

Three-Phase Circuit

= THREE PHASE CIRCUITS.

Generators with two or more windings in which the induced e.m.f are of the same amplitude and frequency but shifted in phase are called (Polyphase Generators). A Circuit Containing polyphase generators and loads is called a Poly phase Circuit.

The most common in practice for the transmission and distribution of electric power is the three-phase system for economical, technical and simplicity reasons.

In a three-phase generator a set of balanced voltages that have identical amplitudes and frequency but are shifted in respect with each other by 120° are produced.

In discussing 3-phase circuits, it is a standard practice to refer to the three phase as:

A, B, and C) or (R, S, and T) or (U, V, and W).

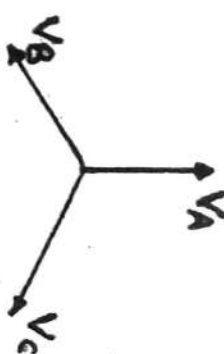
Phase - Sequence

ABC system.
(Positive Sequence)

If $V_A = 50 \angle 20^\circ$ v.o.t.

$\therefore V_B = 50 \angle -30^\circ$ "

$V_C = 50 \angle -150^\circ$ "



CBA system.
(Negative Sequence)

If $V_A = 50 \angle 20^\circ$ v.o.t.

$\therefore V_B = 50 \angle -150^\circ$ "

$V_C = 50 \angle -30^\circ$ "

If $V_A = 100 \angle 0^\circ$ v.o.t.

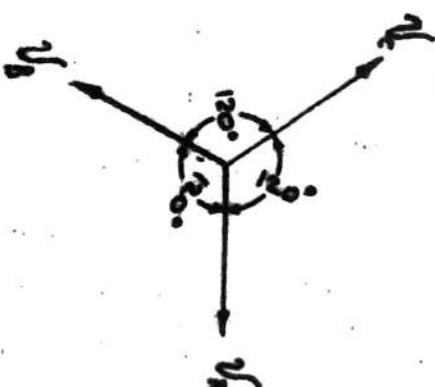
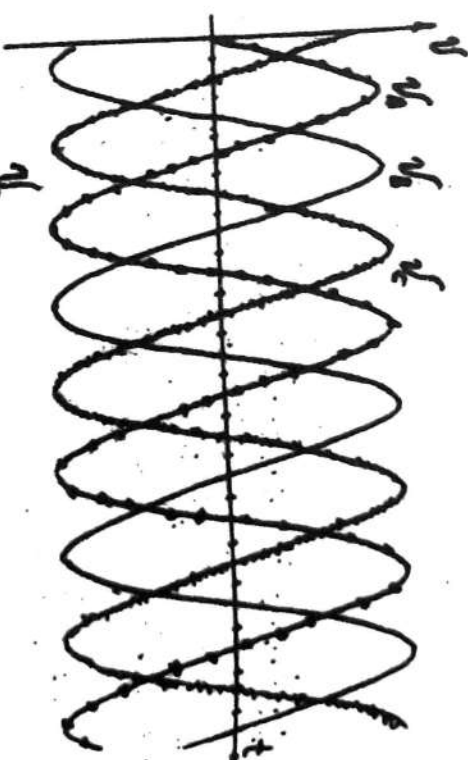
$V_B = 100 \angle -120^\circ$ "

$V_C = 100 \angle +120^\circ$ "

If $V_A = 100 \angle 0^\circ$ v.o.t.

$V_B = 100 \angle +120^\circ$ "

$V_C = 100 \angle -120^\circ$ "



An important characteristic of a set of balanced three-phase voltages is that at any instant of time the sum of the three voltages adds to zero.

$$V_A + V_B + V_C = 0$$

Fig. 5 shows a system in which the power is carried from the generator windings to the load along six wires without interlinking the phases.

Fig. 6 shows a change made for the sake of economy. Instead of having a return wire from each load to each winding, a single wire is used for the return current of all the loads. This wire is called the (Neutral wire).

If the three loads are equal ($Z_A = Z_B = Z_C$), the three currents I_A , I_B , and I_C will be balanced and the sum of these currents is zero.

$$I_N = I_A + I_B + I_C = 0$$

when $Z_A = Z_B = Z_C$.

But when $Z_A \neq Z_B \neq Z_C \rightarrow I_N \neq 0$

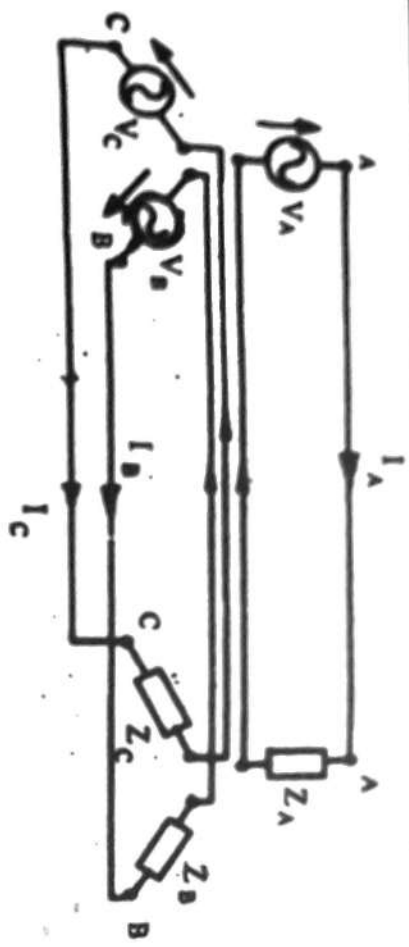


Fig. 5 No-Interlinked 3-phase six wire system

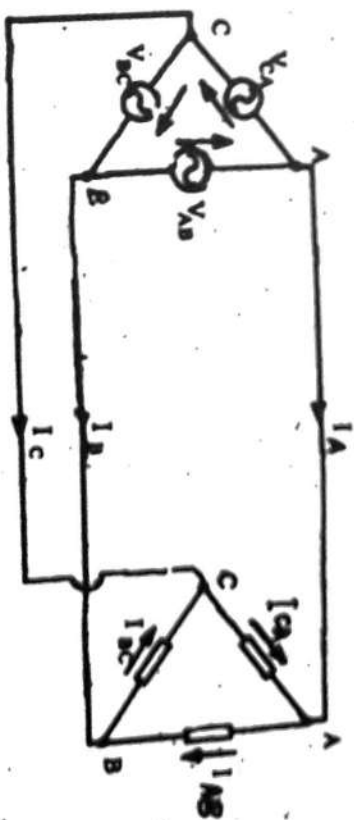


Fig. 7 Three phase THREE wire delta-delta system

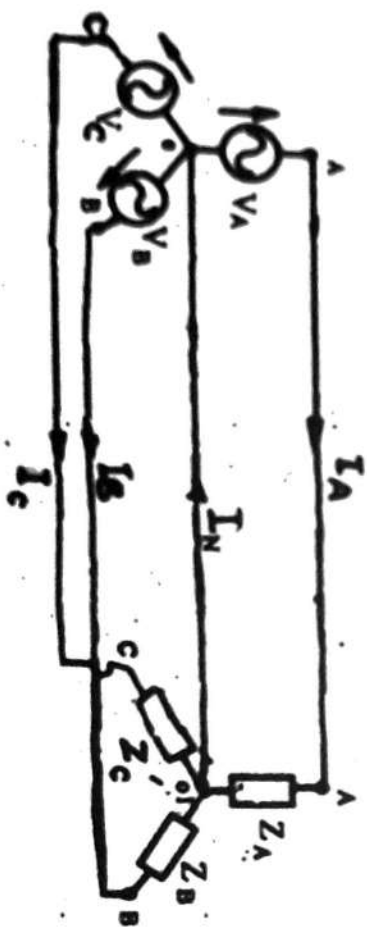


Fig. 6 Three phase FOUR wire star-star system

- A more general method of 3-phase connection which is used for power distribution is shown in Fig. 8.

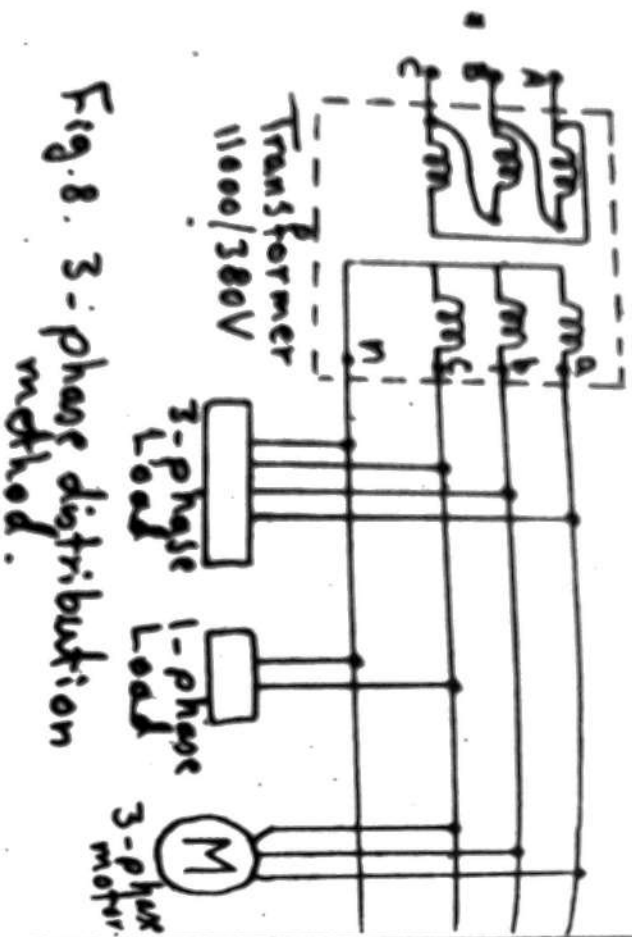


Fig. 8. 3-phase distribution method.

- ** The voltage between any two of the wires (ABC) is called the Line Voltage V_L
- ** The current passing through any one of the three (ABC) wires is called the Line Current I_L
- ** The current passing through any generator winding (or load impedance) is called the (phase current I_{ph}).
- ** The point connecting all the ends of generator windings (XYZ) is called the neutral point of the generator.

** The point connecting all the ends of load impedances is called the neutral point of the load.

** If the phase sequence is not mentioned then a positive ABC system is assumed

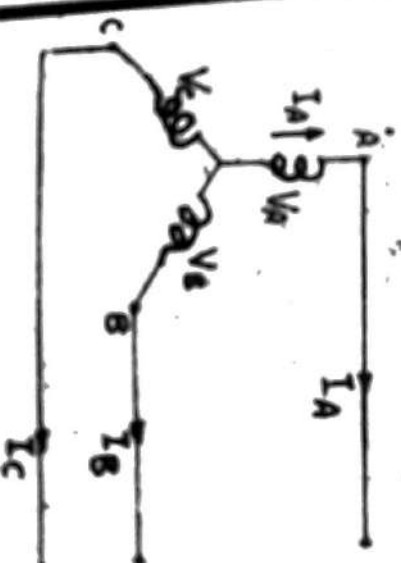
** Unless otherwise specifically stated, a voltage given for a 3-phase system shall be assumed to be the line voltage (V_L)

Relation Ship between Line and Phase Quantities

In Δ Connected generator and in balanced Δ connected load:-

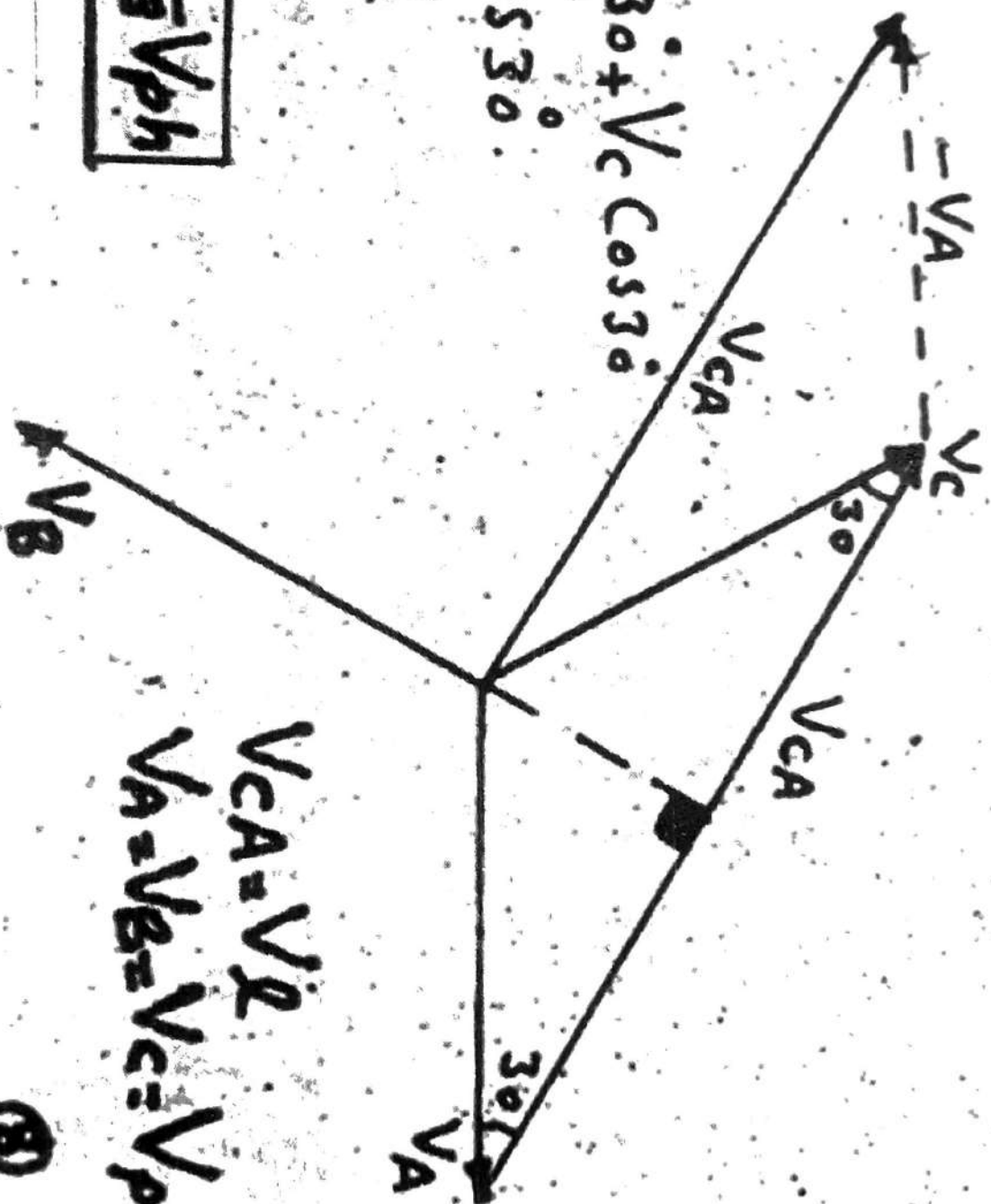
$$I_L = I_{ph}$$

$$V_L = \sqrt{3} V_{ph}$$



$$\begin{aligned}
 E_A &= V_A \cos 30^\circ + V_C \cos 30^\circ \\
 &= 2V_A \cos 30^\circ \\
 &= 2V_A \frac{\sqrt{3}}{2}
 \end{aligned}$$

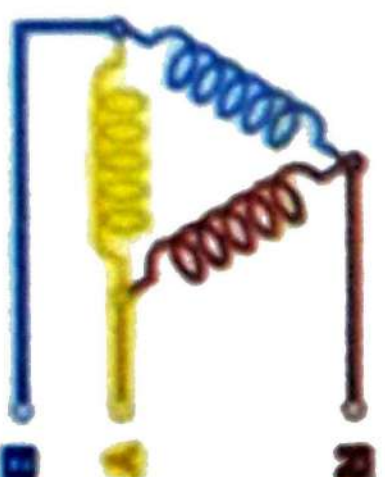
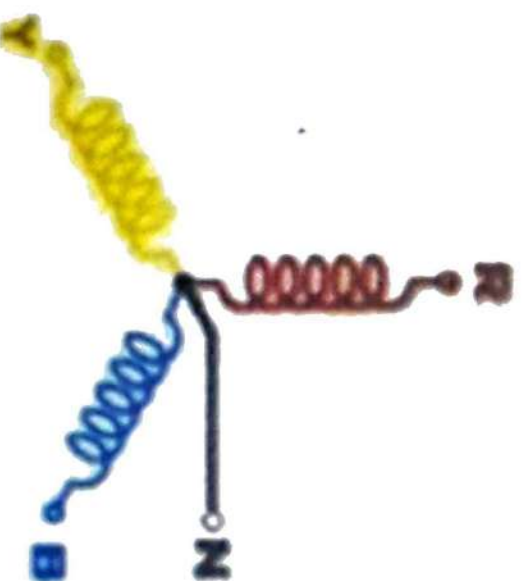
$$\boxed{V_L = \sqrt{3} V_{ph}}$$



$$\begin{aligned}
 V_{CA} &= V_L \\
 V_A = V_B = V_C &= V_{ph}
 \end{aligned}$$

⑧

Types of three-phase lines



Star V/S Delta

Line current = Phase current

$$I_L = I_P$$

Line voltage = $\sqrt{3} \times$ Phase voltage

$$V_L = \sqrt{3} \times V_P$$

Line voltage = Phase voltage

$$V_L = V_P$$

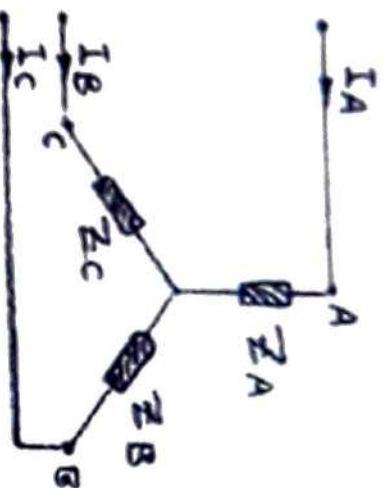
Line current = $\sqrt{3} \times$ Phase current

$$I_L = \sqrt{3} \times I_P$$

1-Y-Connected Load

Power in 3-phase Circuits.

① In Y-Connected Load :-



* When $Z_A \neq Z_B \neq Z_C$.

$$\therefore P = V_A I_A \cos \theta_A + V_B I_B \cos \theta_B + V_C I_C \cos \theta_C$$

* When $Z_A = Z_B = Z_C$.

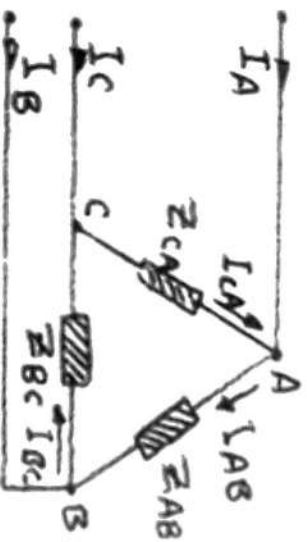
$$\therefore P = 3 V_A I_A \cos \theta_A \\ = \sqrt{3} \cdot \sqrt{3} \cdot \frac{V_L}{\sqrt{3}} \cdot I_L \cos \theta$$

$$\therefore P = \sqrt{3} V_L \cdot I_L \cos \theta$$

$$Q = \sqrt{3} V_L \cdot I_L \sin \theta$$

2-Δ - Connected Load

② In Δ - Connected Load:-



* When $Z_{AB} \neq Z_{BC} \neq Z_{CA}$.

$$P = V_{AB} I_{AB} \cos \theta_{AB} + V_{BC} I_{BC} \cos \theta_{BC} + V_{CA} I_{CA} \cos \theta_{CA}$$

When $Z_{AB} = Z_{BC} = Z_{CA}$

$$P = 3 V_{AB} \cdot I_{AB} \cos \theta_{AB}.$$

$$= \sqrt{3} \cdot \sqrt{3} \cdot V_L \cdot \frac{I_L}{\sqrt{3}} \cos \theta$$

$$P = \sqrt{3} V_L \cdot I_L \cdot \cos \theta.$$

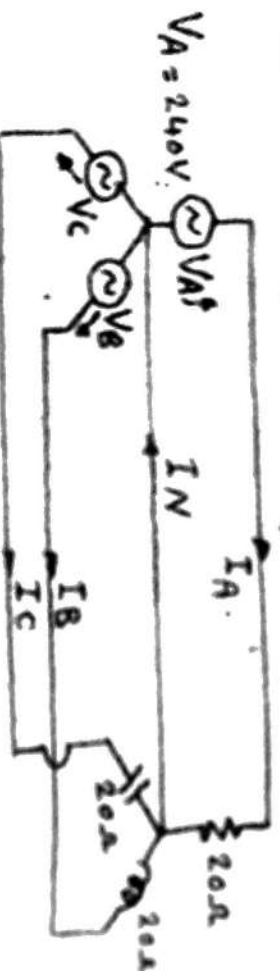
$$Q = \sqrt{3} V_L \cdot I_L \cdot \sin \theta.$$

Example:

In the 3-phase circuit shown:

Find the current distribution in the circuit when the phase sequence is ABC

Draw the phasor diagram of Voltages & Currents



$$I_P V_A = 240 \angle 0^\circ V. \quad \therefore V_B = 240 \angle -120^\circ, V_C = 240 \angle 120^\circ$$

$$I_A = \frac{240 \angle 0^\circ}{20} = 12 \angle 0^\circ A-P.$$

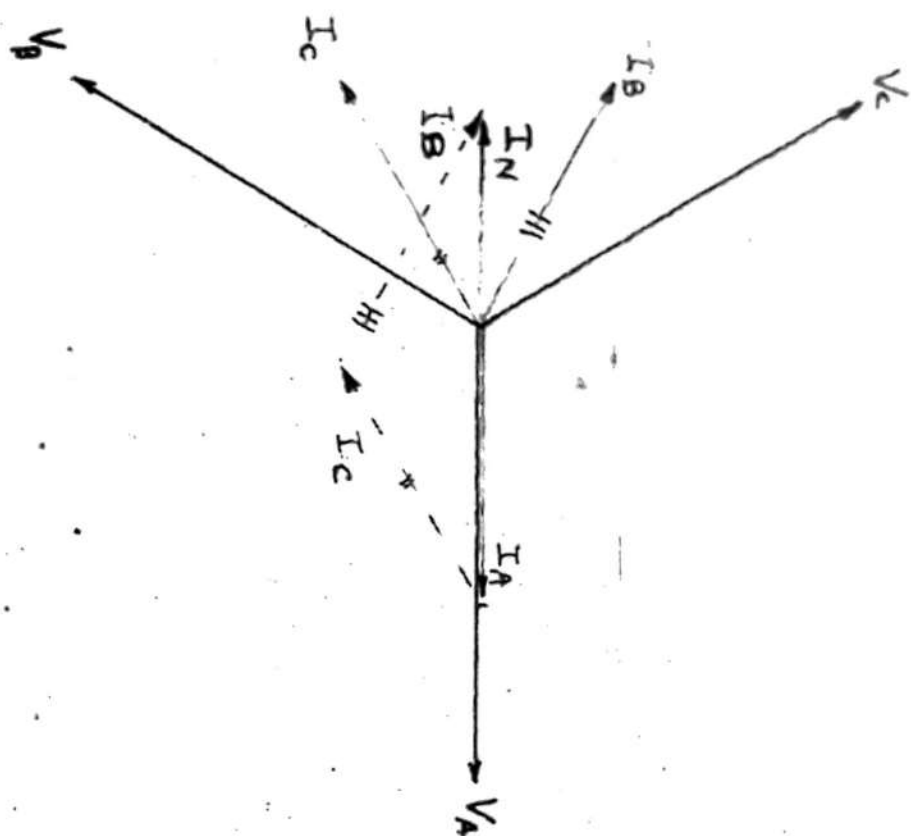
$$I_B = \frac{240 \angle -120^\circ}{20 \angle 90^\circ} = 12 \angle -210^\circ A-P.$$

$$I_C = \frac{240 \angle 120^\circ}{20 \angle -90^\circ} = 12 \angle 210^\circ A-P.$$

$$I_N = I_A + I_B + I_C$$

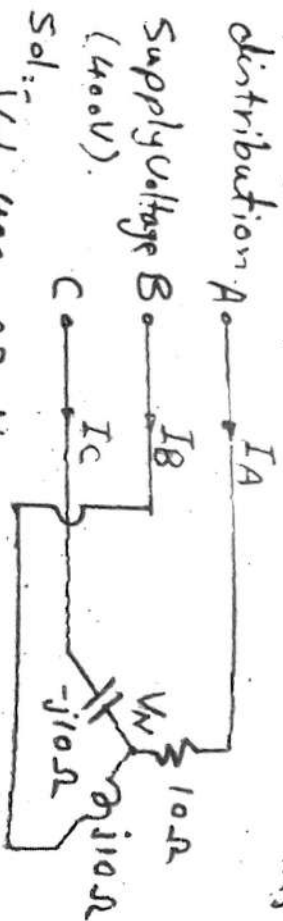
$$= 12 \angle 0^\circ + 12 \angle -210^\circ + 12 \angle 210^\circ = 8.78 \angle -180^\circ = -8.78 A-P.$$

Note: Repeat the same question for -ve Sequence.



Example:

For the circuit shown, find the currents distribution A.



Sol: $V_{ph} = \frac{400}{\sqrt{3}} = 230 \text{ V}$.

$\therefore V_A = 230 \angle 0^\circ$, $V_B = 230 \angle -120^\circ$, $V_C = 230 \angle +120^\circ$.

$$\left(\frac{1}{10} + \frac{1}{j10} + \frac{1}{-j10} \right) V_N - \frac{V_A}{10} - \frac{V_B}{+j10} - \frac{V_C}{-j10} = 0$$

$$(0.1 + j0.1 - j0.1) V_N = 0.1 V_A - j0.1 V_B + j0.1 V_C$$

$$\therefore V_N = \frac{230 \times 0.1 + 230 \angle -120^\circ \times 0.1 \angle -90^\circ + 230 \angle 120^\circ \times 0.1 \angle 90^\circ}{0.1 + j0.1 - j0.1}$$

$$= 230 - 200 + j115 - 200 - j115 = -170 \text{ Volt}$$

$$\therefore I_A = \frac{V_A - V_N}{10}$$

$$= \frac{230 - (-170)}{10} = \frac{400}{10} = 40 \text{ Amp}$$

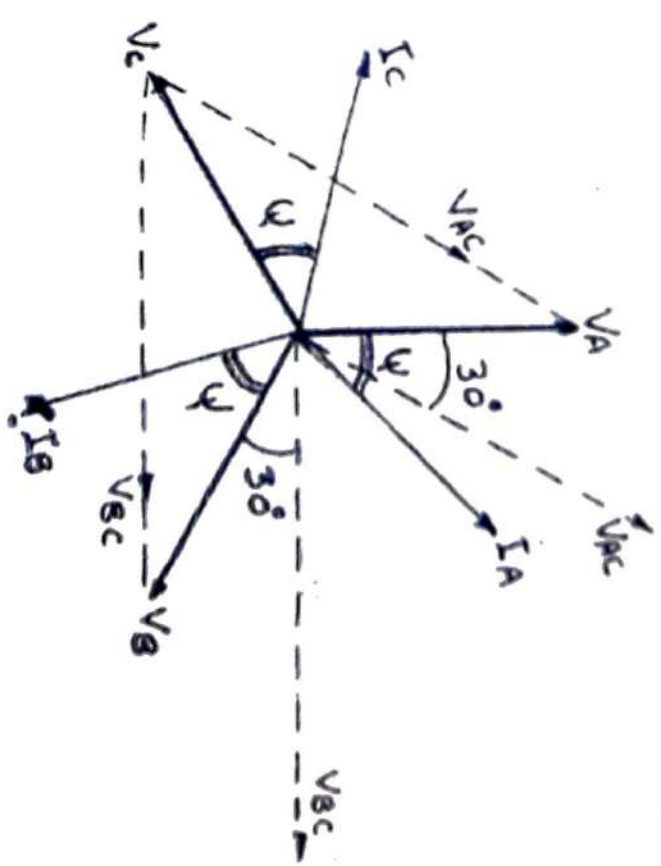
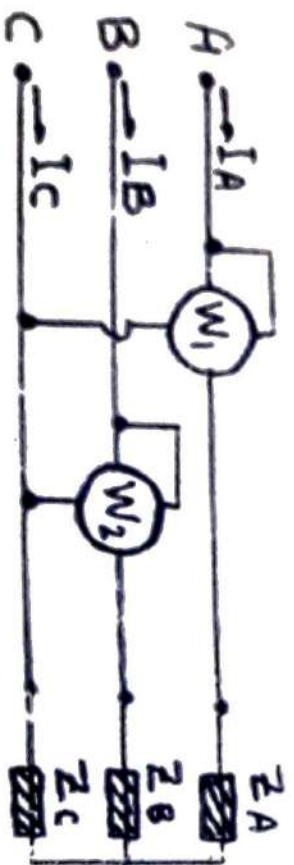
$$I_B = \frac{V_B - V_N}{j10} = \frac{230 \angle -120^\circ - (-170)}{10 \angle 90^\circ} = 20.7 \angle -165^\circ \text{ A}$$

$$I_C = \frac{V_C - V_N}{-j10} = \frac{230 \angle +120^\circ - (-170)}{10 \angle -90^\circ} = 20.7 \angle 165^\circ \text{ A}$$

The Two Wattmeter

The Two Wattmeter method E_{21}, E_{2N}

The active power delivered to a 3-ph three wire, λ or Δ connected balanced or unbalanced load can be found using only two wattmeters if the proper connection is used and if the wattmeters readings are properly interpreted.



$$W_1 = V_{AC} \cdot I_A \cdot \cos(\psi - 30^\circ)$$

$$W_2 = V_{BC} \cdot I_B \cdot \cos(\psi + 30^\circ)$$

$$W_1 + W_2 = V_{AC} \cdot I_A \cdot \cos(\psi - 30^\circ) + V_{BC} \cdot I_B \cdot \cos(\psi + 30^\circ)$$

$$= V_L \cdot I_L \cdot [\cos(\psi - 30^\circ) + \cos(\psi + 30^\circ)]$$

$$= V_L \cdot I_L \cdot [\cos\psi \cos 30^\circ + \sin\psi \sin 30^\circ + \cos\psi \cos 30^\circ - \sin\psi \sin 30^\circ]$$

$$= 2 V_L \cdot I_L \cdot \cos\psi \cos 30^\circ$$

$$W_1 + W_2 = 2 V_L \cdot I_L \cdot \cos\psi \cos 30^\circ$$

$$W_1 + W_2 = \sqrt{3} V_L \cdot I_L \cos\psi$$

which means that the active power delivered to the load is equal to the sum of the two wattmeter readings.

* When $\psi = 0$ (resistive load) $W_1 = W_2$

* When $\psi = 90^\circ$ (pure inductive or capacitive load) $W_1 = -W_2$

$$* \quad W_1 - W_2 = V_L \cdot I_L [\cos 30^\circ \cos\psi + \sin 30^\circ \sin\psi - \cos 30^\circ \cos\psi + \sin 30^\circ \sin\psi]$$

$$= 2 V_L I_L \sin 30^\circ \sin\psi$$

$$W_1 - W_2 = V_L I_L \sin\psi$$

$$\therefore \sqrt{3}(W_1 - W_2) = \sqrt{3} V_L I_L \sin \phi = Q.$$

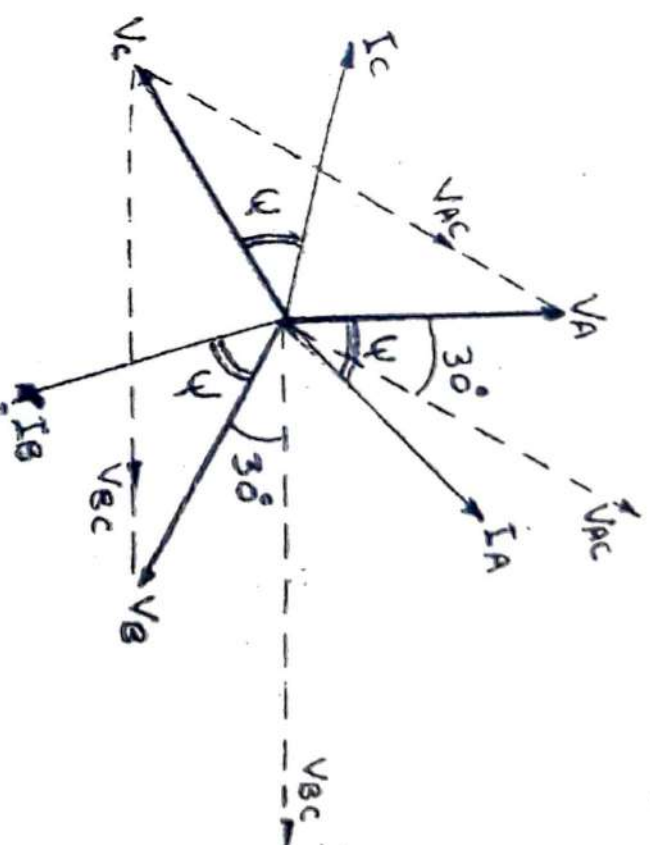
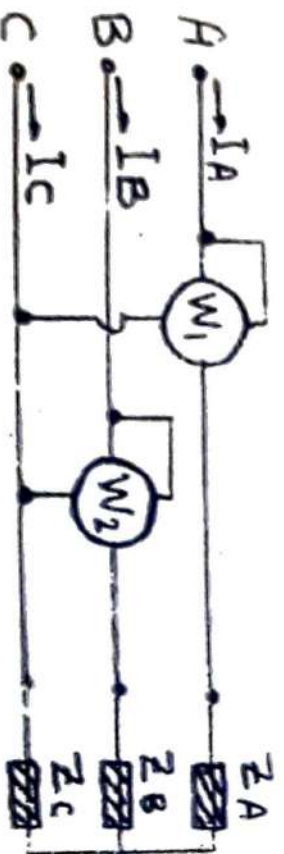
$$\tan \phi = \frac{Q}{P} = \sqrt{3} \frac{W_1 - W_2}{W_1 + W_2}$$

Note: This equation used only for
Balanced load.

The Two Wattmeter

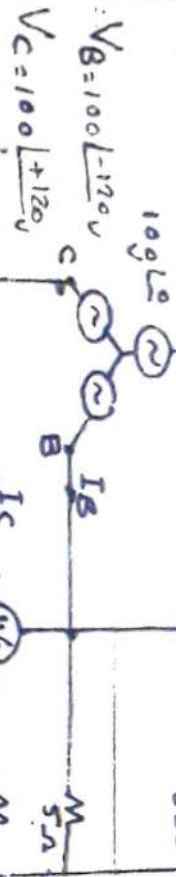
The Two Wattmeter method E_{21}, E_{24}

The active power delivered to a 3-ph three wire, λ or Δ connected balanced or unbalanced load can be found using only two wattmeters if the proper connection is used and if the wattmeters readings are properly interpreted.



Example

For the circuit shown. Find the reading of W_1 & W_2 ?



$$V_B = 100 \angle -120^\circ \text{ V}$$

$$V_C = 100 \angle +120^\circ \text{ V}$$

$$I_A = \frac{V_A}{5\Omega} = \frac{100 \angle 0^\circ}{5} = 20 \angle 0^\circ \text{ A}$$

$$I_B = 20 \angle -120^\circ \text{ A}, \quad I_C = 20 \angle +120^\circ \text{ A}$$

$$W_1 = V_{AB} \cdot I_A \cdot \cos 4^\circ$$

$$= \sqrt{3} \times 100 \times 20 \cos 30^\circ = 3000 \text{ Watts}$$

$$W_2 = V_{CB} \cdot I_C \cdot \cos 4^\circ$$

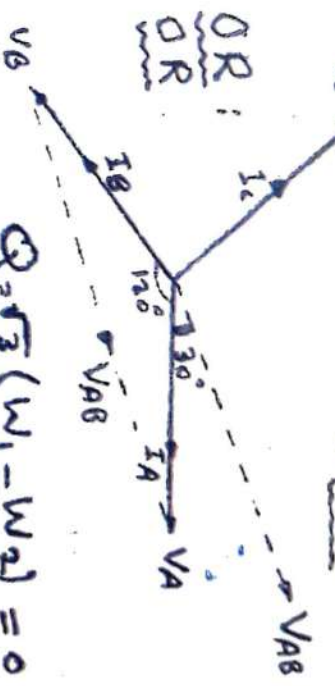
$$= \sqrt{3} \times 100 \times 20 \cos 30^\circ = 3000 \text{ Watts}$$

$$P_T = W_1 + W_2 = 6000 \text{ W}$$

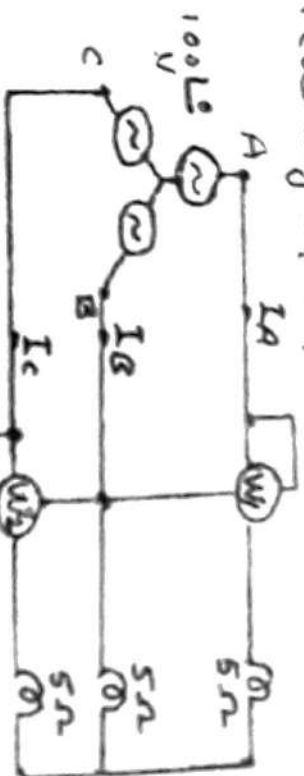
$$P_T = 3 I_L^2 R = 3 \times (20)^2 \times 5 = 6000 \text{ W}$$

$$P_T = \sqrt{3} \cdot V_L \cdot I_L \cos 4^\circ = \sqrt{3} \times \sqrt{3} \times 100 \times 20 \times \cos 0^\circ = 6000 \text{ W}$$

$$Q = \sqrt{3} (W_1 - W_2) = 0 \quad (\text{why})$$



Example
Find the reading of W_1, W_2 for the circuit



$$I_A = \frac{V_A}{j5} = \frac{100 \angle 0^\circ}{5 \angle 90^\circ} = 20 \angle -90^\circ \text{ A-rp}$$

$$I_B = \frac{V_B}{j5} = \frac{100 \angle -120^\circ}{5 \angle 90^\circ} = 20 \angle -210^\circ \text{ A-rp}$$

$$I_C = \frac{V_C}{j5} = 20 \angle 30^\circ \text{ A-rp}$$

$$W_1 = V_{AB} \cdot I_A \cos 4^\circ$$

$$= \sqrt{3} \times 100 \times 20 \cos 120^\circ = \underline{\underline{-1732 \text{ W}}}$$

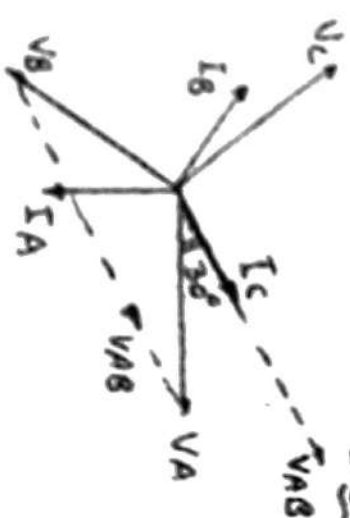
$$W_2 = V_{CB} \cdot I_C \cos 4^\circ$$

$$= \sqrt{3} \times 100 \times 20 \cos 60^\circ = \underline{\underline{1732 \text{ W}}}$$

$$P_T = W_1 + W_2 = 0 \text{ (why)}$$

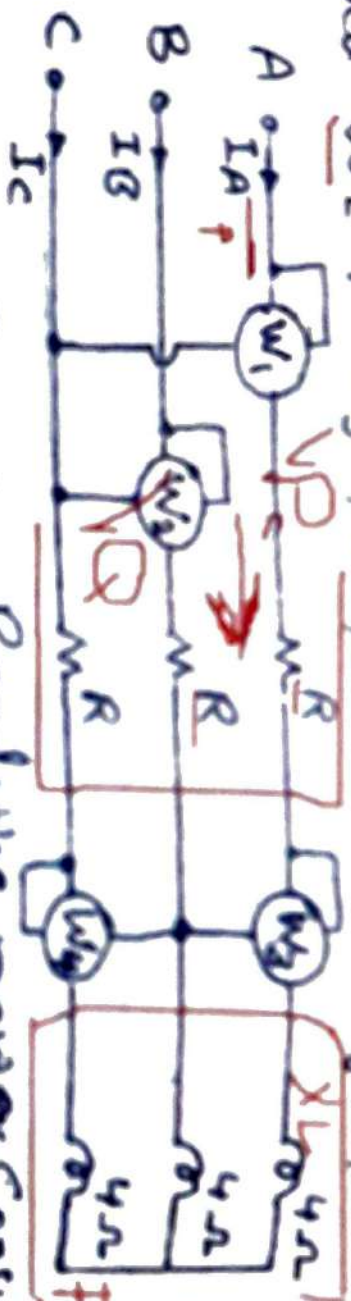
$$Q_T = \sqrt{3} (W_1 - W_2) = \underline{\underline{6000 \text{ VAR}}}$$

$$\text{OR: } Q_T = \sqrt{3} V_L I_C \sin 4^\circ = \sqrt{3} \cdot \sqrt{3} \cdot 100 \times 20 \times \sin 90^\circ = \underline{\underline{6000 \text{ VAR}}}$$



Example 4

For the circuit shown, $W_1 = 12745 \text{ W}$, $W_4 = 5541 \text{ W}$
Find W_2 & W_3 , R , line voltage?



Since W_3 & W_4 Read the power consumed by 4Ω . $\therefore W_3 + W_4 = 0 = P_T$

$$W_3 = -5542 \text{ W}$$

$$P_T = I^2 R$$

Since $(W_1 + W_2)$ & $(W_3 + W_4)$ read the same reactive power (Q) of the load. $P_T = W_1 + W_2 = 0$

$$\therefore \sqrt{3}(W_1 - W_2) = \sqrt{3}(W_3 - W_4) = Q_T$$

$$\therefore W_2 = 1661 \text{ W}$$

$$P = 0 \text{ W}$$

$$Q = \sqrt{3}(W_3 - W_4)$$

3-4

$$* Q_T = \sqrt{3}(W_1 - W_2) = \sqrt{3}(W_2 - W_4) = \underline{19198 \text{ VAR}}$$

$$\therefore Q_{Ph} = 19198/3 = 6399.35 = I_L^2 \cdot X_L$$

$$\therefore I_L = I_A = I_B = I_C = \underline{40 \text{ A rms}}$$

$$Q_{1\phi} = \underline{\quad\quad\quad}$$

$$* P_T = W_1 + W_2 = 14406 \text{ watts}$$

$$P_{Ph} = 14406/3 = 4802 = I_L^2 \cdot R$$

$$\therefore R = \frac{4802}{(40)^2} = \underline{3 \Omega} \quad (40)^2 = \tan^{-1}(4/3) =$$

$$* \tan \phi = \frac{Q_T}{P_T} = \frac{X_L}{R} = \frac{4}{3} \quad \therefore \phi = 53.1^\circ$$

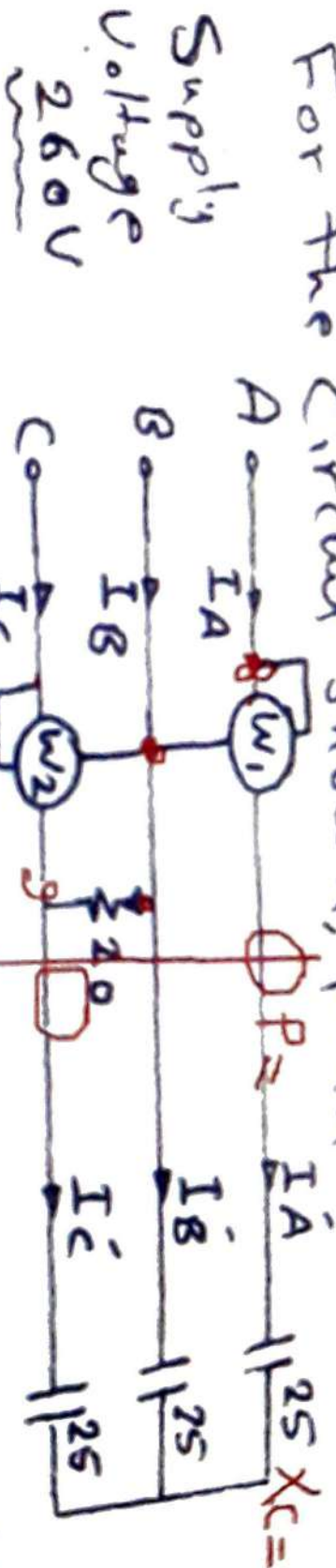
$$P_T = \sqrt{3} V_L I_L \cos \phi$$

$$\therefore \underline{V_L = 346.3 \text{ Volts}}$$

$$P_T = W_1 + W_2$$

Example

For the circuit shown, Find W_1 , ϕ , W_2 .



$$V_A = \frac{260}{\sqrt{3}} = 150 \angle 0^\circ \text{ V}, V_B = 150 \angle -120^\circ \text{ V}, V_C = 150 \angle +120^\circ \text{ V}$$

$$I_A = I_A' = \frac{150 \angle 0^\circ}{25 \angle -90^\circ} = 6 \angle 90^\circ \text{ A}$$

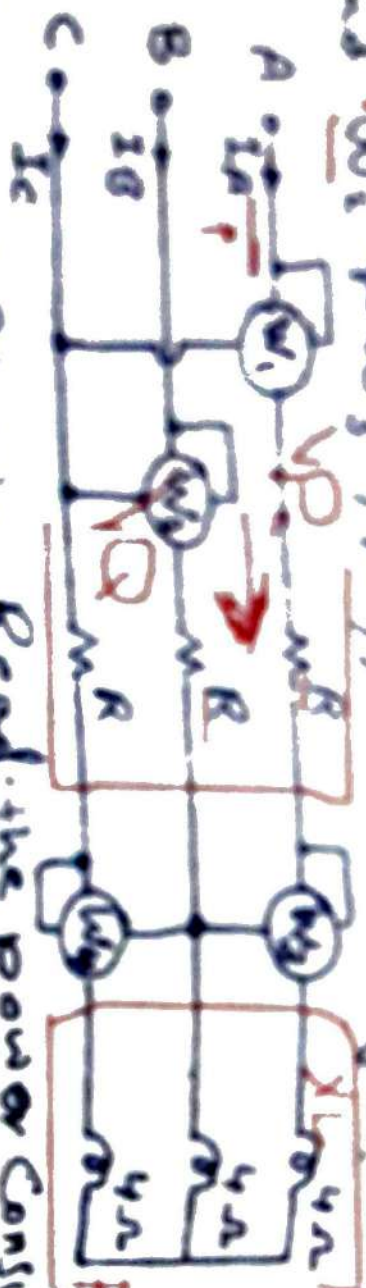
$$I_B = \frac{150 \angle -120^\circ}{25 \angle -90^\circ} = 6 \angle -30^\circ \text{ A}, I_C' = 6 \angle 210^\circ \text{ A}$$

$$I_{BC} = \frac{V_{BC}}{20} = \frac{260 \angle -90^\circ}{20} = 13 \angle -90^\circ \text{ A}$$

$$I_B = I_B + I_{BC} = 6 \angle -30^\circ + 13 \angle -90^\circ = 16.82 \angle -72^\circ \text{ A}$$

$$I_C = I_C' - I_{BC} = 6 \angle 210^\circ - 13 \angle -90^\circ = 11.27 \angle 117.46^\circ \text{ A}$$

Example 2
For the circuit shown, $W_1 = 12745 \text{ W}$, $W_2 = 5542 \text{ W}$
Find W_3 , Q , W_3' , R , ϕ , line voltage?



$P = 0 \text{ W}$
 $Q = \sqrt{3} (W_3' - W_1)$

Since W_3 & W_4 Read the power consumed
by 4Ω : $W_3 + W_4 = 0 = P_T$

$W_3 = -5542 \text{ W}$

$P = I^2 R$

Since $(W_1 + W_2) + (W_3 + W_4)$ read the
same reactive power (Q) of the load. $P_T = W_1 + W_2 = 0$

$\therefore \sqrt{3} (W_1 - W_2) = \sqrt{3} (W_3 - W_4) = Q_T$

$\therefore W_2 = 1661 \text{ W}$

$\times (-1)$

3-4

$$* Q_T = \sqrt{3}(W_1 - W_2) = \sqrt{3}(W_2 - W_4) = 19198 \text{ VAR}$$

$$\therefore Q_{Ph} = 19198/3 = 6399.35 = I_L^2 \cdot X_L$$

$$\therefore I_L = I_A = I_B = I_C = 40 \text{ A m.p.}$$

$$Q_{I_B} = \underline{\hspace{2cm}}$$

$$* P_T = W_1 + W_2 = 14406 \text{ watts}$$

$$P_{Ph} = 14406/3 = 4802 = I_L^2 \cdot R$$

$$\therefore R = \frac{4802}{(40)^2} = 3 \Omega \quad (40)^2 \cdot \tan^{-1}(4/3) =$$

$$* \tan \phi = \frac{Q_T}{P_T} = \frac{X_L}{R} = \frac{4}{3}$$

$$\therefore \phi = 53.1^\circ$$

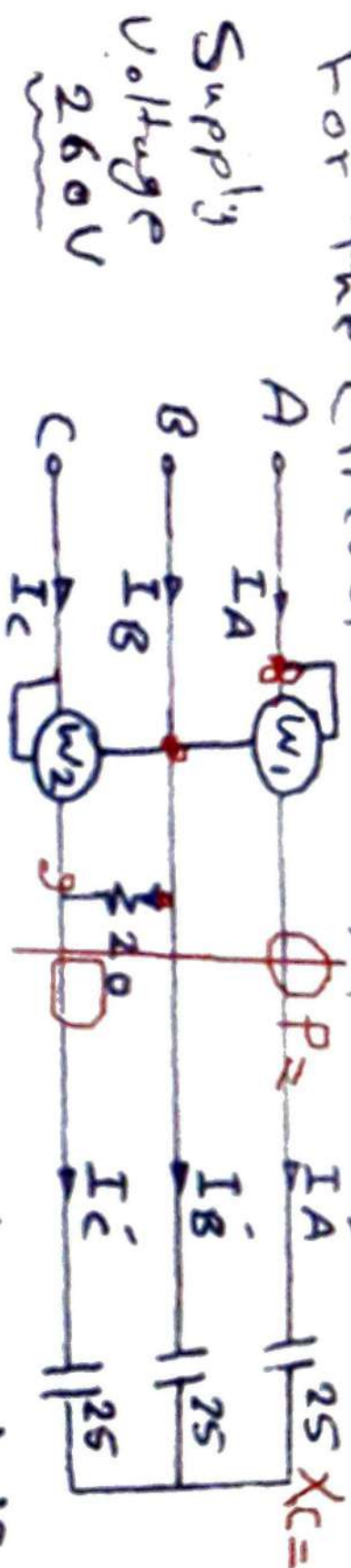
$$P_T = \sqrt{3} V_L I_L \cos \phi$$

$$\therefore V_L = 346.3 \text{ Volts}$$

$$P_T = W_1 + W_2$$

Example

For the circuit shown, Find W_1 , θ , W_2 .



$$V_A = \frac{260}{\sqrt{3}} = 150 \angle 0^\circ \text{ V}, \quad V_B = 150 \angle -120^\circ \text{ V}, \quad V_C = 150 \angle +120^\circ \text{ V}$$

$$I_A = I_A' = \frac{150 \angle 0^\circ}{25 \angle -90^\circ} = 6 \angle 90^\circ \text{ A}$$

$$I_B = \frac{150 \angle -120^\circ}{25 \angle -90^\circ} = 6 \angle -30^\circ \text{ A}, \quad I_C' = 6 \angle 210^\circ \text{ A}$$

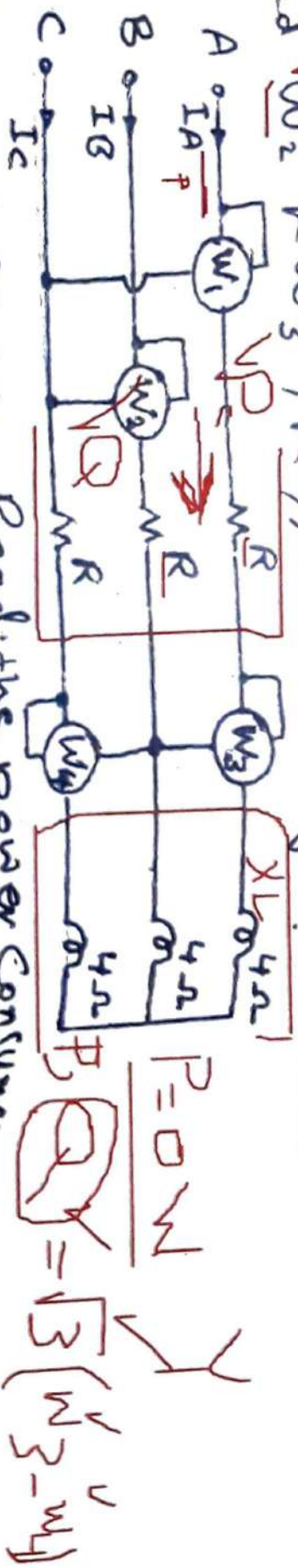
$$I_{BC} = \frac{V_{BC}}{20} = \frac{260 \angle -90^\circ}{20} = 13 \angle -90^\circ \text{ A}$$

$$I_B = I_B + I_{BC} = 6 \angle -30^\circ + 13 \angle -90^\circ = 16.82 \angle -72^\circ \text{ A}$$

$$I_C = I_C' - I_{BC} = 6 \angle 210^\circ - 13 \angle -90^\circ = 11.27 \angle 117.46^\circ \text{ A}$$

Example 6

For the circuit shown, $W_1 = 12745 \text{ W}$, $W_4 = 5542 \text{ W}$
Find W_2 & W_3 , R , line voltage?



Since W_3 & W_4 Read the power consumed by 4-Ω. $\therefore W_3 + W_4 = 0 = P_T$

$$W_3 = -5542 \text{ W}$$

$$P_T = I^2 R$$

Since $(W_1 + W_2) = (W_3 + W_4)$ read the same reactive power (Q) of the load. $P_T = W_1 + W_2 = 0$

$$\therefore \sqrt{3} (W_1 - W_2) = \sqrt{3} (W_3 - W_4) = Q_T$$

$$W_2 = 1661 \text{ W}$$

$$X(-1)$$

3-4

$$P_T = \sqrt{3}(W_1 - W_2) = \sqrt{3}(W_2 - W_1) = 19198 \text{ W}$$

$$Q_{ph} = 19198/3 = 6399.35 = I_R \cdot X_R$$

$$\therefore I_R = I_A = I_B = I_C = 40 \text{ A}$$

$$Q_A = \underline{\hspace{2cm}}$$

$$Q_B = \underline{\hspace{2cm}}$$

$$Q_C = \underline{\hspace{2cm}}$$

$$P_T = W_1 + W_2 = 14406 \text{ watts}$$

$$P_{ph} = 14406/3 = 4802 = I_R \cdot R$$

$$\therefore R = \frac{4802}{(40)^2} = 3 \Omega \quad \tan^{-1}(4/3) =$$

$$\tan \phi = \frac{Q_T}{P_T} = \frac{X_R}{R} = \frac{4}{3}$$

$$\therefore \phi = 53.1^\circ$$

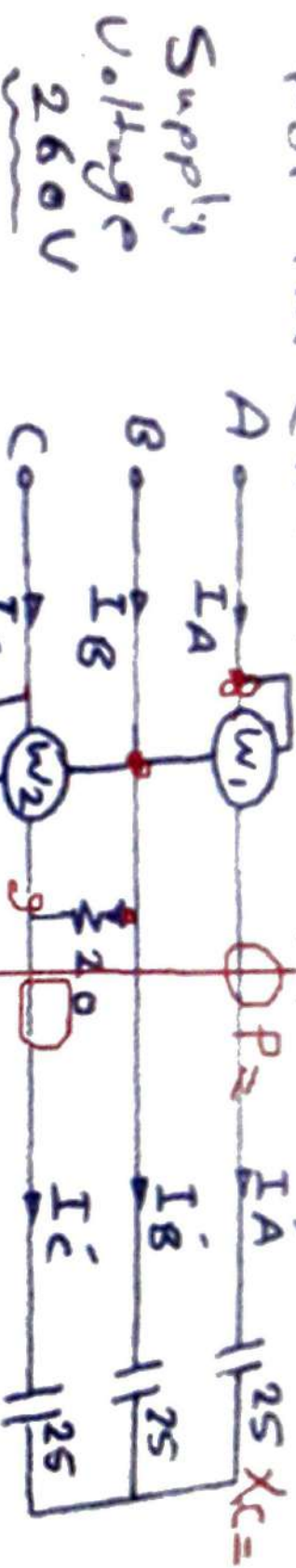
$$P_T = \sqrt{3} V_R I_R \cos \phi$$

$$\therefore V_R = 346.3 \text{ Volts}$$

$$P_T = W_1 + W_2$$

Example

For the circuit shown, Find W_1, W_2



$$V_A = \frac{260}{\sqrt{3}} = 150 \angle 0^\circ \text{ V}, V_B = 150 \angle -120^\circ \text{ V}, V_C = 150 \angle +120^\circ \text{ V}$$

$$I_A = I_A' = \frac{150 \angle 0^\circ}{25 \angle -90^\circ} = 6 \angle 90^\circ \text{ A}$$

$$I_B = \frac{150 \angle -120^\circ}{25 \angle -90^\circ} = 6 \angle -30^\circ \text{ A}, I_C = 6 \angle 210^\circ \text{ A}$$

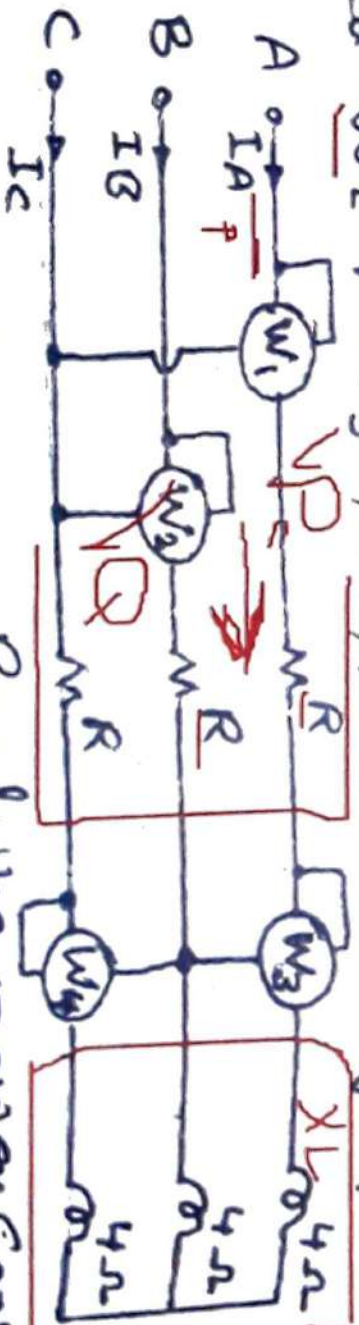
$$I_{BC} = \frac{V_{BC}}{20} = \frac{260 \angle -90^\circ}{20} = 13 \angle -90^\circ \text{ A}$$

$$I_B = I_B + I_{BC} = 6 \angle -30^\circ + 13 \angle -90^\circ = 16.82 \angle -72^\circ \text{ A}$$

$$I_C = I_C - I_{BC} = 6 \angle 210^\circ - 13 \angle -90^\circ = 11.27 \angle 117.46^\circ \text{ A}$$

Example 6

For the circuit shown, $W_1 = 12745 \text{ W}$, $W_4 = 5542 \text{ W}$
Find W_2 & W_3 , R , & line voltage?



Since W_3 & W_4 Read the power consumed
by 4-Ω. $\therefore W_3 + W_4 = 0 = P_T$

$$W_3 = -5542 \text{ W}$$

Since $(W_1 + W_2) + (W_3 + W_4)$ read the
same reactive power (Q) of the load. $P_T = W_1 + W_2 = 0$

$$\therefore \sqrt{3}(W_1 - W_2) = \sqrt{3}(W_3 - W_4) = Q_T$$

$$W_2 = 1661 \text{ W}$$

$$P = 0 \text{ W}$$

$$Q = \sqrt{3}(W_3 - W_4)$$

$$P = I^2 R$$

3-Q

$$* Q_T = \sqrt{3}(W_1 - W_2) = \sqrt{3}(W_3 - W_4) = \underline{19198 \text{ VAR}}$$

$$\therefore Q_{Ph} = 19198/3 = \underline{6399.35 = I_L^2 \cdot X_L}$$

$$\therefore I_L = I_A = I_B = I_C = \underline{40 \text{ A rms}}$$

Q_{I_B} Q_{I_C} Q_A

$$* P_T = W_1 + W_2 = 14406 \text{ watts}$$

$$P_{Ph} = 14406/3 = 4802 = I_L^2 \cdot R$$

$$\therefore R = \frac{4802}{(40)^2} = \underline{3 \Omega} \quad (40)^2 \quad \tan(71.3) =$$

$$* \tan \phi = \frac{Q_T}{P_T} = \frac{X_L}{R} = \frac{4}{3} \quad \therefore \phi = 53.1^\circ$$

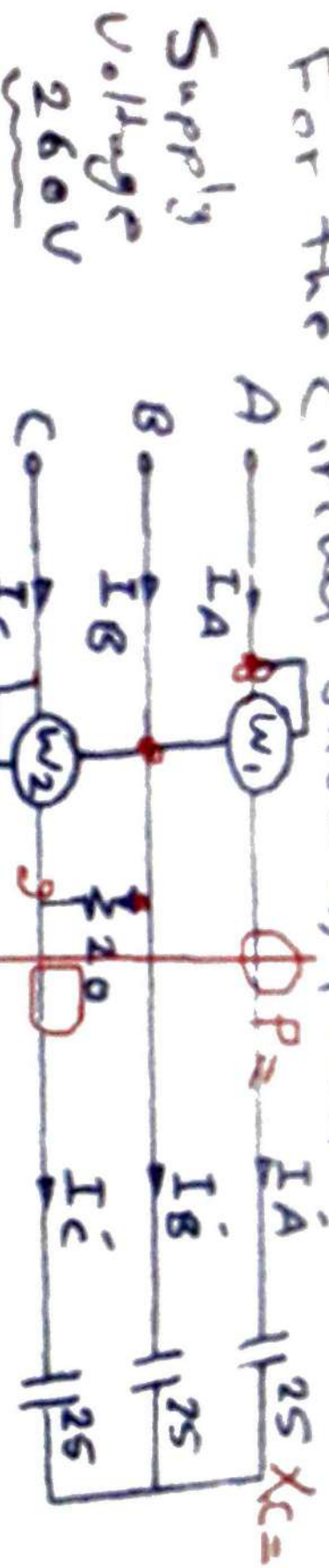
$$P_T = \sqrt{3} V_L I_L \cos \phi$$

$$P_T = W_1 + W_2$$

$$\therefore V_L = \underline{346.3 \text{ Volts}}$$

Example

For the circuit shown, find W_1 , W_2



$$V_A = \frac{260}{\sqrt{3}} = 150 \angle 0^\circ \text{ V}, \quad V_B = 150 \angle -120^\circ \text{ V}, \quad V_C = 150 \angle +120^\circ \text{ V}$$

$$I_A = I_A' = \frac{150 \angle 0^\circ}{25 \angle -90^\circ} = 6 \angle 90^\circ \text{ A}$$

$$I_B = \frac{150 \angle -120^\circ}{25 \angle -90^\circ} = 6 \angle -30^\circ \text{ A}, \quad I_C' = 6 \angle 210^\circ \text{ A}$$

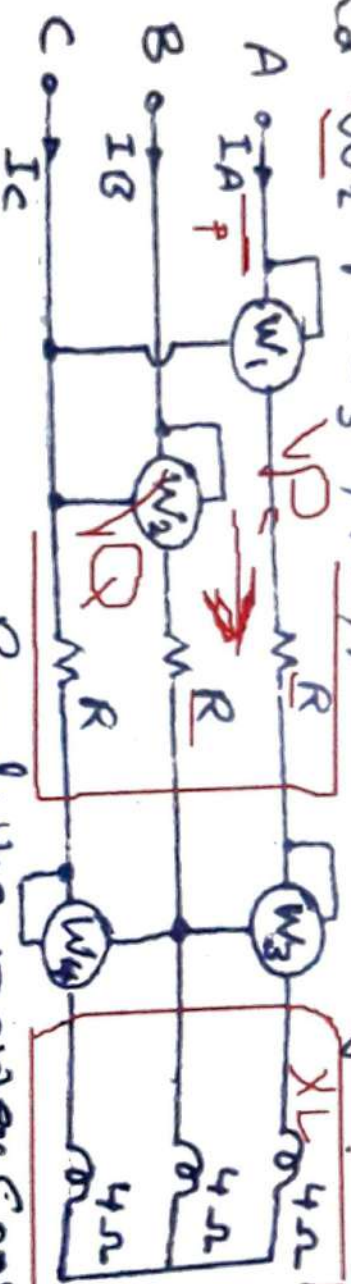
$$I_{BC} = \frac{V_{BC}}{20} = \frac{260 \angle -90^\circ}{20} = 13 \angle -90^\circ \text{ A}$$

$$I_B = I_B + I_{BC} = 6 \angle -30^\circ + 13 \angle -90^\circ = 16.82 \angle -72^\circ \text{ A}$$

$$I_C = I_C' - I_{BC} = 6 \angle 210^\circ - 13 \angle -90^\circ = 11.27 \angle 117.46^\circ \text{ A}$$

Example 6

For the circuit shown, $W_1 = 12745 \text{ W}$, $W_4 = 5542 \text{ W}$
Find W_2 & W_3 , R , line voltage?



Since W_3 & W_4 Read the power consumed
by 4.52. $\therefore W_3 + W_4 = 0 \therefore = P$

$$W_3 = -5542 \text{ W}$$

Since $(W_1 + W_2) + (W_3 + W_4)$ read the

same reactive power (Q) of the load. $P_T = W_1 + W_2 = 0$

$$\therefore \sqrt{3}(W_1 - W_2) = \sqrt{3}(W_3 - W_4) = Q_T$$

$$\therefore W_2 = 1661 \text{ W}$$

$$X(-1)$$

$$P = I^2 R$$

$$P = 0 \text{ W}$$

$$Q = \sqrt{3}(W_3 - W_4)$$

3-~~Q~~

$$* Q_T = \sqrt{3}(W_1 - W_2) = \sqrt{3}(W_3 - W_4) = \underline{19198 \text{ VAR}}$$

Q_A

$$\therefore Q_{Ph} = 19198/3 = 6399.35 = I_L^2 \cdot X_L$$

$$\therefore I_L = I_A = I_B = I_C = \underline{40 \text{ A rms}}$$

Q_{TB}
Q_{TC}

$$* P_T = W_1 + W_2 = 14406 \text{ watts}$$

$$P_{Ph} = 14406/3 = 4802 = I_L^2 \cdot R$$

$$\therefore R = \frac{4802}{(40)^2} = \underline{3 \Omega}$$

$$(40)^2 \cdot \tan^{-1}(4/3) =$$

$$* \tan \phi = \frac{Q_T}{P_T} = \frac{X_L}{R} = \frac{4}{3}$$

$$\therefore \phi = 53.1^\circ$$

$$P_T = \sqrt{3} V_L I_L \cos \phi$$

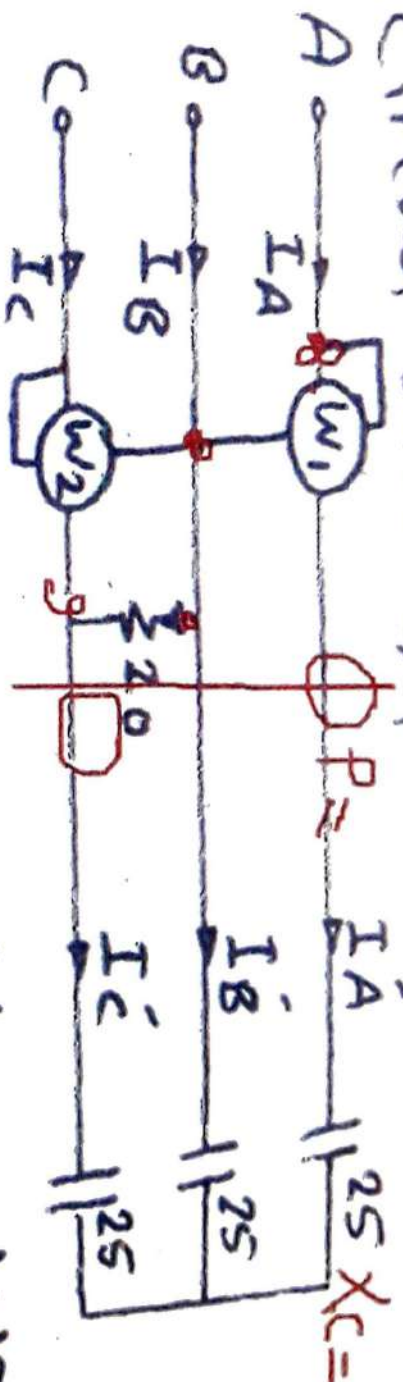
$$\therefore V_L = 346.3 \text{ Volts}$$

$$P_T = W_1 + W_2$$

Example

For the circuit shown, Find W_1 , W_2

Supply Voltage
260V



$$V_A = \frac{260}{\sqrt{3}} = 150 \angle 0^\circ \text{ V}, V_B = 150 \angle -120^\circ \text{ V}, V_C = 150 \angle +120^\circ \text{ V}$$

$$I_A = I_A' = \frac{150 \angle 0^\circ}{25 \angle -90^\circ} = 6 \angle 90^\circ \text{ A}$$

$$I_B = \frac{150 \angle -120^\circ}{25 \angle -90^\circ} = 6 \angle -30^\circ \text{ A}, I_C' = 6 \angle 210^\circ \text{ A}$$

$$I_{BC} = \frac{V_{BC}}{20} = \frac{260 \angle -90^\circ}{20} = 13 \angle -90^\circ \text{ A}$$

$$I_B = I_B + I_{BC} = 6 \angle -30^\circ + 13 \angle -90^\circ = 16.82 \angle -72^\circ \text{ A}$$

$$I_C = I_C' - I_{BC} = 6 \angle 210^\circ - 13 \angle -90^\circ = 11.27 \angle 117.46^\circ \text{ A}$$

$$W_1 = I_A V_{AB} \cos \psi_1$$

$$= 6 \times 260 \cos 60 = \underline{780 \text{ W}}$$

$$W_2 = I_C V_{CB} \cos \psi_2$$

$$= 11.27 \times 260 \cos 27.46$$

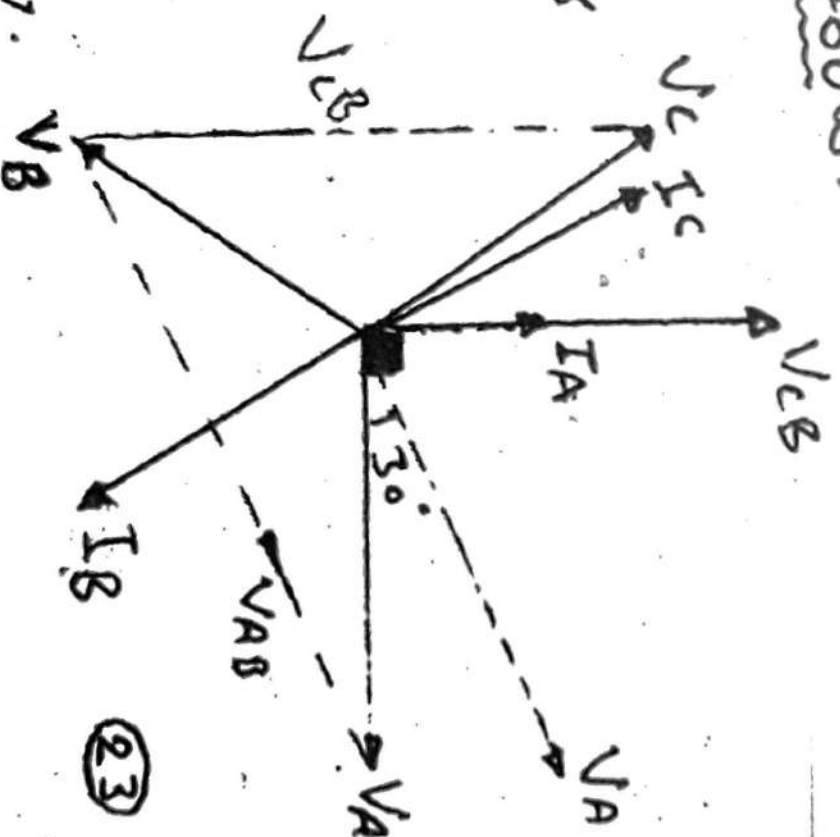
$$= \underline{2600 \text{ W}}$$

$$P_T = W_1 + W_2$$

$$= 3380 \text{ W}$$

$$\text{Check } P_T = P_{20\Omega}$$

$$= (13)^2 \times 20 = 3380 \text{ W}$$

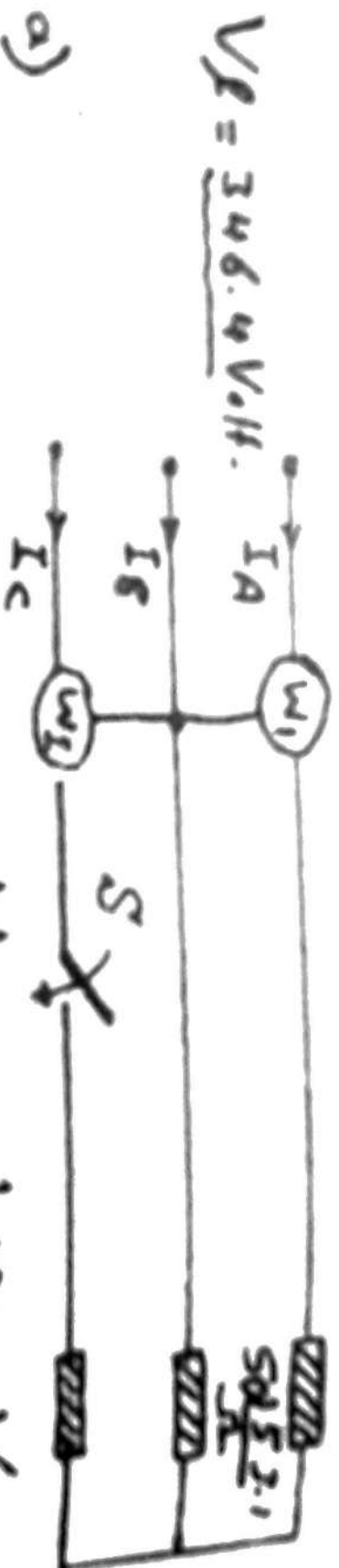


(23)

Example

For the circuit, find W_1 & W_2 when:

a) Switch (S) closed. b) Switch (S) open.



a) $V_A = \frac{346.4 \angle 0^\circ}{\sqrt{3}} = 200 \angle 0^\circ$, $V_B = 200 \angle -120^\circ$, $V_C = 200 \angle +120^\circ$

$$I_A = \frac{V_A}{Z} = \frac{200 \angle 0^\circ}{50 \angle 53.1^\circ} = \underline{4 \angle -53.1^\circ} \text{ A}$$

$$\therefore I_B = 4 \angle -173.1^\circ \text{ A}, \quad \underline{I_C = 4 \angle 66.9^\circ} \text{ A}$$

$$W_1 = I_A V_{AB} \cos(30^\circ + 53.1^\circ) \\ = 4 \times 346.4 \cos 83.1^\circ = \underline{166.46 \text{ W}}$$

$$W_2 = I_C V_{CB} \cos(90^\circ - 66.9^\circ)$$

b) S is open.

$$\therefore W_2 = 0 \quad (I_C = 0).$$

$$\therefore I_A = \frac{V_{AB}}{60 + j80} = \frac{346.4 \angle 130^\circ}{100 \angle 53.1^\circ} = \underline{\underline{3.464 \angle -23.1^\circ \text{ A}}}.$$

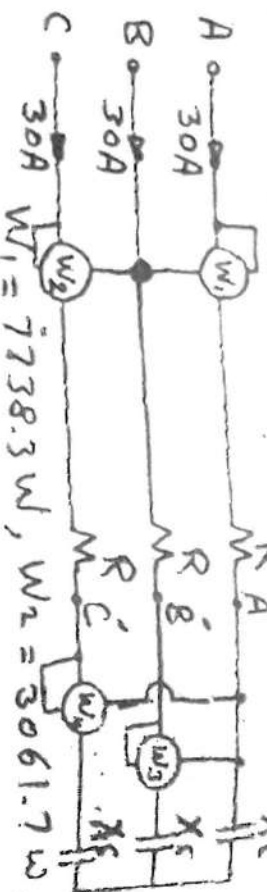
$$W_1 = I_A V_{AB} \cos(30^\circ + 23.1^\circ).$$

$$= 3.464 \times 346.4 \cos 53.1^\circ \\ = \underline{\underline{720 \text{ W}}}$$

$$\text{check} \quad P = I_A^2 \times (30 + 30)$$

$$= (3.464)^2 \times 60 = 720 \text{ W}$$

Example: For the circuit, find the value of $(R \text{ or } X_c)$ then the reading of W_3 or W_4 .



Ans: $\tan \psi = \sqrt{3} \frac{(W_1 - W_2)}{(W_1 + W_2)} = \sqrt{3} \frac{4676.6}{10800} = 0.75 = \frac{X_c}{R}$

$\psi = 36.86^\circ$

$P_T = W_1 + W_2 = 10800 = \sqrt{3} \times V_L \times I_L \cos \psi = \sqrt{3} \times V_L \times 30 \cos 36.86^\circ$

$\therefore V_L = 259.8 \text{ Volt} \quad \therefore V_{ph} = \frac{259.8}{\sqrt{3}} = 150 \text{ Volt}$

$\therefore V_A = 150 \angle 0^\circ \text{ or } I_A = 30 \angle +36.86^\circ \text{ A}$

$Z_A = Z_B = Z_C = \frac{V_A}{I_A} = \frac{150 \angle 0^\circ}{30 \angle +36.86^\circ} = 5 \angle -36.86^\circ$

$\therefore Z_A = (4 - j3) \Omega \quad \therefore R = 4 \Omega \quad X_c = 3 \Omega$

$\therefore V_A = 30 \angle 36.86^\circ \times 3 \angle -90^\circ = 90 \angle -53.14^\circ \text{ V}$

$\therefore V_B = 90 \angle -173.14^\circ \text{ V}, V_C = 90 \angle 66.86^\circ \text{ V}$

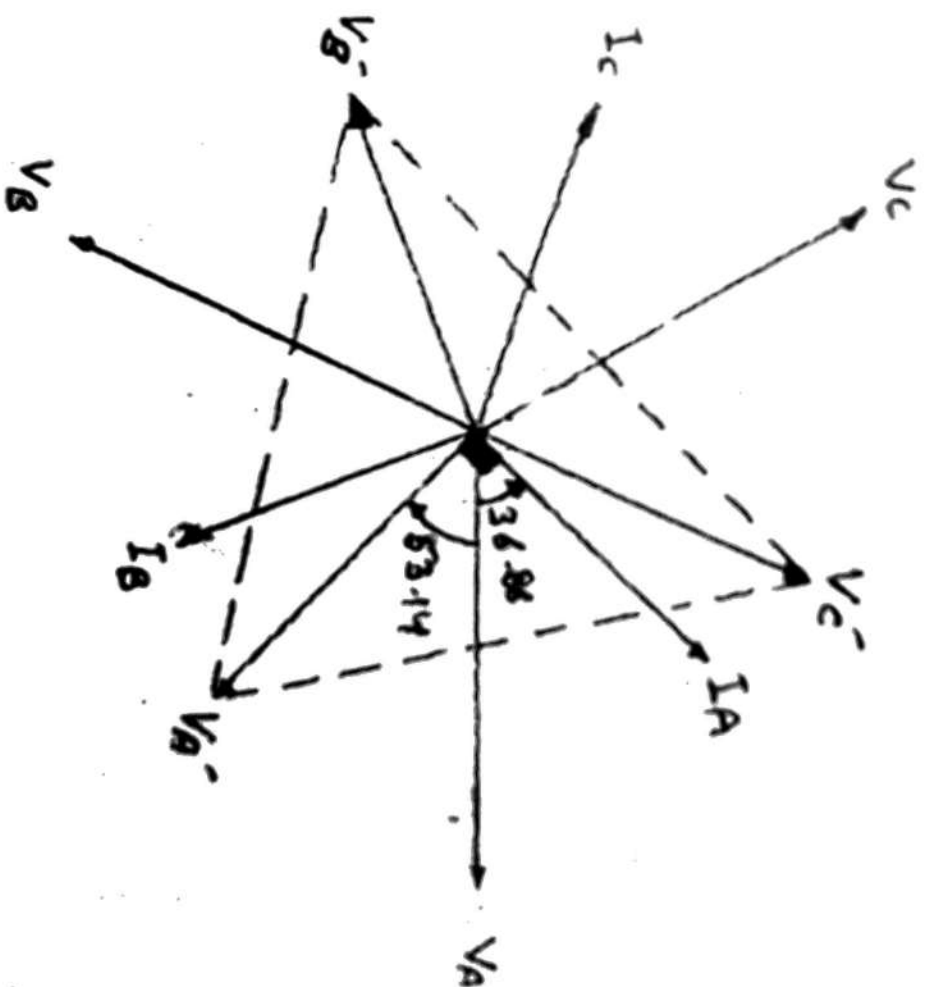
$W_3 = I_B V_B \cos \alpha = \sqrt{3} \times 90 \times 30 \cos 120^\circ = -2338.2 \text{ Watts}$

$W_4 = I_C V_C \cos \gamma = \sqrt{3} \times 90 \times 30 \cos 60^\circ = 2338.2 \text{ Watts}$

$$Q_T = \sqrt{3}(W_1 - W_2) = 8100 \text{ VAR.}$$

Check $Q_T = 3(I_A)^2 \times 3 = 8100 \text{ VAR.}$

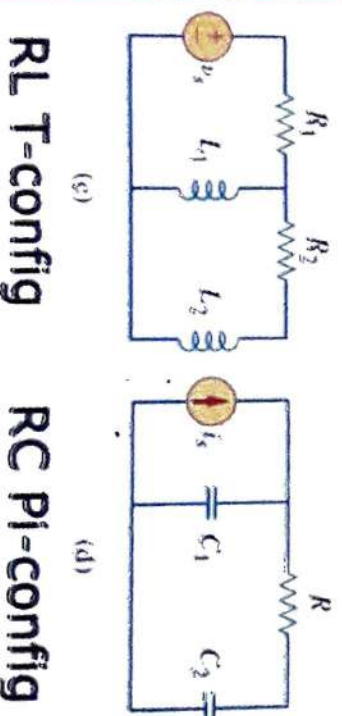
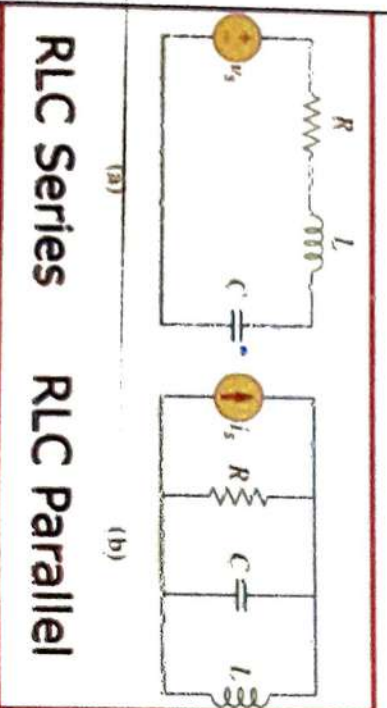
$$I_A = 30 \sqrt{3686}, \quad I_B = 30 \sqrt{15686}, \quad I_C = 30 \sqrt{15686}$$

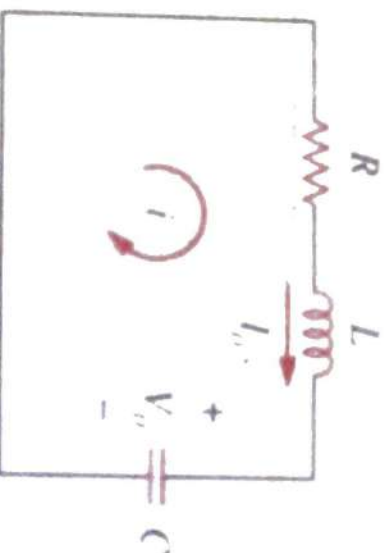


8.1 Second Order RLC circuits (1)

What is a 2nd order circuit?

A second-order circuit is characterized by a second-order differential equation. It consists of resistors and the equivalent of two energy storage elements.





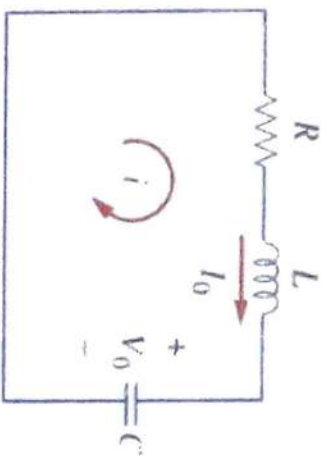
➤ The solution of the source-free series RLC circuit is called as the natural response of the circuit.

➤ The circuit is excited by the energy initially stored in the capacitor and inductor.

The 2nd order
of expression

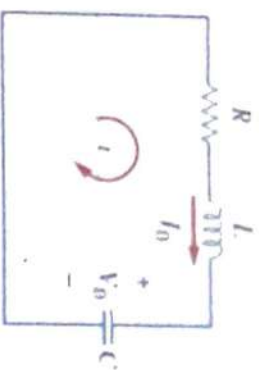
$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = 0$$

How to derive and how to solve?



For Capacitor : $v(0) = v(0^+) = v(0^-) = V_0$

For Inductor : $i(0) = i(0^+) = i(0^-) = I_0$



Initial Conditions $i(0) = I_0$ $v(0) = \frac{1}{C} \int_{-\infty}^0 i dt = V_0$

Apply KVL

$$Ri + L \frac{di}{dt} + \frac{1}{C} \int_{-\infty}^t i dt = 0$$

$$R \frac{di}{dt} + L \frac{d^2 i}{dt^2} + \frac{i}{C} = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = 0$$

To solve such a 2nd order diff ea. We need 2 initial conditions, such as $i(0)$ and $\frac{di(0)}{dt}$ (from $v(0)$)

We get the initial value of the derivative of i from equation after applying KVL; that is,

$$Ri(0) + L \frac{di(0)}{dt} + v(0) = 0$$

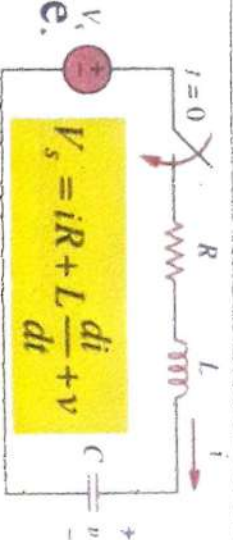
$$RI_0 + L \frac{di(0)}{dt} + V_0 = 0$$

2 Initial Conditions

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0)$$

$$i(0) = I_0$$

The step response is obtained by the sudden application of a dc source.



$$i = C \frac{dv}{dt}$$

$$V_s = RC \frac{dv}{dt} + LC \frac{d^2v}{dt^2} + v$$

The 2nd order of expression

$$\frac{d^2v}{dt^2} + \frac{R}{L} \frac{dv}{dt} + \frac{v}{LC} = \frac{V_s}{LC}$$

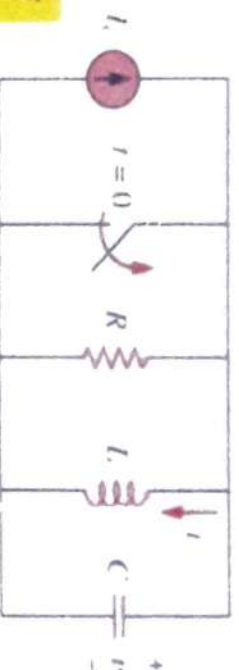
The above equation has the same form as the equation for source-free series RLC circuit.

- The same coefficients (important in determining the frequency parameters).
- Different circuit variable in the equation.

The step response is obtained by the sudden application of a dc source.

$$C \frac{dv}{dt} + \frac{v}{R} + i = I_s$$

$$v = L \frac{di}{dt}$$



$$LC \frac{d^2 i}{dt^2} + \frac{L}{R} \frac{di}{dt} + i = I_s$$

The 2nd order of expression

$$\frac{d^2 i}{dt^2} + \frac{1}{RC} \frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}$$

It has the same form as the equation for source-free parallel RLC circuit.

- The same coefficients (important in determining the frequency parameters).
- Different circuit variable in the equation.