lecture 10

4.5 Classification of Resistances

Based upon the value, the resistances can be classified into the following three categories:

1. **Low resistance.** The resistance with values less than or equal to 1 Ω are called low resistance. Armatures winding of machines, ammeter shunts, cables, contacts etc. all have a low resistance value.
2. **Medium resistance.** The resistance with values ranging from 1 Ω to 100 kΩ are called medium resistance. The resistors employed in electronic circuits usually are of medium resistance type.
3. **High resistance.** The resistance with values above 100 kΩ are high resistances. Some of the electrical and electronic circuits do employ resistors with high resistance values.

4.6 Measurement of Resistance

There are several methods used for the measurement of resistance. Fig. 4.2 shows the methods that are used for the measurement the low-, medium- and high-resistance values.

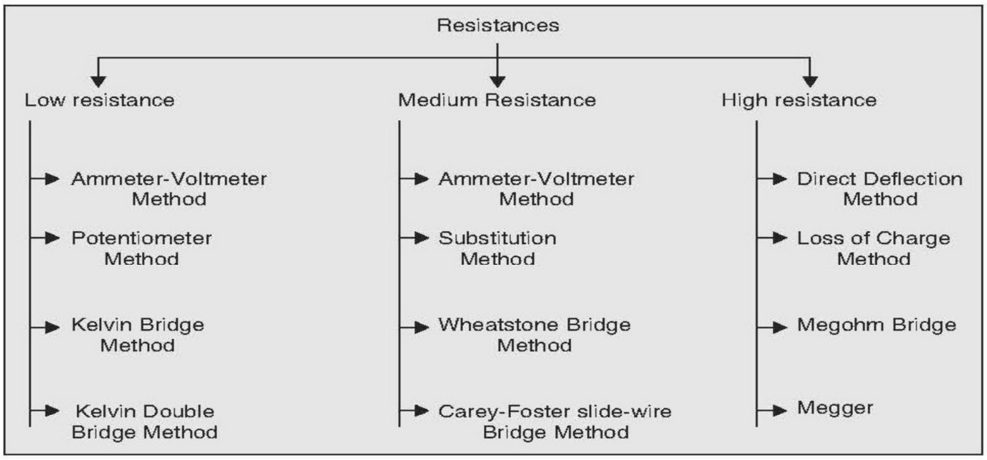

Fig. 4.2.

4.1 Measurements of Low Resistance

There are several methods for the measurement of low resistance value. But the following ones are important from the subject point of view :

1. Ammeter-Voltmeter Method
2. Potentiometer Method
3. Kelvin Bridge Method
4. Kelvin Double Bridge Method

We shall now discuss all the above mentioned methods one by one in the following pages.

4.2 Ammeter-Voltmeter Method

This method is used for measuring low resistance value when accuracy of the order of 1 % is sufficient. The ammeter-voltmeter method employs the simple ohm's law to determine the value of an unknown resistance.

Fig. 4.3 shows the circuit arrangement of ammeter-voltmeter method. The current through the unknown resistor (R_X) and potential drop across it are measured simultaneously. The readings are obtained by ammeter and voltmeter respectively. The required range of instrument to be used and the voltage of the supply required will depend on the size and rating of the resistance under test. A high-value resistor will require high-voltage source, a high-range voltmeter and a low-range ammeter whereas, a low-value resistor will require in most cases, a low–voltage, high-current source, a low-range voltmeter and a high-range ammeter. The exact requirement will, of course, depend also on the rating of the resistor, as well as the instruments available. There are two ways to connect the voltmeter as discussed below.

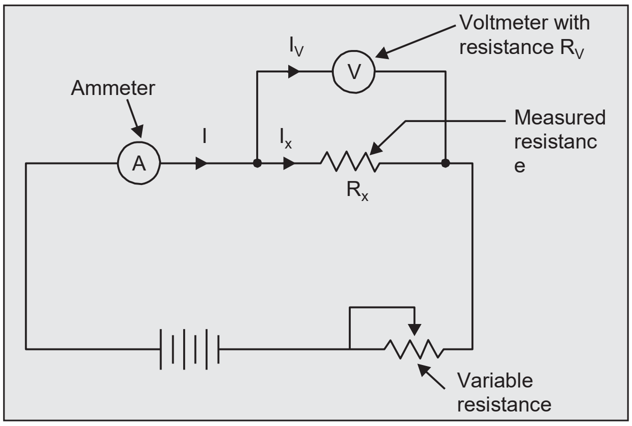

Fig. 4.3.

1. *Voltmeter is connected directly across the resistor only*

When the voltmeter is connected directly across the resistor, the ammeter measures the current flowing through the unknown resistance R_X and the voltmeter.

Current through ammeter

= Current through unknown resistance (X) + Current through voltmeter

$$I = I_X + I_V$$

$$I_X = I - I_V$$

the value of unknown resistance,

$$R_X = \frac{V}{I_X} = \frac{V}{I - I_V} = \frac{V}{I - V/R_V}$$

$$= \frac{V}{I\left(1 - \frac{V}{I R_V}\right)} \quad \text{----(i)}$$

where V = voltmeter reading

R_V = resistance of the voltmeter

I = current indicated by ammeter

The value of unknown resistance is determined by equation (*i*).

Substituting the value, $V/I = R_m$ in the equation (*i*) we get,

$$R_X = R_m \left(\frac{1}{1 - \frac{R_m}{R_V}} \right)$$

From the above equation we see that the true value of unknown resistance is equal to measured value of unknown resistance provided that voltmeter is of infinite resistance. However if the voltmeter is of very large resistance as compared to the resistance under measurement then,

i.e. $R_V >> R_m$

$$R_X = 1 + \frac{R_m}{R_V}$$

Thus the measured value of unknown resistance, R_m is lesser than its true value.

2. *Voltmeter is connected directly across the ammeter and resistor:*

Figure 4.4 shows the voltmeter connected directly across the ammeter and unknown resistance, R_X, the voltmeter measures the voltage drop across the ammeter and unknown resistance. The ammeter is connected so that it indicates only the current flowing through the unknown resistance.

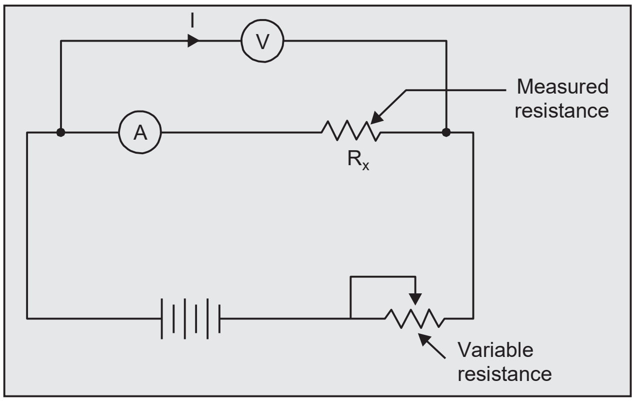

Fig. 4.4.

$$V = I R_A + I R_X = I (R_A + R_X)$$

$$R_X = \frac{V}{I} - R_A \qquad \ldots(ii)$$

where R_A is the resistance of the ammeter. The value of unknown resistance is determined by the equation (*ii*).The ammeter-voltmeter method of measuring resistance is capable of fair accuracy, depending on care in taking the reading and on the accuracy and range of the instruments used for measurement of voltage and current. This method is useful in some laboratory work in which high accuracy is not required.

4.3 Advantages and Disadvantages of Ammeter-Voltmeter Method

Advantages

Some of the main advantages of ammeter-voltmeter method are given below :

1. It does not require skilled operation.
2. Accuracy of the order ± 1 % can be achieved.

Disadvantages

Some of the main disadvantages of ammeter-voltmeter are given below :

1. A correction factor needs to apply on the measured value to obtain the true value of the resistance.
2. The low values of resistances invariably have a high percentage of error.

4.4 Potentiometer Method

In potentiometer method the unknown resistance is compared with a standard resistance of the same order of magnitude. Fig. 4.5 shows the circuit diagram of potentiometer method. As seen from the diagram, the unknown resistance R_X, ammeter A, a rheostat R (to limit the current) and a standard resistance are connected in series with low voltage high current supply source. The value of standard resistance should be known.

The current through the circuit is adjusted by a rheostat so that a potential difference across the resistor is about 1V. The voltage drop across the potentiometer and the standard resistor is measured by a potentiometer. The ratio of the two potentiometer reading gives the ratio of R_X to S.

$$\frac{R_X}{S} = \frac{\text{Potentiometer reading across } R_X}{\text{Potentiometer reading across } S} = \frac{V_X}{V_S}$$

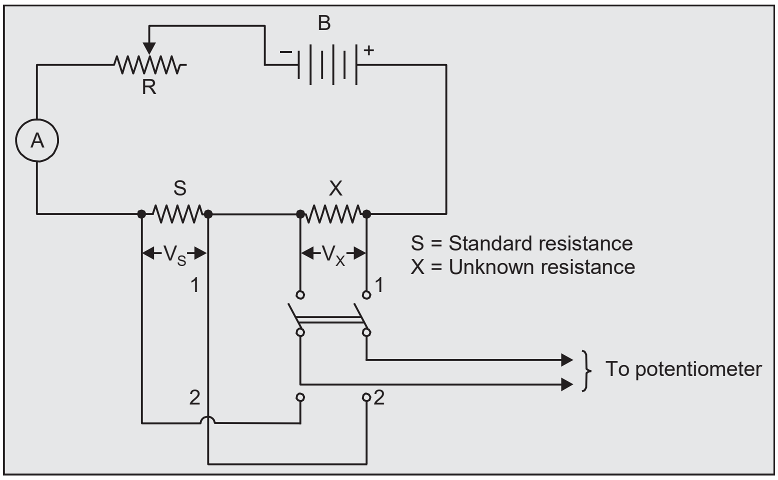

Fig. 4.5.

The accuracy of this method depends upon there being no change in current between the two readings. The source to supply current through the circuit should be extremely stable.

4.5 Advantages and Disadvantages of Potentiometer Method

Advantages of Potentiometer Method

Though there are numerous advantages of potentiometer method, yet some of the important are given below:

1. Inexpensive
2. Simple to handle
3. Useful for the measurement of large amplitudes of displacement
4. Electrical efficiency is very high.

Disadvantages of Potentiometer Method

Though the potentiometer method has a number of disadvantages, yet some of them are given below:

1. Force is required to move the sliding contacts.
2. Sliding contacts can wear out, become misaligned and generate noise.

4.6 Kelvin Bridge

Fig. 4.6 shows the circuit diagram of a Kelvin Bridge. This circuit provides great accuracy in the measurement of low value resistance generally below 1 Ω. It is used for measuring resistance values ranging from microohms to 1 ohm.

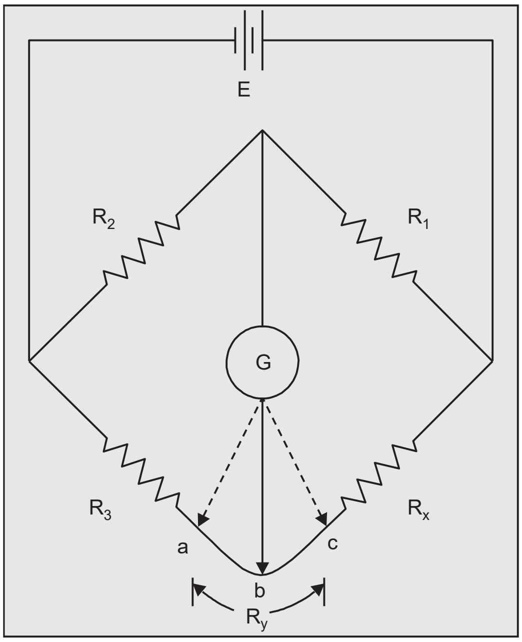

Fig. 4.6. Kelvin Bridge

The resistance R_y represents the resistance of the conducting lead from R_3 to R_x. The resistance R_x is the unknown resistance to be measured. The galvanometer can be connected either to point '*c*' or to point '*a*'. When it is connected to point '*a*', the resistance R_y of the connecting lead is added to the unknown resistance R_x. The measurement value of the resistance is too high than the actual value.

When the galvanometer is connected to the point '*c*', the resistance R_y of the connecting lead is added to the known resistance R_3. The actual value of R_3 is higher than the normal value by the resistance R_y and the resulting measurement of R_x is lower than the actual value.

If the galvanometer is connected to point '*b*', in between points '*c*' and '*a*', in such a way that the ratio of the resistance from '*c*' to '*b*' and that from '*a*' to '*b*' equals the ratio of resistance R_1 and R_2 then,

$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$$

Balance equation for the bridge is given by relation,

$$\frac{R_x + R_{cb}}{R_3 + R_{ab}} = \frac{R_1}{R_2}$$

$$(R_x + R_{cb}) = \frac{R_1}{R_2}\,(R_3 + R_{ab}) \qquad \ldots(i)$$

We know that

$$R_{ac} + R_{bc} = R_y$$

and $$\frac{R_{bc}}{R_{ac}} = \frac{R_1}{R_2} \qquad \ldots(ii)$$

Adding 1 on the both side of equation (*ii*) we get

$$\frac{R_{bc}}{R_{ac}} + 1 = \frac{R_1}{R_2} + 1$$

$$\frac{R_{bc} + R_{ac}}{R_{ac}} = \frac{R_1 + R_2}{R_2}$$

$$\frac{R_y}{R_{ac}} = \frac{R_1 + R_2}{R_2} \qquad \ldots(\; R_{ac} + R_{bc} = R_y)$$

$$R_{ac} = \frac{R_2\; R_y}{R_1 + R_2} \qquad \ldots(iii)$$

$$R_{bc} = R_y - R_{ac}$$

$$= R_Y - \frac{R_2\; R_Y}{R_1 + R_2}$$

$$= \frac{R_1\; R_y}{R_1 + R_2} \qquad \ldots(iv)$$

Substituting the equation (*iii*) and (*iv*) in equation (*i*),

$$R_x + \frac{R_1\, R_y}{R_1 + R_2} = \frac{R_1}{R_2}\left(R_3 + \frac{R_2\, R_y}{R_1 + R_2}\right)$$

$$R_x = \frac{R_1\; R_3}{R_2}$$

This is the standard equation of the bridge balance. The equation does not depend on the resistance of connecting lead from R_3 to R_x. The effect of lead and contact resistances is completely eliminated by connecting the galvanometer to the intermediate position '*b*'.

4.7 Double Kelvin Bridge

Fig. 4.7 shows the circuit diagram of Kelvin double bridge. This bridge contains another set of ratio arms hence called double bridge. The second set of arms labeled '*l*' and '*m*'. The galvanometer is connected to point '*f*'. The ratio of the resistances of arms '*l*' and '*m*' is same as the ratio of R_1 and R_2.

The galvanometer indicates "zero" when the potential at '*a*' equals the potential at '*f*', i.e.,

$$E_{ab} = E_{bcf}$$

According to the *Voltage Divider Rule* the voltage across the '*ab*',

$$E_{ab} = \frac{R_2}{R_1 + R_2} \times E \qquad \ldots(i)$$

The value of E is given by,

$$E = I\,[R_3 + R_x + (l + m) \parallel R_y]$$

$$E = I\left[R_3 + R_x + \frac{(l + m)\, R_y}{(l + m) + R_y}\right]$$

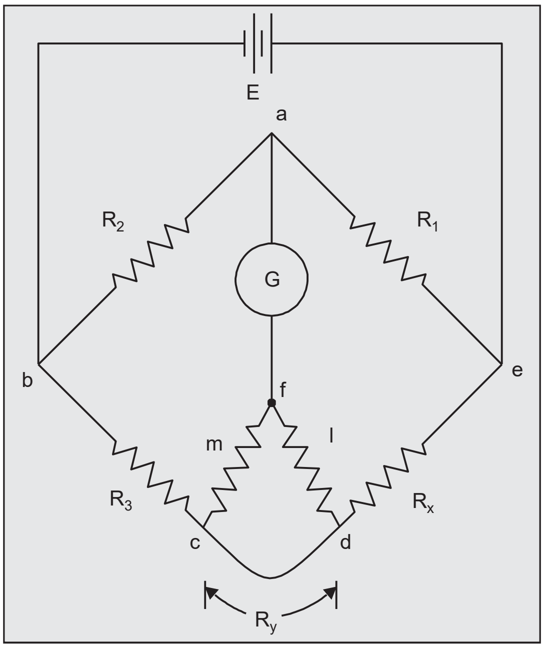

Fig. 4.7. Double Kelvin Bridge

Substituting the value of E in equation (i) we get,

$$E_{ab} = \frac{R_2}{R_1 + R_2} \times I \left[R_3 + R_x + \frac{(l+m)\, R_y}{(l+m) + R_y} \right]$$

Similarly,

$$E_{bcf} = I \left[R_3 + \frac{m}{l+m} \left\{ \frac{(l+m)\, R_y}{(l+m) + R_y} \right\} \right]$$

but,

$$E_{ab} = E_{bcf} = \frac{R_2}{R_1 + R_2} \times I \left[R_3 + R_x + \frac{(l+m)\, R_y}{(l+m) + R_y} \right]$$

Rearranging the above equation we get,

$$R_x = \frac{R_1\, R_3}{R_2} + \frac{m R_1\, R_y}{R_2\, (a + b + R_y)} - \frac{l R_y}{l + m + R_y}$$

$$R_x = \frac{R_1\, R_3}{R_2} + \frac{mR_y}{l + m + R_y} \left(\frac{R_1}{R_2} - \frac{l}{m} \right) \qquad \ldots(ii)$$

The ratio of the resistances of arms 'l' and 'm' is same as the ratio of R_1 and R_2, i.e.,

$$\frac{R_1}{R_2} = \frac{l}{m} \qquad \ldots(iv)$$

From equation (iv) and (iii) we get the value of R_x,

$$R_x = \frac{R_1\, R_3}{R_2}$$

This is the equation for Kelvin Bridge. It indicates that the resistance of the connecting lead R_Y, has no effect on the measurement, provided that the ratios of the resistances of the two sets of ratio arms are equal. Fig. 4.8 shows the Kelvin double bridge. This bridge is mostly used for industrial and laboratory purpose.

Fig. 4.8. Kelvin double bridge used in industry

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Example 4.3. *In a Kelvin double bridge, there is error due to mismatch between the ratio of outer and inner arm resistances. The following data relates to this bridge,*

Standard resistance = 100.03 μΩ

Inner arms = 100.31 Ω and 200 Ω

Outer arms = 100.24 Ω and 200 Ω

The resistance of connecting leads from standard to unknown resistance is 680 μΩ. Determine the value of unknown resistance.

Solution. Given: $R_3 = 100.03\ \mu\Omega = 100.03 \times 10^{-6}\ \Omega$; $l = 100.31\ \Omega$; $m = 200\ \Omega$; $R_1 = 100.24\ \Omega$; $R_2 = 200\ \Omega$ and $R_y = 680\ \mu\Omega = 680 \times 10^{-6}\ \Omega$.

We know that the value of unknown resistance

$$R_x = \frac{R_1 R_3}{R_2} + \frac{m R_y}{l + m + R_y}\left(\frac{R_1}{R_2} - \frac{l}{m}\right)$$

$$= \frac{100.24 \times (100.03 \times 10^{-6})}{200} + \frac{200 \times (680 \times 10^{-6})}{100.31 + 200 + (680 \times 10^{-6})}\left(\frac{100.24}{200} - \frac{100.31}{200}\right)$$

$$= (50.135 \times 10^{-6}) + (4.528 \times 10^{-4}) \times (-3.5 \times 10^{-4})$$
$$= 49.97 \times 10^{-6}\ \Omega$$