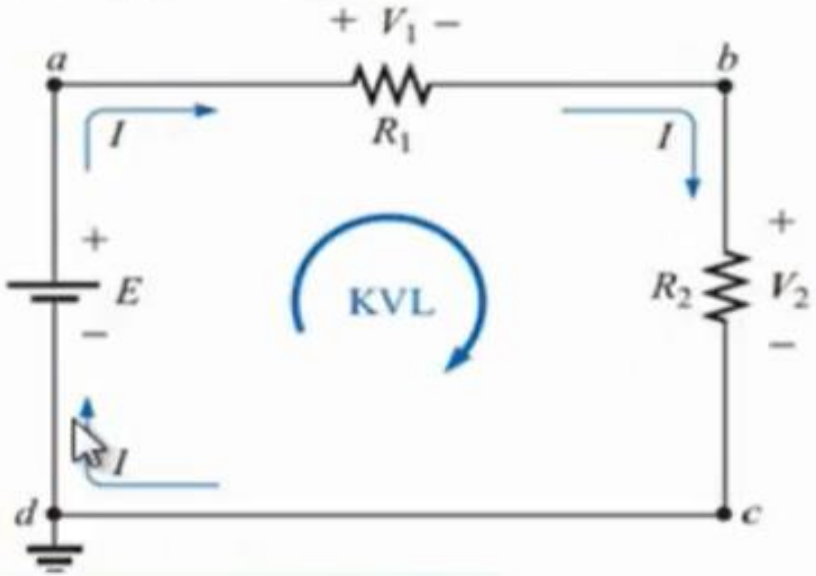
$$\Sigma_{\circlearrowright} V = 0$$

(Kirchhoff's voltage law in symbolic form) continue

KIRCHHOFF'S VOLTAGE LAW

$$+E - V_1 - V_2 = 0$$

Or $E = V_1 + V_2$



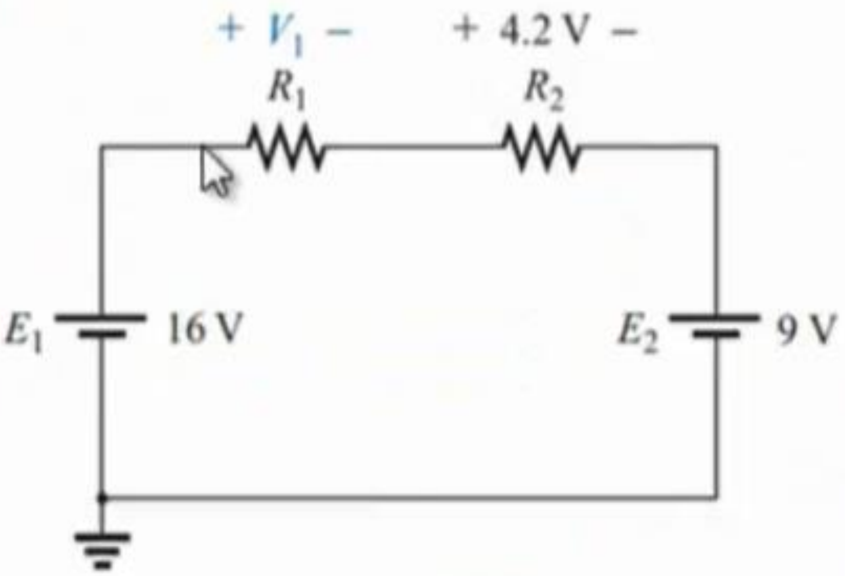
Revealing that
the applied voltage of a series circuit equals the sum of the voltage drops across the series elements.

Kirchhoff's voltage law can also be stated in the following form:

$$\Sigma_{\circlearrowright} V_{rises} = \Sigma_{\circlearrowright} V_{drops}$$

EXAMPLES

EXAMPLE 5.4 Determine the unknown voltages for the networks of Fig. 5.14.



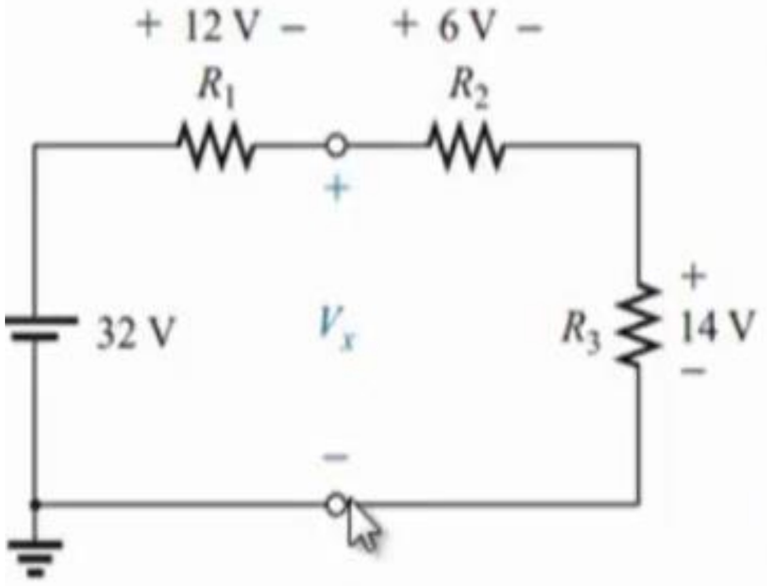
Solution

$$+E_1 - V_1 - V_2 - E_2 = 0$$

$$V_1 = E_1 - V_2 - E_2 = 16\text{ V} - 4.2\text{ V} - 9\text{ V}$$
$$= \mathbf{2.8\text{ V}}$$

EXAMPLES

EXAMPLE 5.4 Determine the unknown voltages for the networks of Fig. 5.14.



(b)

Using the clockwise direction for the other loop involving R_2 and R_3 will result in

Solution

$$+E - V_1 - V_x = 0$$

$$V_x = E - V_1 = 32\text{ V} - 12\text{ V} = \mathbf{20\text{ V}}$$

$$+V_x - V_2 - V_3 = 0$$

$$V_x = V_2 + V_3 = 6\text{ V} + 14\text{ V} = \mathbf{20\text{ V}}$$

Examples

EXAMPLE 11 Using Kirchhoff's voltage law, determine the unknown voltage for the circuit in Fig. 30.

Solution: Note that in this circuit, there are various polarities across the unknown elements since they can contain any mixture of components. Applying Kirchhoff's voltage law in the clockwise direction results in

$$+60\text{ V} - 40\text{ V} - V_x + 30\text{ V} = 0$$

and $$V_x = 60\text{ V} + 30\text{ V} - 40\text{ V} = 90\text{ V} - 40\text{ V}$$

with $$V_x = \mathbf{50\text{ V}}$$

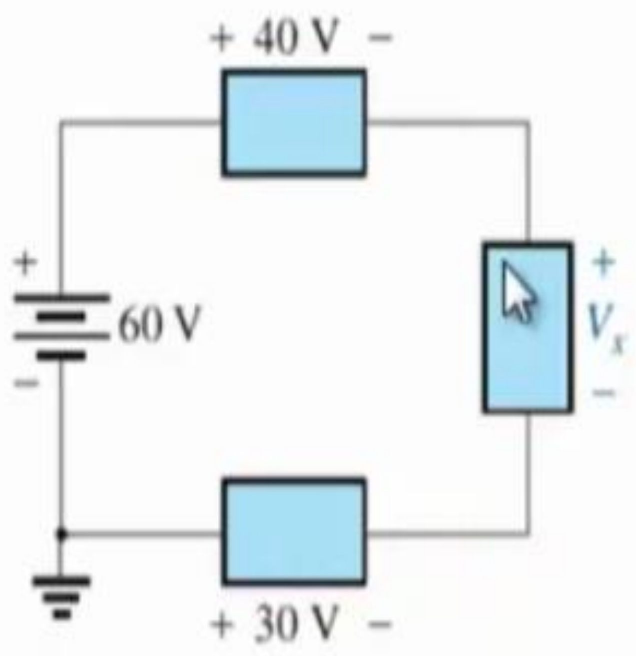


FIG. 30

Examples

EXAMPLE 13 For the series circuit in Fig. 32.

a. Determine V_2 using Kirchhoff's voltage law.
b. Determine current I_2.
c. Find R_1 and R_3.

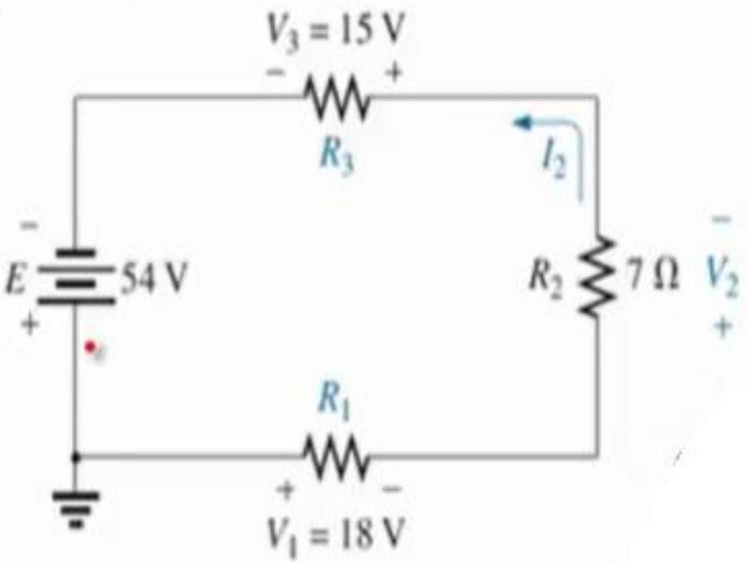


FIG. 32

Solutions:

a. Applying Kirchhoff's voltage law in the clockwise direction starting at the negative terminal of the supply results in

$$-E + V_3 + V_2 + V_1 = 0$$

and $$E = V_1 + V_2 + V_3 \text{ (as expected)}$$

so that $$V_2 = E - V_1 - V_3 = 54\text{ V} - 18\text{ V} - 15\text{ V}$$

and $$V_2 = \mathbf{21\text{ V}}$$

Voltage Division In a Series Circuit

VOLTAGE DIVISION IN A SERIES CIRCUIT

The previous section demonstrated that the sum of the voltages across the resistors of a series circuit will always equal the applied voltage.

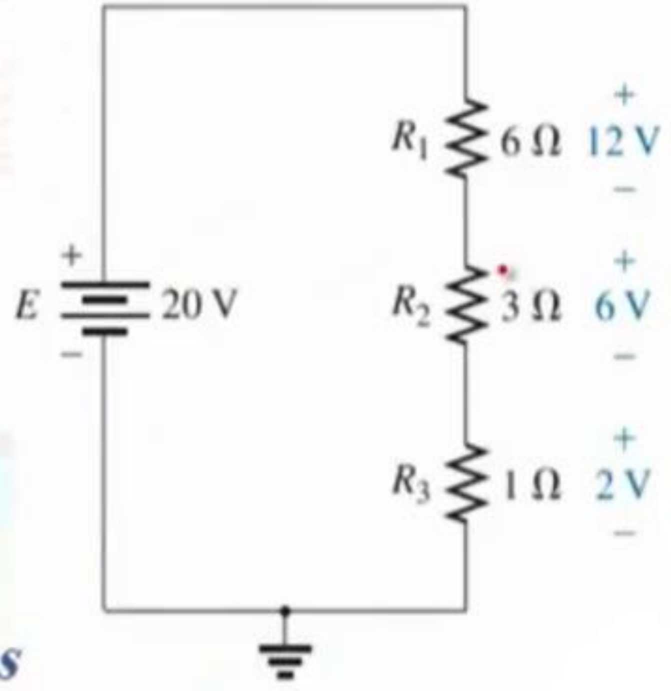


FIG. 33

It cannot be more or less than that value.

The next question is, how will a resistor's value affect the voltage across the resistor?

It turns out that ***the voltage across series resistive elements will divide as the magnitude of the resistance levels.***

KIRCHHOFF'S VOLTAGE LAW

In other words, ***in a series resistive circuit, the larger the resistance, the more of the applied voltage it will capture.***

In addition, ***the ratio of the voltages across series resistors will be the same as the ratio of their resistance levels.***

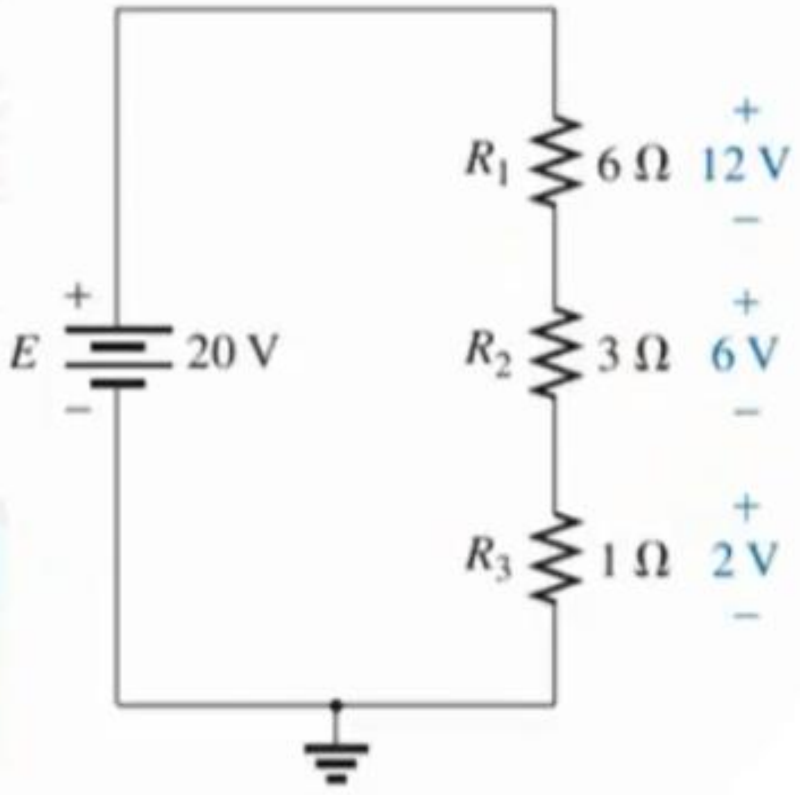
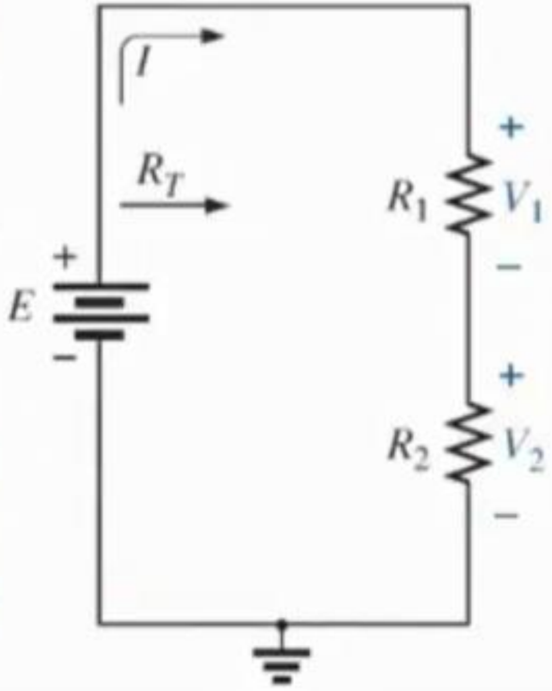


FIG. 33

Voltage Divider Rule

Voltage Divider Rule (VDR)

The **voltage divider rule (VDR)** permits the determination of the voltage across a series resistor without first having to determine the current of the circuit.



The voltage divider rule states that :

the voltage across a resistor in a series circuit is equal to the value of that resistor times the total applied voltage divided by the total resistance of the series configuration.

continue

Voltage Divider Rule (VDR)

The mathematic formula of VDR is :

$$V_x = R_x \frac{E}{R_T}$$

where V_x is the voltage across the resistor R_x , E is the impressed voltage across the series elements, and R_T is the total resistance of the series circuit.

Examples

Electric Circuits Analysis

Voltage Divider Rule (VDR)

EXAMPLE 15 For the series circuit in Fig. 37.

a. Without making any calculations, how much larger would you expect the voltage across R_2 to be compared to that across R_1?
b. Find the voltage V_1 using only the voltage divider rule.
c. Using the conclusion of part (a), determine the voltage across R_2.
d. Use the voltage divider rule to determine the voltage across R_2, and compare your answer to your conclusion in part (c).
e. How does the sum of V_1 and V_2 compare to the applied voltage?

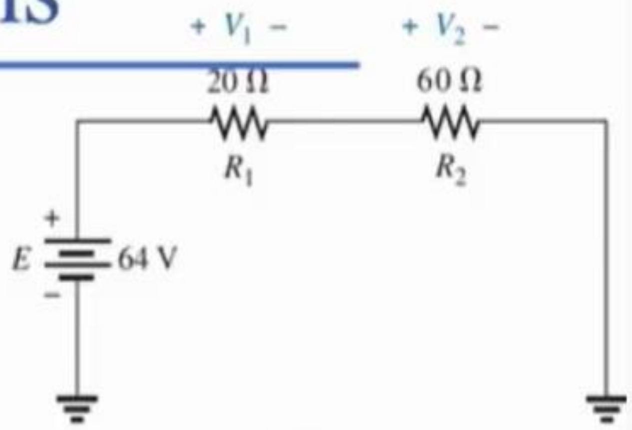


FIG. 37

Examples

Electric Circuits Analysis

Voltage Divider Rule (VDR)

Solutions:

a. Since resistor R_2 is three times R_1, it is expected that $V_2 = 3V_1$.

b. $V_1 = R_1 \frac{E}{R_T} = 20\,\Omega\left(\frac{64\text{ V}}{20\,\Omega + 60\,\Omega}\right) = 20\,\Omega\left(\frac{64\text{ V}}{80\,\Omega}\right) = \mathbf{16\text{ V}}$

c. $V_2 = 3V_1 = 3(16\text{ V}) = \mathbf{48\text{ V}}$

d. $V_2 = R_2 \frac{E}{R_T} = (60\,\Omega)\left(\frac{64\text{ V}}{80\,\Omega}\right) = \mathbf{48\text{ V}}$

The results are an exact match.

e. $E = V_1 + V_2$

$64\text{ V} = 16\text{ V} + 48\text{ V} = \mathbf{64\text{ V}}$ (checks)

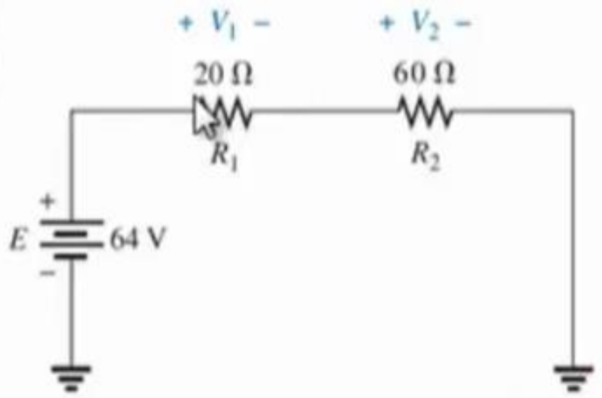


FIG. 37

continue

Electric Circuits Analysis

Voltage Divider Rule (VDR)

EXAMPLE 18 Given the voltmeter reading in Fig. 39, find voltage V_3.

Solution: Even though the rest of the network is not shown and the current level has not been determined, the voltage divider rule can be applied by using the voltmeter reading as the full voltage across the series combination of resistors. That is,

$$V_3 = R_3\frac{(V_{\text{meter}})}{R_3 + R_2} = \frac{3\ \text{k}\Omega(5.6\ \text{V})}{3\ \text{k}\Omega + 1.2\ \text{k}\Omega}$$

$$V_3 = \mathbf{4\ V}$$

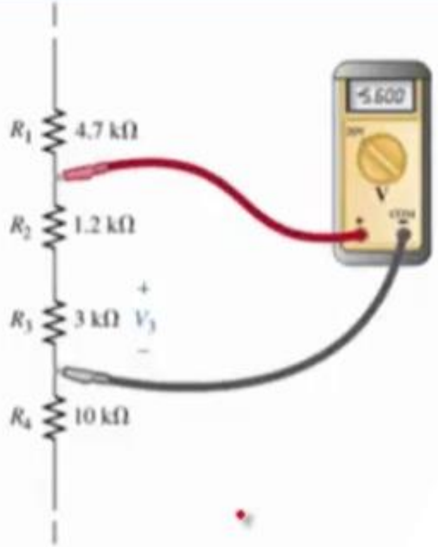


FIG. 39

Interchanging series elements

Electric Circuits Analysis

INTERCHANGING SERIES ELEMENTS

The elements of a series circuit can be interchanged without affecting the total resistance, current, or power to each element.

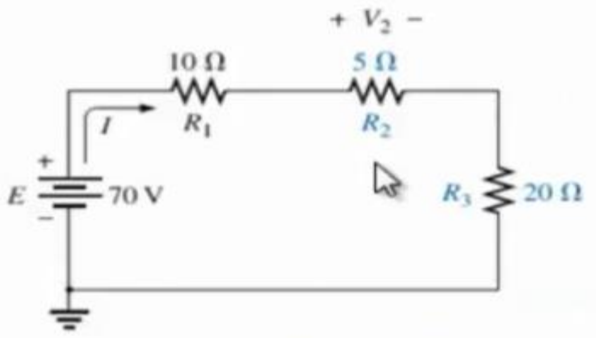


FIG. 41

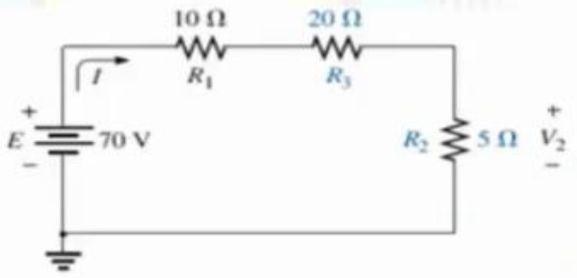


FIG. 42

Example

Electric Circuits Analysis

INTERCHANGING SERIES ELEMENTS

EXAMPLE 20 Determine I and the voltage across the 7 the network in Fig. 43.

Solution: The network is redrawn in Fig. 44.

$$R_T = (2)(4\ \Omega) + 7\ \Omega = 15\ \Omega$$

$$I = \frac{E}{R_T} = \frac{37.5\ \text{V}}{15\ \Omega} = \mathbf{2.5\ A}$$

$$V_{7\Omega} = IR = (2.5\ \text{A})(7\ \Omega) = \mathbf{17.5\ V}$$

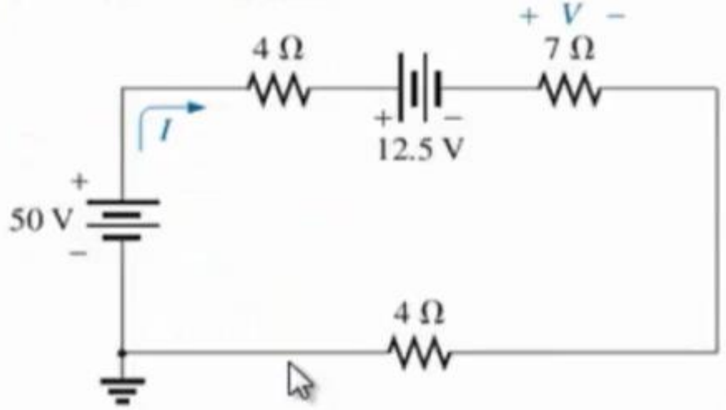
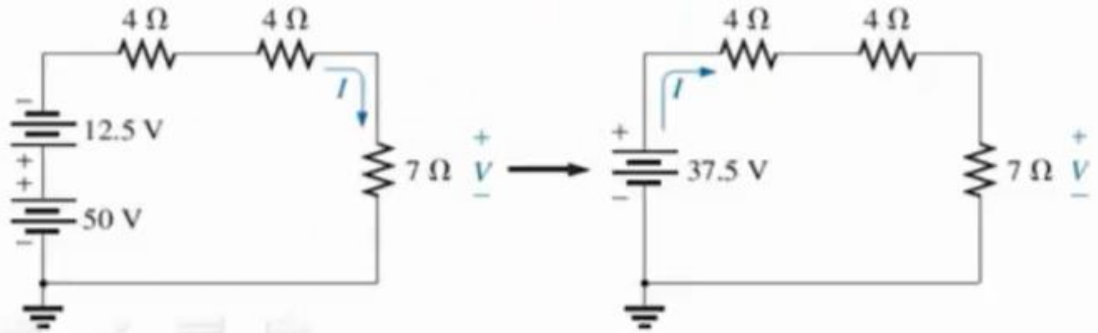


FIG. 43



Voltage source and ground

Electric Circuits Analysis

Voltage Sources and Ground

Except for a few special cases, electrical and electronic systems are grounded for reference and safety purposes.

The symbol for the ground connection appears in Fig. 5.45 with its defined potential level—zero volts.

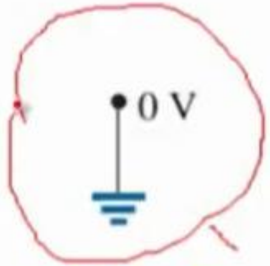


FIG. 45

Ground potential.

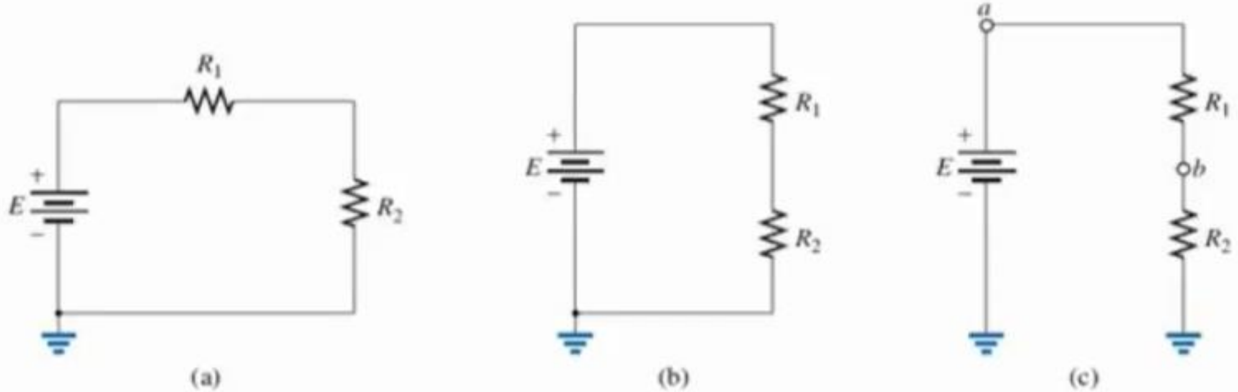


FIG. 46

Three ways to sketch the same series dc circuit.

Double Subscript Notation

Electric Circuits Analysis

Double-Subscript Notation

The fact that voltage is an across variable and exists between two points has resulted in a double-subscript notation that defines the first subscript as the higher potential.

$$V_{ab} = V_a - V_b$$

Example

Electric Circuits Analysis

Double-Subscript Notation

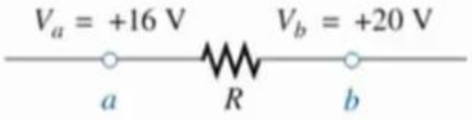


FIG. 52

EXAMPLE 21 Find the voltage V_{ab} for the conditions in Fig. 52.

Solution: Applying Eq. (12) gives

$$V_{ab} = V_a - V_b = 16\text{ V} - 20\text{ V}$$
$$= \mathbf{-4\ V}$$

Note the negative sign to reflect the fact that point ***b*** is at a higher potential than point ***a***

Example

Electric Circuits Analysis

Double-Subscript Notation

EXAMPLE 25 Determine V_{ab}, V_{cb}, and V_c for the network in Fig. 59.

Solution

$$I = \frac{54\ \text{V}}{45\ \Omega} = 1.2\ \text{A}$$

$$V_{ab} = IR_2 = (1.2\ \text{A})(25\ \Omega) = \mathbf{30\ V}$$

$$V_{cb} = -IR_1 = -(1.2\ \text{A})(20\ \Omega) = \mathbf{-24\ V}$$

$$V_c = E_1 = \mathbf{-19\ V}$$

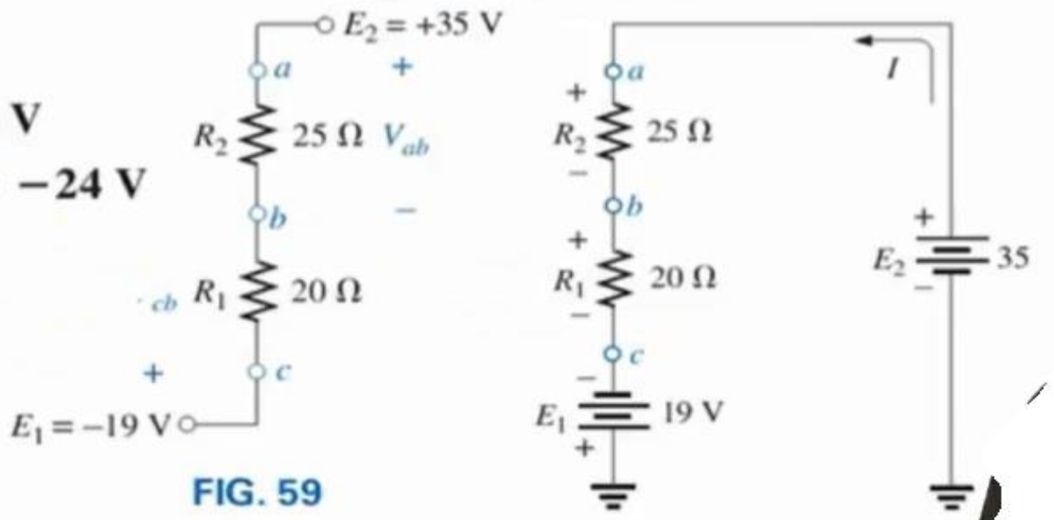


FIG. 59

Conclusion

Conclusion

This lecture revealed the concept of KIRCHHOFF'S VOLTAGE LAW. Application of the law requires that we define a closed path of investigation, permitting us to start at one point in the network, travel through the network, and find our way back to the original starting point.