# Linear Momentum and Collisions

## Linear Momentum

The linear momentum of a particle with mass m moving with velocity v is defined as

$$p = mv$$

Linear momentum is a vector. When giving the linear momentum of a particle you must specify its magnitude and direction. We can see from the definition that its units must be

$$\frac{kg \cdot m}{s}$$

The momentum of a particle is related to the net force on that particle in a simple way; since the mass of a particle remains constant, if we take the time derivative of a particle's momentum we find

$$dp/\,dt \;=\; m \qquad (1)$$

$$dv\,/dt \;=\; ma \;=\; F_{net} \qquad (2)$$

so that

$$Fnet \;= dp\,/dt \qquad (3)$$

## Impulse, Average Force

When a particle moves freely then interacts with another system for a (brief) period and then moves freely again, it has a definite change in momentum; we define this change as the impulse I of the interaction forces:

$$I \;=\; p_f \;-\; p_{\,i} = \Delta p \qquad (4)$$

Impulse is a vector and has the same units as momentum. When we integrate Eq. 7.2 we can show:

$$I \;=\; \int_{t_i}^{t_f} F\;dt \;=\; \Delta p \qquad (5)$$

We can now define the average force which acts on a particle during a time interval Δt. It is:

$$F \;=\; \Delta p/\Delta t = \frac{I}{\Delta t} \qquad (6)$$

The value of the average force depends on the time interval chosen.

## Conservation of Linear Momentum

Linear momentum is a useful quantity for cases where we have a few particles (objects) which interact with each other. Such a system is called an isolated system.

We often have reason to study systems where a few particles interact with each other very briefly, with forces that are strong compared to the other forces in the world that they may experience. In those situations, and for that brief period of time, we can treat the particles as if they were isolated.
We can show that when two particles interact only with each other (they are isolated) then their total momentum remains constant:

$$p_{1i} + p_{2i} = p_{1f} + p_{2f} \qquad (7)$$

or, in terms of the masses and velocities,

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad (8)$$

Or,

$$p_{1i} + p_{2i} = p_{1f} + p_{2f} \qquad (9)$$

$$P_i = P_f \qquad (10)$$

## Collisions

When we talk about a collision in physics (between two particles, say) we mean that two particles are moving freely through space until they get close to one another; then, for a short period of time they exert strong forces on each other until they move apart and are again moving freely.
For such an event, the two particles have well-defined momenta $p_{1i}$ and $p_{2i}$ before the collision event and $p_{1f}$ and $p_{2f}$ afterwards. But the sum of the momenta before and after the collision is conserved.

If two objects collide, stick together, and move off as a combined mass, we call this a perfectly inelastic collision. One can show that in such a collision more kinetic energy is lost than if the objects were to bounce off one another and move off separately.

When two particles undergo an elastic collision then we also know that

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

In the special case of a one-dimensional elastic collision between masses m1 and m2 we can relate the final velocities to the initial velocities. The result is

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

This result can be useful in solving a problem where such a collision occurs,

## The Center of Mass

For a system of particles, there is a special point in space known as the center of mass which is of great importance in describing the overall motion of the system. This point is a weighted average of the positions of all the mass points. If the particles in the system have masses $m_1$, $m_2$, . . . $m_N$, with total mass

$$\sum_i^N m_i = m_1 + m_2 + \cdots + m_N = M$$

and respective positions $r_1$, $r_2$, . . . ,$r_N$, then the center of mass $r_{CM}$ is:

$$\mathbf{r}_{\mathrm{CM}} = \frac{1}{M}\sum_i^N m_i \mathbf{r}_i$$

Which means that the x, y and z coordinates of the center of mass are

$$x_{\mathrm{CM}} = \frac{1}{M}\sum_i^N m_i x_i \qquad y_{\mathrm{CM}} = \frac{1}{M}\sum_i^N m_i y_i \qquad z_{\mathrm{CM}} = \frac{1}{M}\sum_i^N m_i z_i$$

For an extended object (i.e. a continuous distribution of mass) the definition of $r_{CM}$ is given by an integral over the mass elements of the object:

$$\mathbf{r}_{\mathrm{CM}} = \frac{1}{M}\int \mathbf{r}\, dm$$

Which means that the x, y and z coordinates of the center of mass are now:

$$x_{\mathrm{CM}} = \frac{1}{M}\int x\, dm \qquad y_{\mathrm{CM}} = \frac{1}{M}\int y\, dm \qquad z_{\mathrm{CM}} = \frac{1}{M}\int z\, dm$$

When the particles of a system are in motion then in general their center of mass is also in motion. The velocity of the center of mass is a similar weighted average of the individual velocities:

$$\mathbf{v}_{\mathrm{CM}} = \frac{d\mathbf{r}_{\mathrm{CM}}}{dt} = \frac{1}{M}\sum_i^N m_i \mathbf{v}_i$$

In general the center of mass will accelerate; its acceleration is given by

$$\mathbf{a}_{\mathrm{CM}} = \frac{d\mathbf{v}_{\mathrm{CM}}}{dt} = \frac{1}{M}\sum_i^N m_i \mathbf{a}_i$$

If P is the total momentum of the system and M is the total mass of the system, then the motion of the center of mass is related to P by:

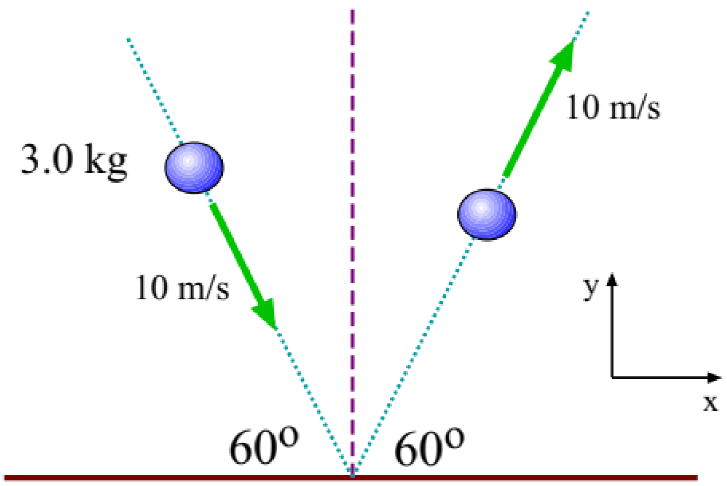
$$\mathbf{v}_{\mathrm{CM}} = \frac{\mathbf{P}}{M} \qquad \text{and} \qquad \mathbf{a}_{\mathrm{CM}} = \frac{1}{M}\frac{d\mathbf{P}}{dt}$$

# Solved Problems

**1. A $3.00\,\mathrm{kg}$ particle has a velocity of $(3.0\mathbf{i} - 4.0\mathbf{j})\,\frac{\mathrm{m}}{\mathrm{s}}$. Find its $x$ and $y$ components of momentum and the magnitude of its total momentum.**

Using the definition of momentum and the given values of $m$ and $\mathbf{v}$ we have:

$$\mathbf{p} = m\mathbf{v} = (3.00\,\mathrm{kg})(3.0\mathbf{i} - 4.0\mathbf{j})\,\frac{\mathrm{m}}{\mathrm{s}} = (9.0\mathbf{i} - 12.\mathbf{j})\,\frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$




So the particle has momentum components

$$p_x = +9.0\,\frac{\mathrm{kg \cdot m}}{\mathrm{s}} \qquad \text{and} \qquad p_y = -12.\,\frac{\mathrm{kg \cdot m}}{\mathrm{s}}\ .$$

The magnitude of its momentum is

$$p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.0)^2 + (-12.)^2}\,\frac{\mathrm{kg \cdot m}}{\mathrm{s}} = 15.\,\frac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

**2. A child bounces a superball on the sidewalk. The linear impulse delivered by the sidewalk is $2.00\,\mathrm{N\cdot s}$ during the $\frac{1}{800}\,\mathrm{s}$ of contact. What is the magnitude of the average force exerted on the ball by the sidewalk.**

The magnitude of the change in momentum of (impulse delivered to) the ball is $|\Delta\mathbf{p}| = |\mathbf{I}| = 2.00\,\mathrm{N\cdot s}$. (The *direction* of the impulse is upward, since the initial momentum of the ball was downward and the final momentum is upward.)

Since the time over which the force was acting was

$$\Delta t = \tfrac{1}{800}\,\mathrm{s} = 1.25\times10^{-3}\,\mathrm{s}$$

then from the definition of average force we get:

$$|\overline{\mathbf{F}}| = \frac{|\mathbf{I}|}{\Delta t} = \frac{2.00\,\mathrm{N\cdot s}}{1.25\times10^{-3}\,\mathrm{s}} = 1.60\times10^{3}\,\mathrm{N}$$