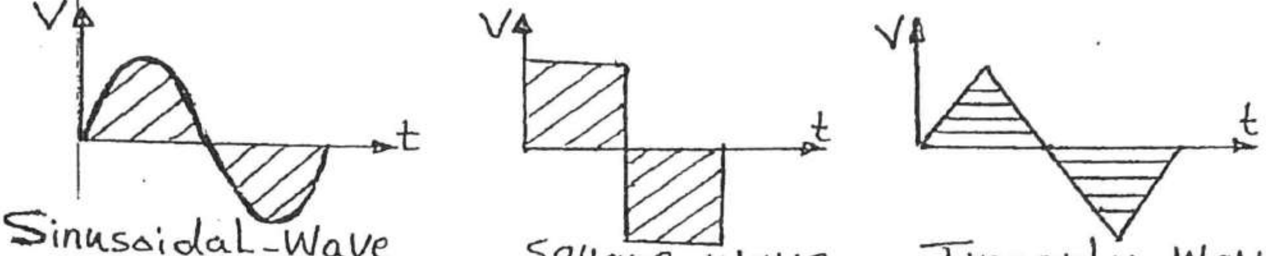
Alternating Current (A-C) Circuits

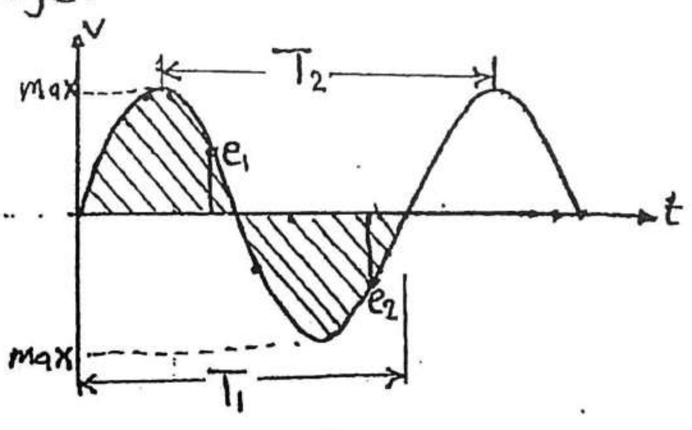
The analysis of networks in which the magnitude of the Source of e.m.f Varies in a set manner



Sinusoidal-Wave   square-wave   Trangular-Wave

The term alternating indicates only that the wave-form alternates between two prescribed Levels, the term Sinusoidal, square, trangular must also be applied. The pattern of particular interest is the Sinusoidal a-c Voltage.

Definitions:



** Wave form : The path traced by a quantity, Such as the e.m.f in fig-above. plotted as a function of some Variable such as time (above), degree, radian, and so on.

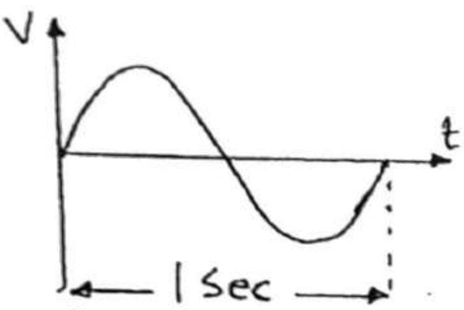
** Instantanous Value: The magnitude of a waveform at any instant of time, denoted by Lower-case letters (e_1 & e_2)

** Amplitude or peak Value: The maximum Value of a Wave form, denoted by Upper-case Letters (max).

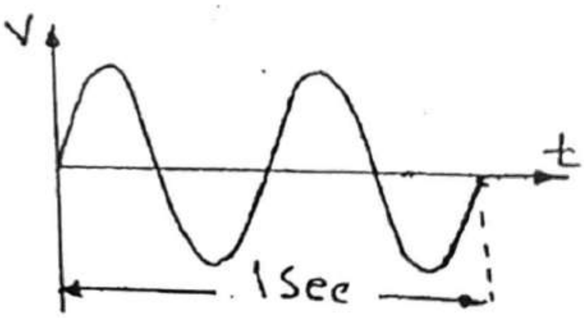
** periodic Waveform: A waveform that continually repeats it self after the same time interval.

** Period (T): The time interval between two successive repetitions of a periodic wave form ($T_1 = T_2$).

** Frequency: The number of cycles that occur in 1 sec. The unit of frequency is cycle/sec (cps) or Hertz (Hz)



$f = 1\,cps = 1\,Hz$ } $T = 1\,sec$



$f = 2\,cps = 2\,Hz$ } $T = \frac{1}{2}\,sec.$

Since the frequency is inversly propotional to the period. The two can be related by the following equation

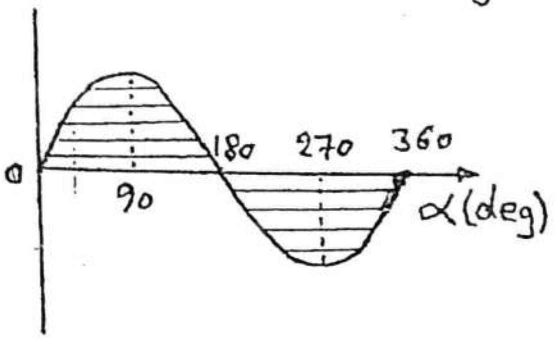
$$\boxed{f = \frac{1}{T}}$$

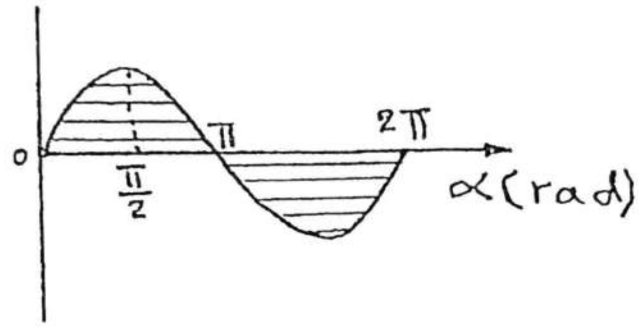
The Sine Wave

The basic mathematical format for sinusoidal wave form is $(A \sin\alpha)$.

Where: A is the max. or peak Value.

α is the unit of measurement of horizontal axis, it may be in degree or radian.





$$2\pi\ (\text{rad}) = 360\ (\text{Deg}).$$

OR

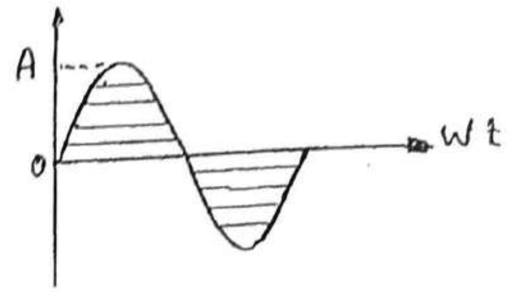
$$\text{Rad} = \frac{\pi}{180}\ \text{Deg}.$$

phase relations

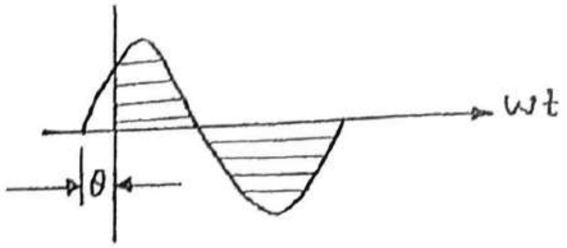
If the wave form is shifted to the right or Left of Zero, the expression becomes:

$A \sin(\omega t \mp \theta)$.

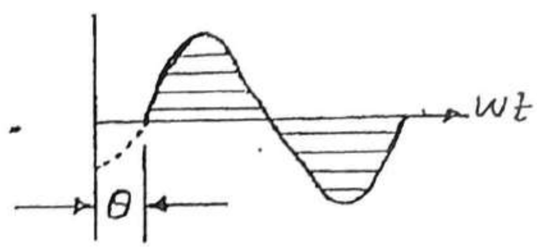
$A \sin \omega t$



$A \sin(\omega t + \theta)$

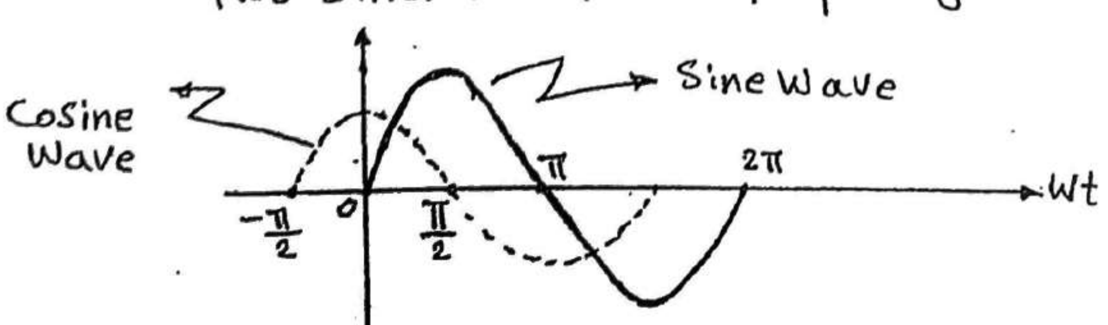


$A \sin(\omega t - \theta)$.



The Cosine wave is said to **Lead** the sine wave by 90°, and the Sine wave is said to **Lag** the Cosine wave by 90°.

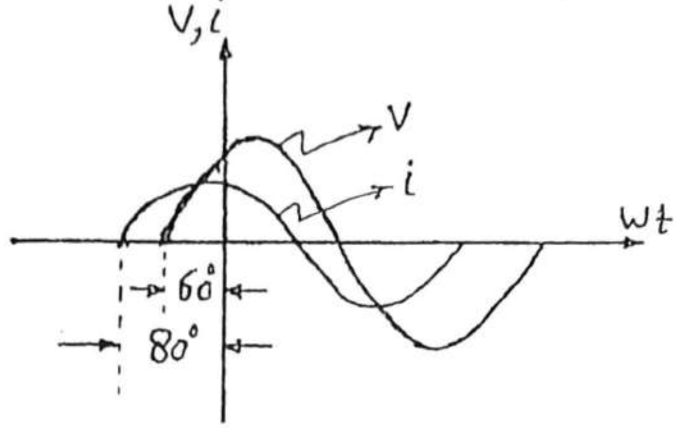
Note:- Lead and Lag indicate the relation ship between two Sinewave of Same frequency.



Ex: What is the phase relationship between V & i.

$V = 10 \sin(\omega t + 60)$.

$i = 5 \sin(\omega t + 80)$.



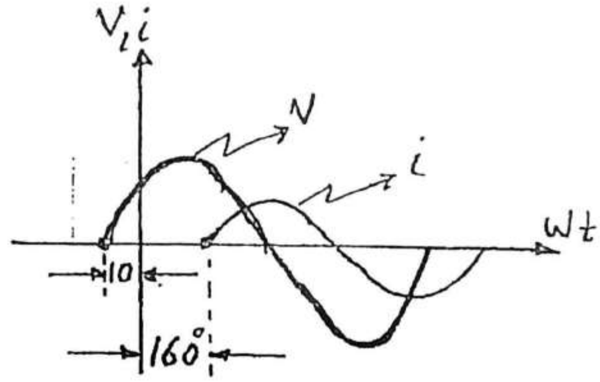
$\therefore$ i Lead V by 20°.

or V Lags i by 20°.

Ex: Find the phase relation between V & i.

$i = -2 \sin(\omega t + 20)$.

$V = 5 \sin(\omega t + 10)$.



$i = -2 \sin(\omega t + 20)$.

$= 2 \sin(\omega t + 20 - 180)$

$= 2 \sin(\omega t - 160)$.

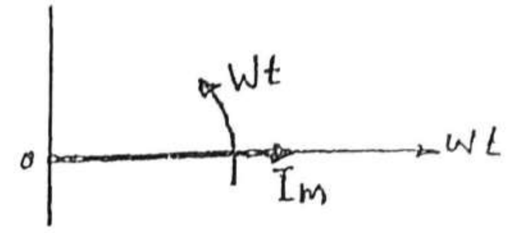
$\therefore$ V Lead i by 170°

or i Lags V by 170°.

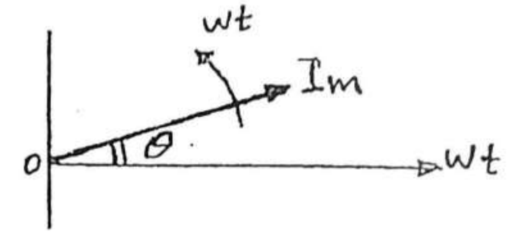
Phasors:-

A rotating radius vector having a constant magnitude (Length) with one end fixed at origin, during its rotation it produces the Sine wave, for example we can express the instantanous value of current (i).

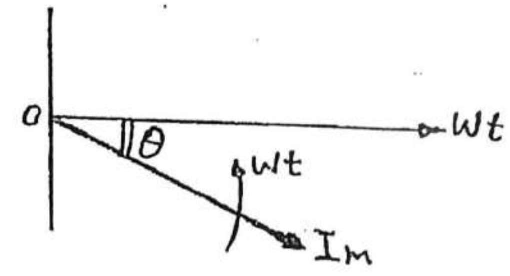
$i = I_m \sin Wt$



$i = I_m \sin(Wt + \theta)$.



$i = I_m \sin(wt - \theta)$.



$$W = \text{Angular Velocity} = \frac{\text{Distance (Deg or Rad)}}{\text{Time}}.$$

$$= \frac{\alpha}{t} = \frac{2\pi}{T}.$$

$$\therefore \quad \boxed{W = 2\pi f} \quad (\text{rad/sec}).$$

Average (Mean) Value

The average value of any current or Voltage is the value indicated on a (d-c) meter.

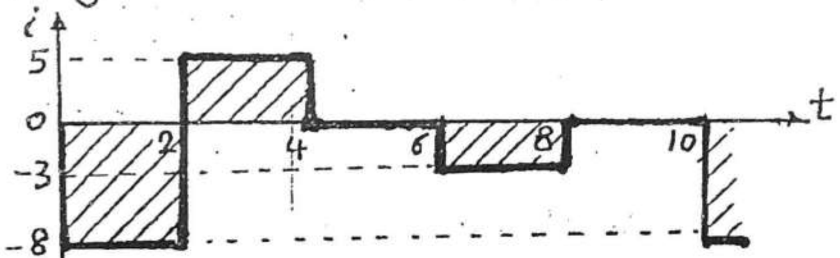
$$I \text{ or } V(\text{average value}) = \frac{\text{algebriac Sum of areas}}{\text{Length of curve}}.$$

OR

$$\boxed{I(\text{average}) = \frac{1}{T}\int_0^T i\,dt.}$$

$$\boxed{V(\text{average}) = \frac{1}{T}\int_0^T V\,dt.}$$

Ex: Find the average value of the following waveform over one full cycle.



$$I(av) = \frac{(-8\times 2)+(5\times 2)+(-3\times 2)}{10} = \frac{-12}{10} = -1.2 \text{ Amp}.$$

OR

$$I(av) = \frac{1}{T}\int_0^T i\,dt$$

$$= \frac{1}{10}\left[\int_0^2 -8\,dt + \int_2^4 5\,dt + \int_4^6 0\,dt + \int_6^8 -3\,dt + \int_8^{10} 0\,dt\right]$$

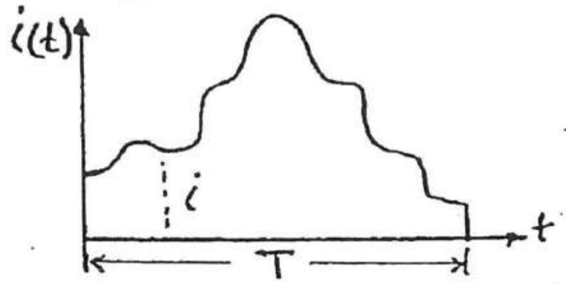
$$= \frac{1}{10}\left[-8\times 2 + 5\times 2 + (-3\times 2)\right] = \frac{-12}{10} = -1.2 \text{ Amp}$$

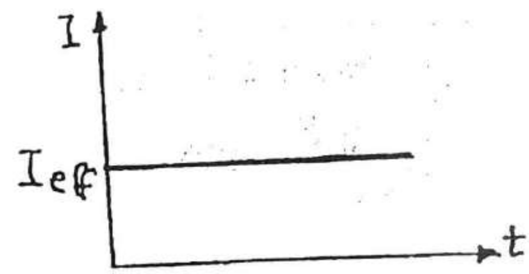
Effective Value (r.m.s).

- The effective value of alternating current is measured interms of the direct constants current that produced the the same heating effect in the same resistance for the same period of time.

$\therefore$ Heat generated by current (I) for time (t) in a resistance (R) is:

$$\text{Heat generated} = I^2 \cdot R \cdot t \quad \text{(Joules)}.$$





** Heat generated by (a-c) $= \int_0^T i^2 \cdot R \cdot dt. \quad (J).$

$$= \underbrace{R\int_0^T i^2 \cdot dt}. \quad (J).$$

** Heat generated by constant current $= \underbrace{I_{eff}^2 \cdot R \cdot T}$

$$\therefore \; I_{eff}^2 \cdot \cancel{R} \cdot T = \int_0^T i^2 \cdot \cancel{R} \cdot dt$$

$$\therefore \; I_{eff}^2 = \frac{1}{T}\int_0^T i^2 \cdot dt.$$

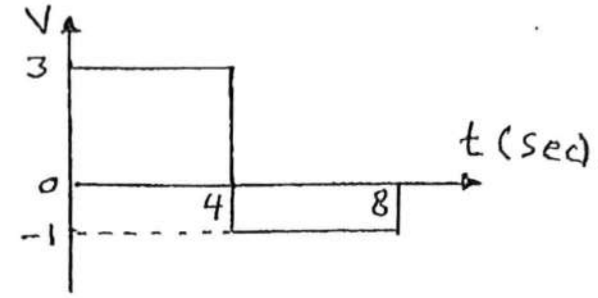
OR

square

$$\boxed{I_{eff} = I_{r.m.s} = \sqrt{\underbrace{\frac{1}{T}\int_0^T}_{\text{mean}} i^2 \cdot dt}}$$

root

Ex: Find the effective value (r.m.s) of the voltage shown.



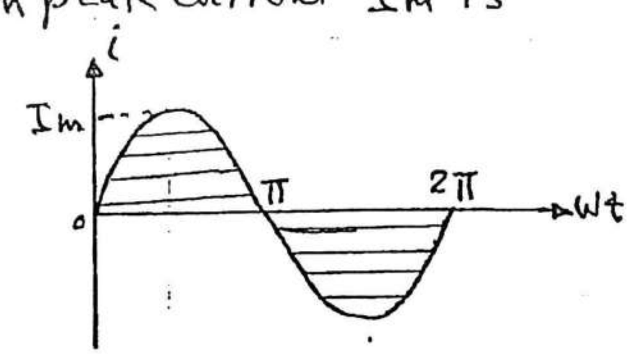
$$V_{eff} = V_{r.m.s} = \sqrt{\frac{1}{T}\int_0^T V^2 \cdot dt}.$$

$$= \sqrt{\frac{1}{8}\left[\int_0^4 3^2 dt + \int_4^8 (-1)^2 dt\right.}$$

$$= \sqrt{5} = 2.23 \text{ Volt}.$$

Ex: Show that the effective value or (r.m.s) of sine wave current with peak current I_m is

$$I_{r.m.s} = \frac{I_m}{\sqrt{2}}$$



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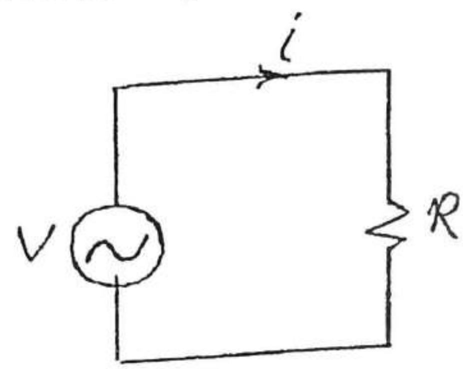
Response of basic R, L and C elements to a Sinusoidal Voltage or Current

To find the response, we can use the ohm's Law and the basic equations for (R, L and C).

① The Resistor (R).



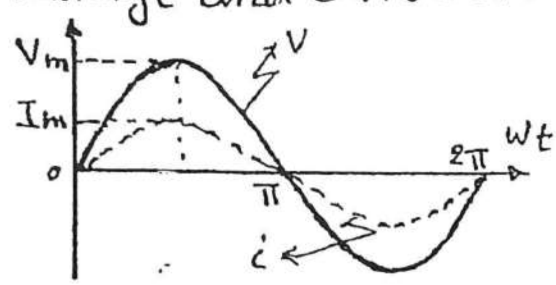
Let $V = V_m \sin \omega t$.

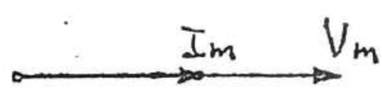
then $i = \frac{V}{R} = \frac{V_m}{R} \sin \omega t$.

$= I_m \sin \omega t$.

Where : $I_m = \frac{V_m}{R}$

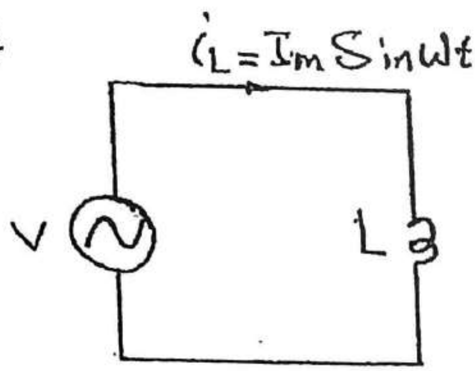
∴ The voltage and current are in phase.





② The Inductor (L).

For the inductor we know that



$$V_L = L \frac{di}{dt}$$

let $i_L = I_m \sin \omega t$.

$\therefore V_L = L \frac{d}{dt}(I_m \sin \omega t)$.

$\therefore V_L = L(\omega I_m \cos \omega t) = \omega L \cdot I_m \cdot \cos \omega t$.

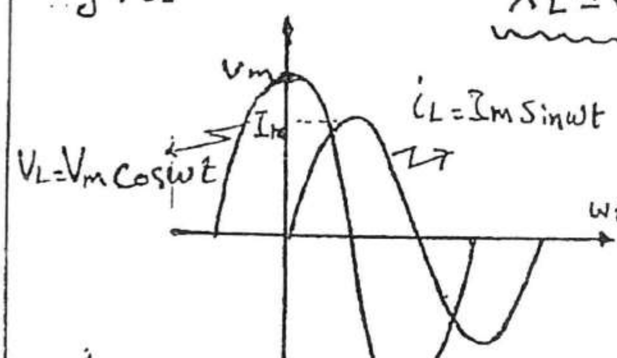
$\therefore V_L = V_m \cos Wt = V_m \sin(Wt + 90^\circ)$

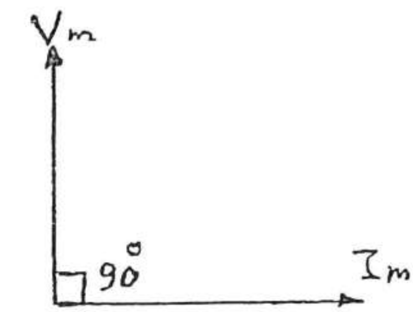
Where $V_m = WL \cdot I_m$.

$$\boxed{\therefore \frac{V_m}{I_m} = WL = 2\pi f L = X_L \ (\Omega)}$$

The quantity (WL) is called the reactance of an inductor and measured in ohm, it is symbolically represented by X_L

$X_L = WL$





From the phasor diagram, shows that V_L is Lead I_L by 90°, or I_L is Lags V_L by 90°.

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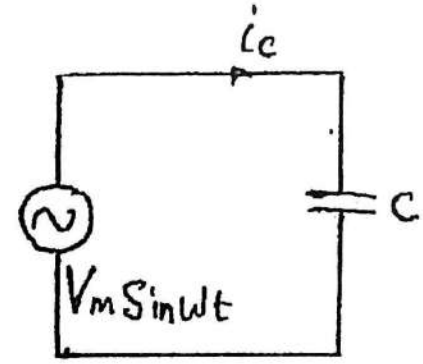
③ The Capacitor (C)

For the Capacitor We have:



$$i_c = C \frac{dV_c}{dt}$$

let $V_c = V_m \sin Wt$.

$$\therefore i_c = C \frac{d}{dt} (V_m \sin Wt).$$

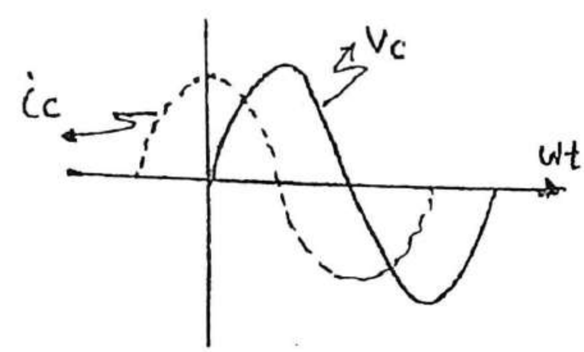
$\therefore \; i_c = C\,(W V_m \cos wt) = W c V_m \cos Wt$.

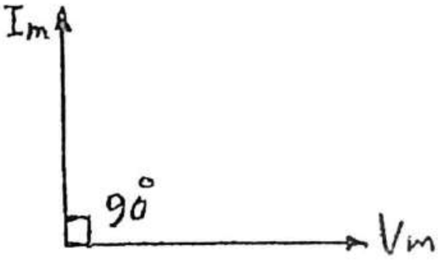
or $\quad i_c = I_m \cos Wt = I_m \sin(Wt + 90^\circ)$.

where, $\quad I_m = W c V_m$.

$$\boxed{\therefore \; \frac{V_m}{I_m} = \frac{1}{Wc} = \frac{1}{2\pi f c} = X_c \quad (\Omega)}$$

The quantity $\left(\frac{1}{Wc}\right)$ is called the reactance of Capacitance and symbolically represented by X_C and measured in ohm.

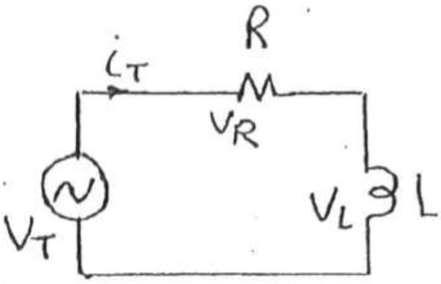




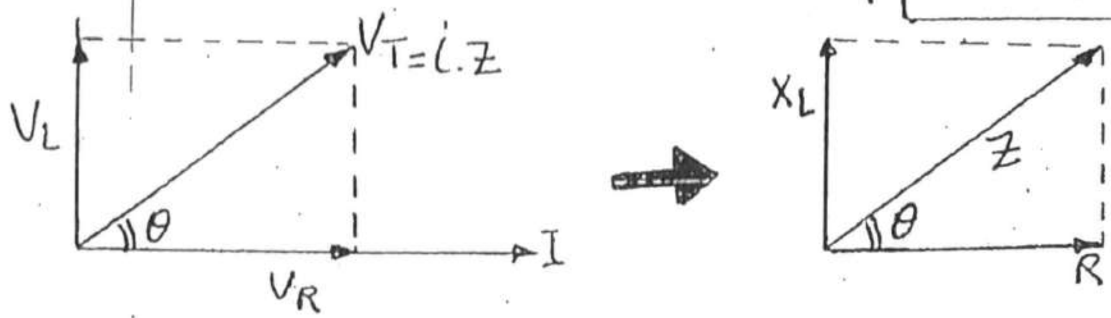
From the phasor diagram, shows that i_c is Lead V_c by 90°, or V_c is Lags i_c by 90°.

R_L and C Connections

① R_L in Series;



Let $i_T = I_m \sin \omega t$.

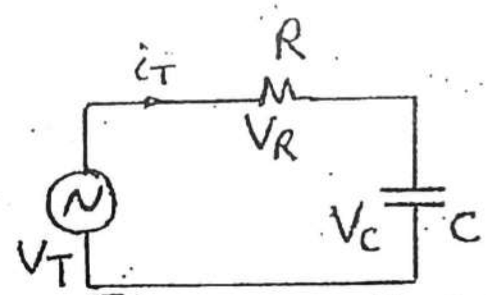


$\therefore |V_T| = \sqrt{V_R^2 + V_L^2}$ and $\theta = \tan^{-1} \frac{V_L}{V_R}$

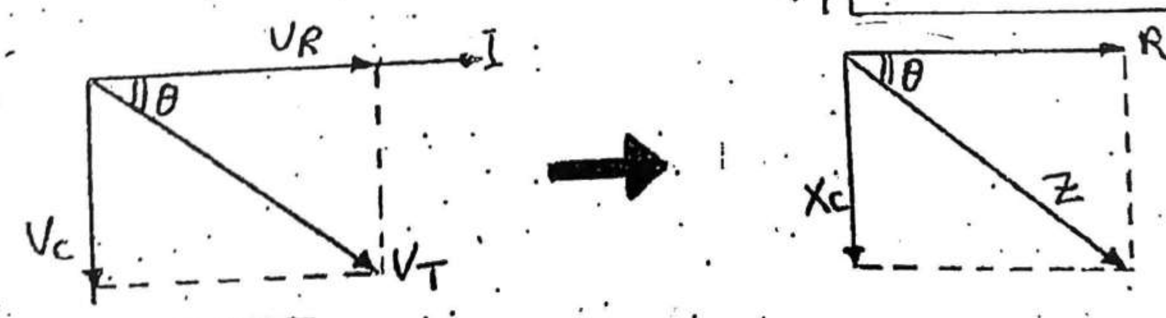
and $Z = \sqrt{R^2 + X_L^2}$. or $\theta = \tan^{-1} \frac{X_L}{R}$.

V_T lead i_T by angle θ, or i_T lags V_T by θ.

② R—C in Series:



Let $i_T = I_m \sin \omega t$.



$V_T = \sqrt{V_R^2 + V_C^2}$

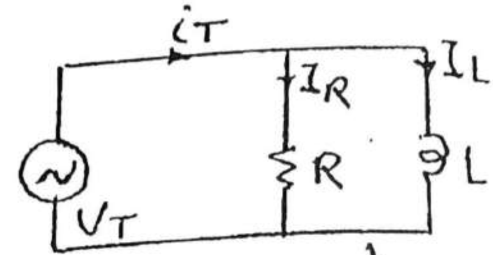
and $Z = \sqrt{R^2 + X_C^2}$ and $\theta = \tan^{-1} \frac{V_C}{V_R} = \tan^{-1} \frac{X_C}{R}$.

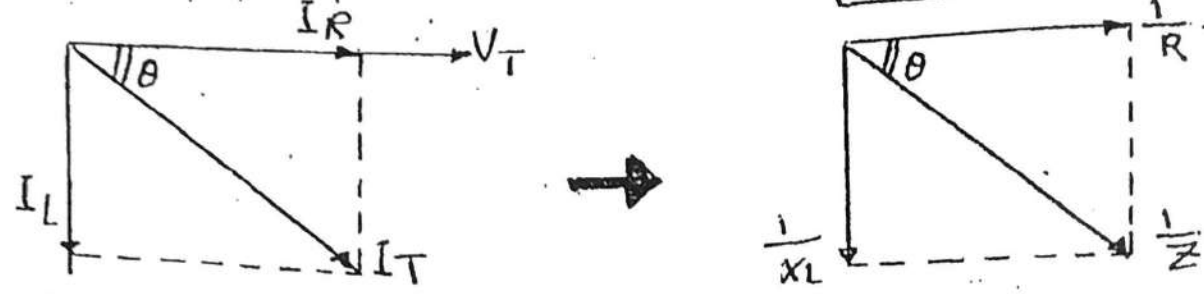
V_T lags i_T by angle θ, or i_T lead V_T by θ.

Note: Z is the total impedance, and its unit is (Ω).

③ R–L in parallel:

Let $V_T = V_m \sin \omega t$



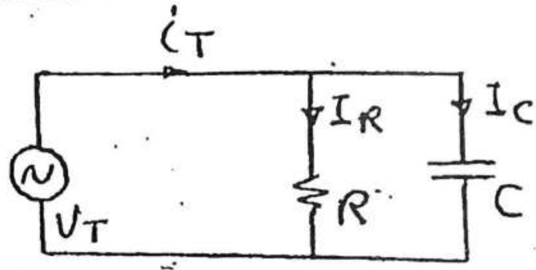


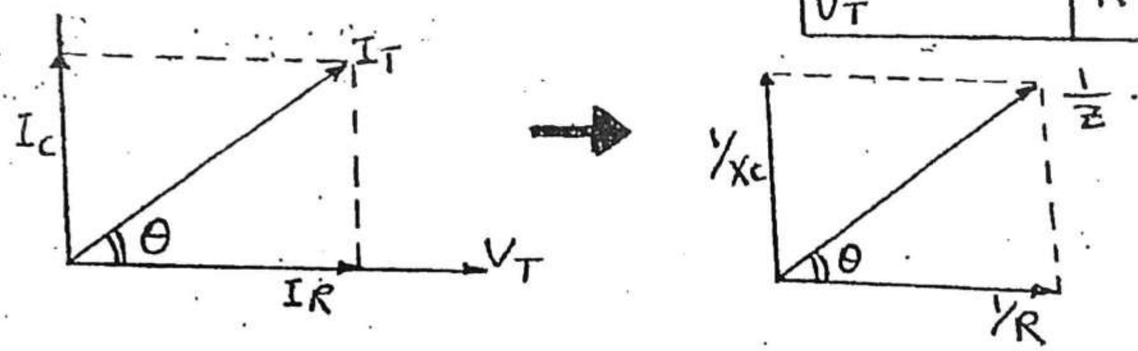
$\therefore i_T = \sqrt{I_R^2 + I_L^2}$

and $\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \frac{1}{X_L^2}}$ and $\theta = \tan^{-1} \frac{I_L}{I_R} = \tan^{-1} \frac{R}{X_L}$.

④ R–C in parallel:

Let $V_T = V_m \sin \omega t$.





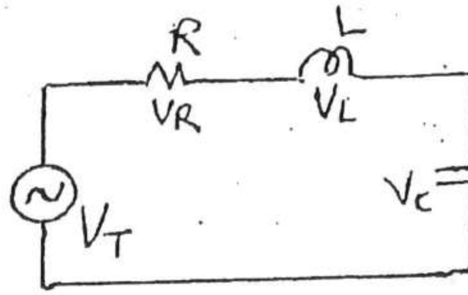
$\therefore i_T = \sqrt{I_R^2 + I_C^2}$.

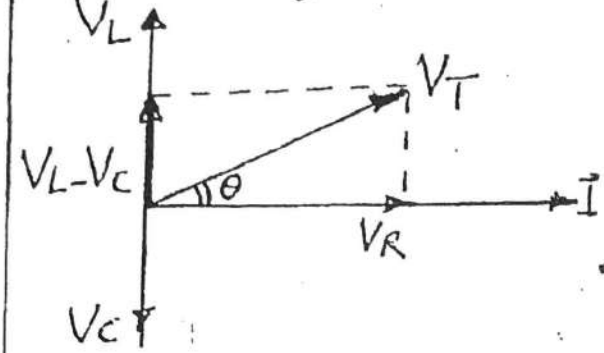
and $\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \frac{1}{X_C^2}}$ and $\theta = \tan^{-1} \frac{I_C}{I_R} = \tan^{-1} \frac{R}{X_C}$.

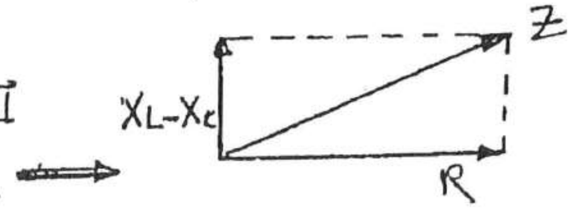
⑤ R-L and C in Series.

Let $i_T = I_m \sin\omega t$.

If $X_L > X_C$







$\therefore V_T = \sqrt{V_R^2 + (V_L - V_C)^2}$

and $Z = \sqrt{R^2 + (X_L - X_C)^2}$. and $\theta = \tan^{-1}\dfrac{V_L - V_C}{V_R} = \tan^{-1}\dfrac{X_L - X_C}{R}$

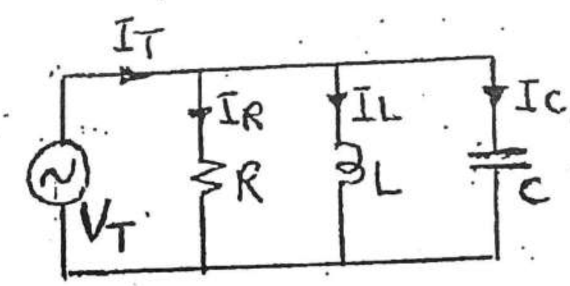
** If $X_L > X_C$ $\therefore V_T$ lead i_T by θ.

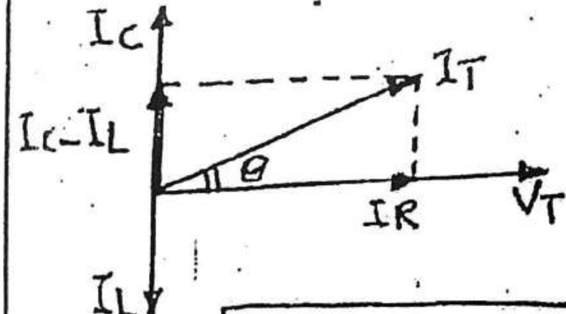
$X_L < X_C$ $\therefore V_T$ lag i_T by θ.

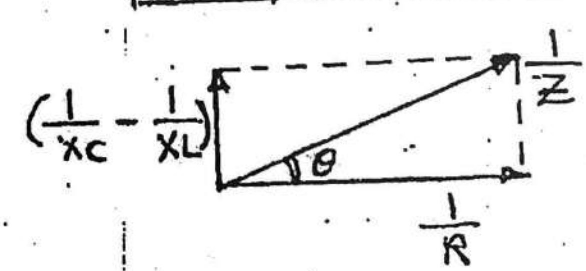
⑥ R-L and C in parallel:

Let $V_T = V_m \sin\omega t$.

If $X_L > X_C$







$\therefore I_T = \sqrt{I_R^2 + (I_C - I_L)^2}$.

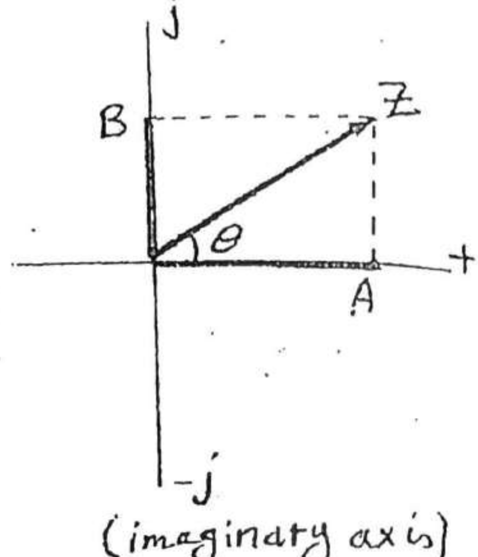
and $\dfrac{1}{Z} = \sqrt{\left(\dfrac{1}{R}\right)^2 + \left(\dfrac{1}{X_C} - \dfrac{1}{X_L}\right)^2}$. and $\theta = \tan^{-1}\dfrac{I_C - I_L}{I_R}$

** If $X_L > X_C$ $\therefore V_T$ lag I_T by θ.

$X_L < X_C$ $\therefore V_T$ lead I_T by θ.

Complex Number

There are two form used to represent a complex number (C), the rectangular form, and polar form



* Rectangular form:

$$C = A + jB$$

* polar form:

$$C = Z\angle\theta$$

Conversion between forms

① Rectangular → polar.

If $C = A + jB$

$\therefore Z = \sqrt{A^2 + B^2}$ $\qquad \theta = \tan^{-1}\frac{B}{A}$

$\therefore C = Z\angle\theta = \sqrt{A^2+B^2}\ \angle \tan^{-1}\frac{B}{A}$

$$\boxed{\begin{aligned} j &= \angle 90^\circ = \sqrt{-1} \\ j \times j &= -1 \\ j\cdot j\cdot j &= -j \\ j\cdot j\cdot j\cdot j &= j^4 = 1 \end{aligned}}$$

② polar → Rectangular.

If $C = Z\angle\theta$.

$A = Z\cos\theta$.

$B = Z\sin\theta$.

$\therefore C = A + jB = Z(\cos\theta + j\sin\theta)$.

Mathematical operations with Complex number:

If $C_1 = A_1 + jB_1 = Z_1\angle\theta_1$

$C_2 = A_2 + jB_2 = Z_2\angle\theta_1$

① Addition:

$$C = C_1 + C_2 = (A_1 + A_2) + j(B_1 + B_2).$$

② Subtraction:

$$C = C_1 - C_2 = (A_1 - A_2) + j(B_1 - B_2).$$

③ Multiplication:

$$C = C_1 \cdot C_2$$

* In polar form: $C = C_1 \cdot C_2 = Z_1 \cdot Z_2 \angle\theta_1 + \theta_2$
* In rectangular form: $C = C_1 \cdot C_2 = (A_1 + jB_1)(A_2 + jB_2)$

$$= A_1A_2 + jB_1A_2 + jA_1B_2 - B_1B_2.$$

$$= (A_1A_2 - B_1B_2) + j(B_1A_2 + A_1B_2).$$

④ Division:

$$C = \frac{C_1}{C_2}$$

* In polar form: $C = \frac{C_1}{C_2} = \frac{Z_1}{Z_2}\angle\theta_1 - \theta_2.$
* In regtangular form: $C = \frac{C_1}{C_2} = \frac{A_1 + jB_1}{A_2 + jB_2}.$

$$= \frac{A_1 + jB_1}{A_2 + jB_2} \times \left(\frac{A_2 - jB_2}{A_2 - jB_2}\right)$$

$$= \frac{(A_1 + jB_1)(A_2 - jB_2)}{A_2^2 + B_2^2}$$

Where: $(A_2 - jB_2)$ is the conjugate of the $(A_2 + jB_2)$.

Ex: If $C_1 = 6+j8 = 10\angle 53.13$.
$C_2 = 3+j4 = 5\angle 53.13$. Find:-

① Addition: $C = C_1 + C_2 = 9+j12$.

② Subtraction: $C = C_1 - C_2 = (6-3)+j(8-4) = 3+j4$.

③ Multiplication: $C = C_1 \cdot C_2$

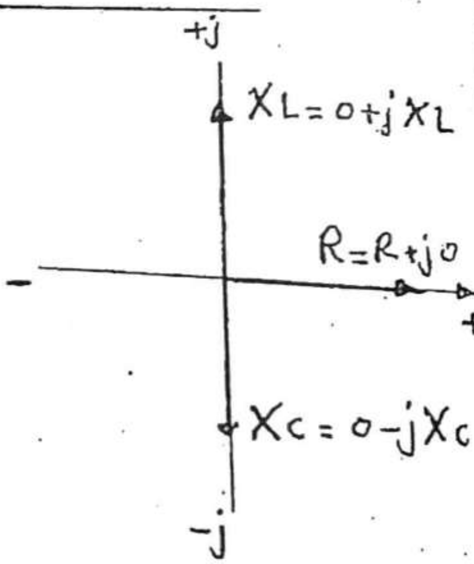
* In polar: $C = 10 \times 5 \angle 53.13 + 53.13 = 50\angle 106.26$.
* In rectangular: $C = (3\times 6 - 8\times 4) + j(8\times 3 + 4\times 6) = -14 + j48$.

Check: Same result.

④ Division: $C = \frac{C_1}{C_2}$.

* In polar: $C = \frac{10}{5}\angle 53.13 - 53.13 = 2\angle 0 = 2+j0$
* In rectangular $= \frac{(6+j8)(3-j4)}{3^2+4^2} = \frac{18 - \cancel{j24} + \cancel{j24} + 32}{25}$
$= 50/25 = 2$.

In General:



* Horizontal axis Called Real axis or (resistance axis).
* Vertical axis Called imaginary axis or (reactance axis).

∴ The resistance (R) represented as

$\boxed{R = R + j0}$

The Inductance (XL) represented as

$\boxed{X_L = 0 + jX_L}$

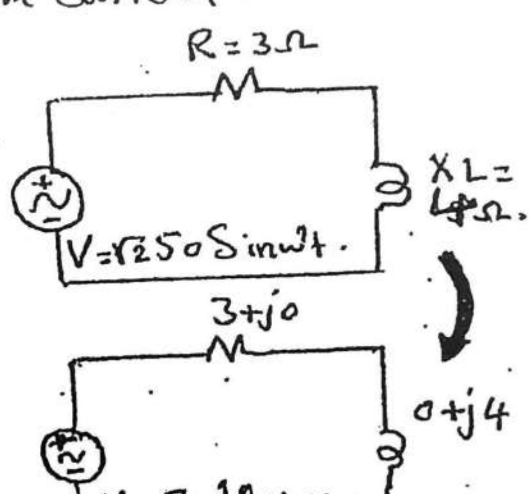
The Capacitor reactor (X_c) represented as

$\boxed{X_C = 0 - jX_C}$

Conversion between Time to phasor Domain

Time Domain ⇌	phasor Domain
$\sqrt{2}\,50 \sin\omega t$	$50\angle 0$
$45 \cos\omega t = 45 \sin(\omega t + 90)$	$\frac{45}{\sqrt{2}}\angle 90$
$\sqrt{2}\,100 \sin(\omega t + 30)$	$100\angle 30^\circ$
$\sqrt{2}\,10 \sin(\omega t + 70)$	$10\angle 70$
$\sqrt{2}\,120 \sin(\omega t - 80)$	$120\angle -80$.
⋮	⋮

Ex: Find the total impedance and the current.



Since R in series with XL.

$\therefore Z = (3+j0) + (0+j4)$

$= (3+j4)\,\Omega$

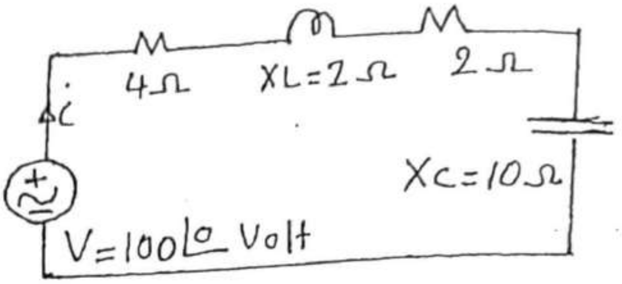
$= 5\angle 53.13\ \Omega.$

$I = \frac{V_T}{Z} = \frac{50\angle 0}{5\angle 53.13} = 10\angle -53.13$ Amp.

and the current is $(\sqrt{2} \cdot 10 \sin(\omega t - 53.13))$. Amp.

Ex: Find the current in the Circuit.

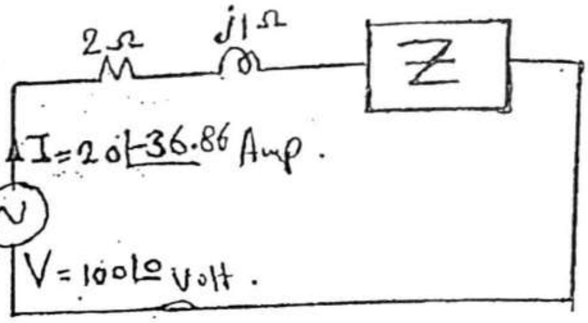
$$Z = (6 - j8)\,\Omega = 10\angle{-53.13}\,\Omega.$$



$$\therefore i = \frac{V}{Z} = \frac{100\angle 0}{10\angle{-53.13}}$$

$$= 10\angle 53.13 \text{ Amp.}$$

Ex: Find the Value of (Z) for the Circuit.
If $I = 20\angle{-36.86}$ Amp.

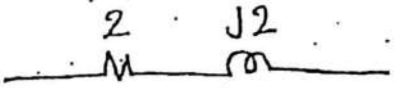


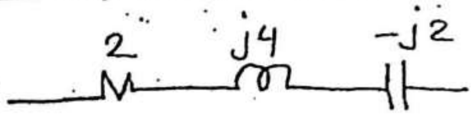
$$\text{Total impedance} = \frac{V}{I}$$

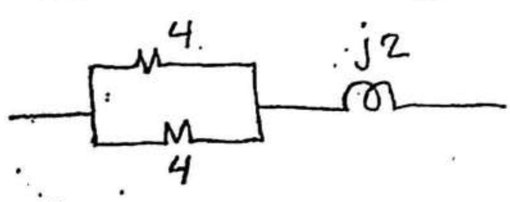
$$= \frac{100\angle 0}{20\angle{-36.86}} = 5\angle 36.86\,\Omega.$$

$$= (4 + j3)\,\Omega.$$

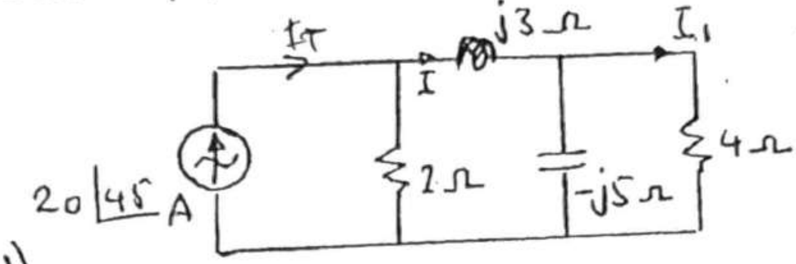
$\therefore$ total $R = 4\,\Omega$, total inductor $= 3\,\Omega$.

$\therefore Z =$ 

OR $Z =$ 

OR $Z =$ 

Ex: For the circuit shown, find (I) & (I_1)?



$Z = 2 \,||\, \left(j3 + (-j5 \,||\, 4\Omega)\right) \Omega.$

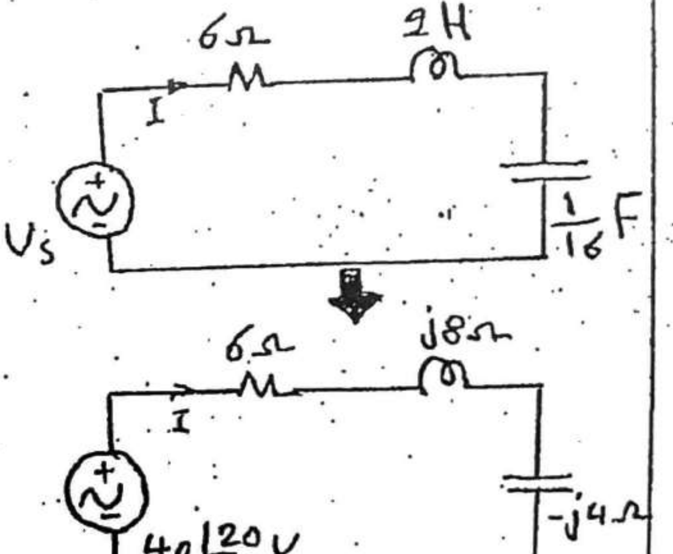
$= 2 \,||\, \left(j3 + \frac{4(-j5)}{4-j5}\right) = 2 \,||\, (j3 + 3.12\angle{-38.7}) \ \Omega.$

$= 2 \,||\, 2.65\angle{23.3} \ \Omega.$

$I = 20\angle{45} \ \frac{2}{2 + 2.65\angle{23.3}} = 8.77\angle{31.7} \, A$ (By current divider Rule)

$I_1 = 8.77\angle{31.7} \ \frac{-j5}{4-j5} = 6.85\angle{-7} \, A$ (" " " ").

Ex: Find (I) in time domain?



$V_s = \sqrt{2} \times 40 \sin(4t + 20)$ volt.

$\therefore V_s = 40\angle{20}$ volt.

$X_L = \omega L = 4 \times 2 = 8 \Omega.$

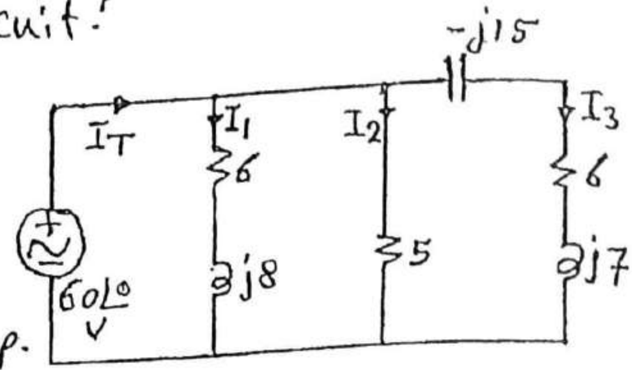
$X_C = \frac{1}{\omega C} = \frac{1}{4 \times 1/16} = 4\Omega.$

$\therefore I = \frac{V}{Z} = \frac{40\angle{20}}{6 + j4}$

$= \frac{40\angle{20}}{7.211\angle{33.7}} = 5.54\angle{-13.7} \ A.$

$\therefore I = \sqrt{2} \times 5.54 \sin(4t - 13.7) \, A.$

Ex: Find the (I_T) of the Circuit?



Solution:

$$\underline{I_1} = \frac{60\angle 0}{6+j8} = \frac{60\angle 0}{10\angle 53.1}$$
$$= 6\angle -53.1 = 3.6 - j4.8 \text{ Amp.}$$

$$\underline{I_2} = \frac{60\angle 0}{5} = 12 \text{ Amp.}$$

$$\underline{I_3} = \frac{60\angle 0}{6-j8} = \frac{60\angle 0}{10\angle -53.1} = 3.6 + j4.8 \text{ Amp.}$$

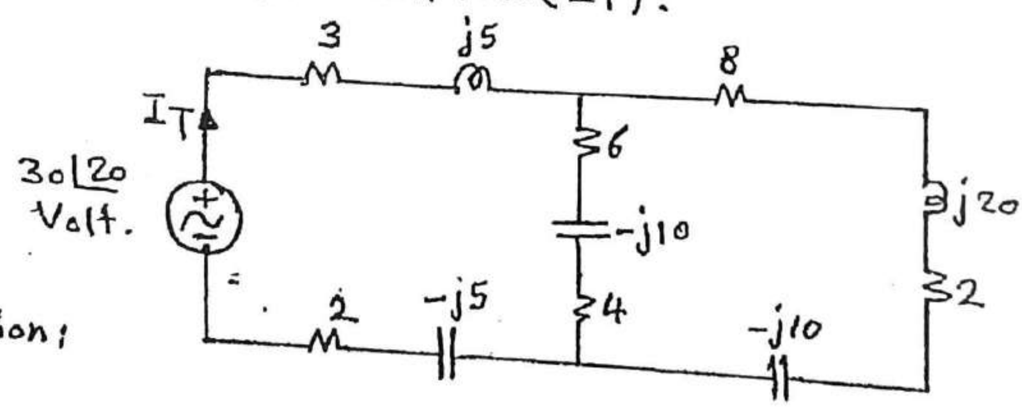
BT $I_T = I_1 + I_2 + I_3 = 19.2\angle 0$ Amp.

OR

$$Z_T = (6+j8) // 5 // (6-j8) \text{ - then;}$$

$$I_T = \frac{V_T}{Z_T} = \frac{60\angle 0}{Z_T} = 19.2\angle 0 \text{ Amp} \longrightarrow \text{Check -}$$

Ex: For the circuit shown find (I_T)?



Solution;

$$Z_T = \left[(10+j10) // (10-j10)\right] + 3 + 2 + j5 - j5$$
$$= 10 + 5 = 15\,\Omega.$$

$$I_T = \frac{V_T}{Z_T}$$

$$\therefore I_T = \frac{30\angle 20}{15} = 2\angle 20 \text{ Amp.}$$

Kirchhoff's Law

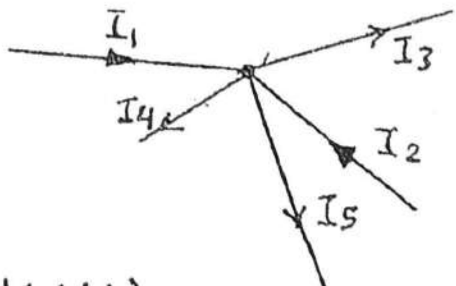
(4)

① Kirchhoff's current Law (K.C.L) :-

It states as (The algebraic sum of the incoming currents and Outgoing currents at a node (point) is Zero).

$$\sum I_{inComing} = \sum I_{outgoing}.$$

or $I_1 + I_2 = I_3 + I_4 + I_5$.

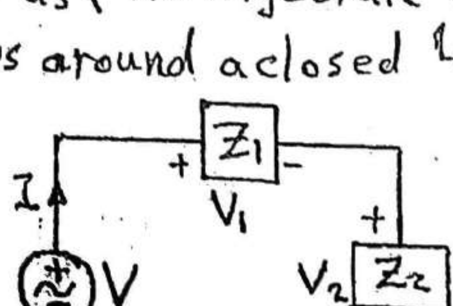


② Kirchhoff's Voltage Law (K.V.L) :-

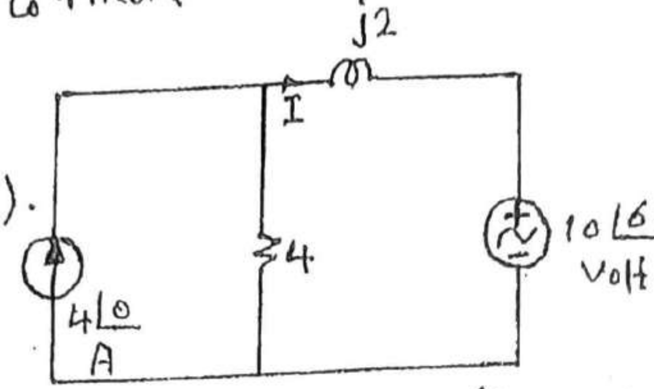
Kirchhoff's Voltage Law states as (The algebraic sum of the potential rises and drops around a closed loop (or Circuit) is Zero).

$$\sum V = 0.$$

OR $V = V_1 + V_2$.



Ex: Use Kirchhoff's Law to find (I)?



Solution:

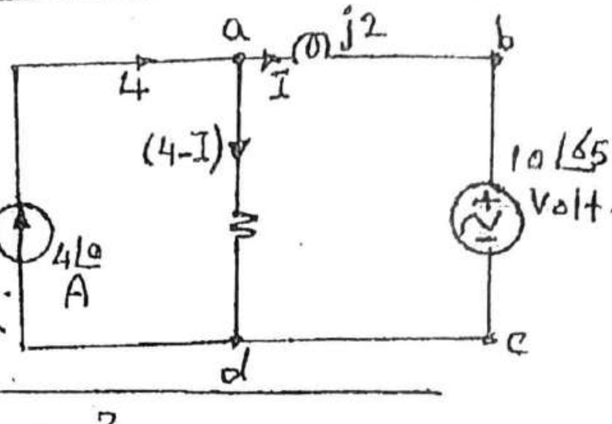
From the circuit (a, b, c, d).

$4(4-I)=j2I+10\angle 65$.

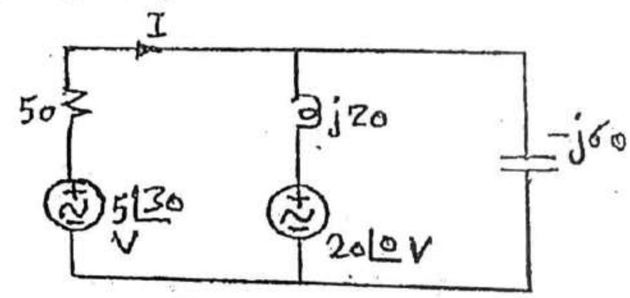
$16-4I=j2I+10\angle 65$.

$\therefore\ 16-10\angle 65=I(4+j2)$.

$\therefore I=\dfrac{16-10\angle 65}{4+j2}=3.32\angle -64.2$ Amp.



Ex: Find (I) for the circuit shown?



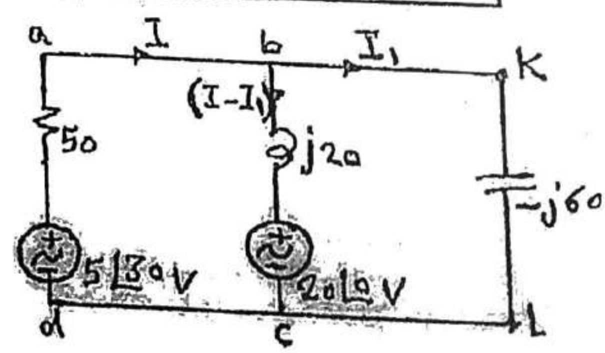
Solution:

For the circuit of (a, b, c, d).

$5\angle 30=50I+j20(I-I_1)+20\angle 0$ ---------- (1)

For the circuit (a, K, L, d)

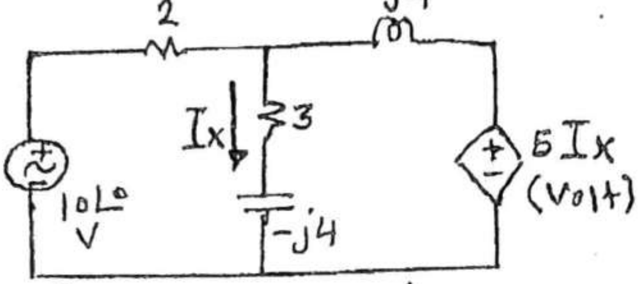
$5\angle 30=50I+(-j60)I_1$. ---------- (2)



Solving eq (1) and eq (2)

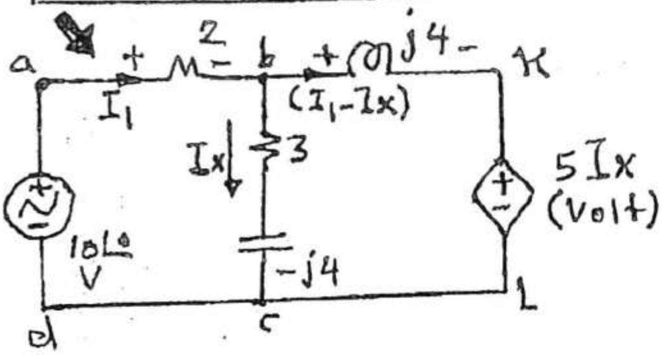
$I=0.442\angle 144$ Amp.

Ex: For the circuit shown find (I_x)?



solution:

From the circuit (a, b, c, d)



$$10\angle 0° = 2I_1 + (3 - j4)\,I_x \quad \text{--------- (1)}$$

and for the circuit (a, K, L, d)

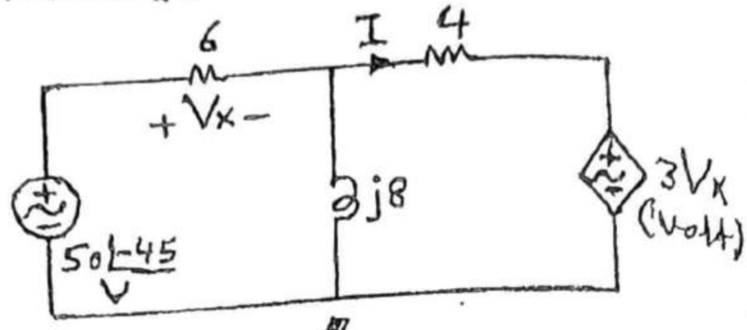
$$10\angle 0° = 2I_1 + j4\,(I_1 - I_x) + 5I_x$$

$$10\angle 0° = (2 + j4)\,I_1 + (5 - j4)\,I_x \quad \text{--------- (2)}$$

Solving the eq(1) & eq(2)

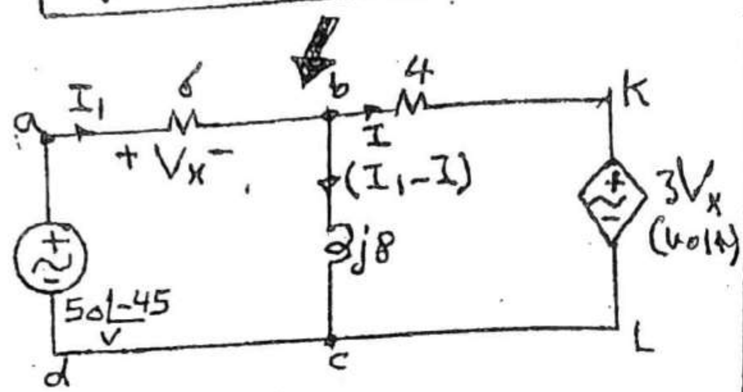
$$I_x = \frac{10 + j10}{6} = \underline{2.35\angle 45} \text{ Amp.}$$

Ex: For the Circuit Use Kirchhoff's Laws to find (I)?



Solution:

For the closed circuit (abc & d):-



$50\angle{-45} = 6I_1 + j8(I_1 - I)$.

$50\angle{-45} = (6+j8)I_1 - j8I$ ---------- (1).

From Circuit (aKL & d),

$50\angle{-45} = 6I_1 + 4I + 3V_x$ ---------- (2).

$\boxed{\text{But}}$ $V_x = 6I_1$

$\therefore 50\angle{-45} = 6I_1 + 4I + 3 \times 6I_1$

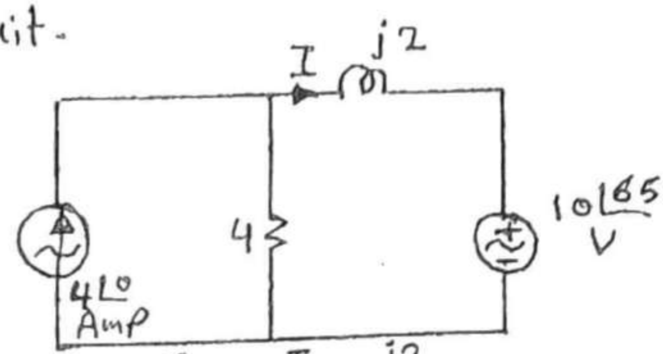
$50\angle{-45} = 24I_1 + 4I$

Solving eq(1) & eq(2)

$I = 4.37\angle{27.2}$ Amp

Mesh Analysis (Loop Current mehod)

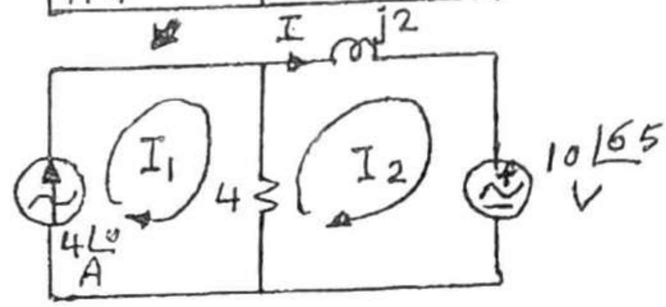
Ex: Find (I) for the circuit.



Solution:-

From the circuit

$I_1 = 4\angle 0$ Amp.



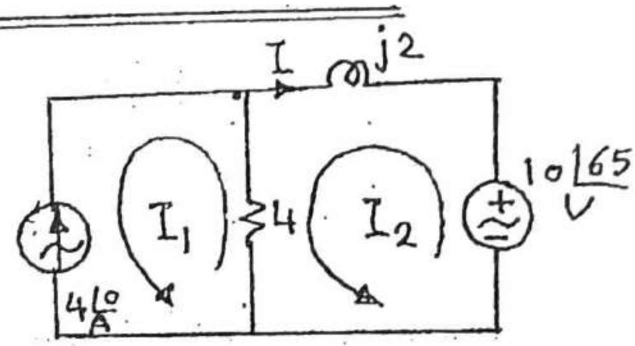
loop(2):

$-10\angle 65 = (4+j2)I_2 - 4I_1$

$-10\angle 65 = (4+j2)I_2 - 4\times 4$.

$\therefore I_2 = I = \dfrac{16 - 10\angle 65}{4+j2} = 3.32\angle -64.2$ Amp.

OR



From the circuit

$I_1 = -4\angle 0$ Amp.

Loop(2):

$10\angle 65 = (4+j2)I_2 - 4I_1$

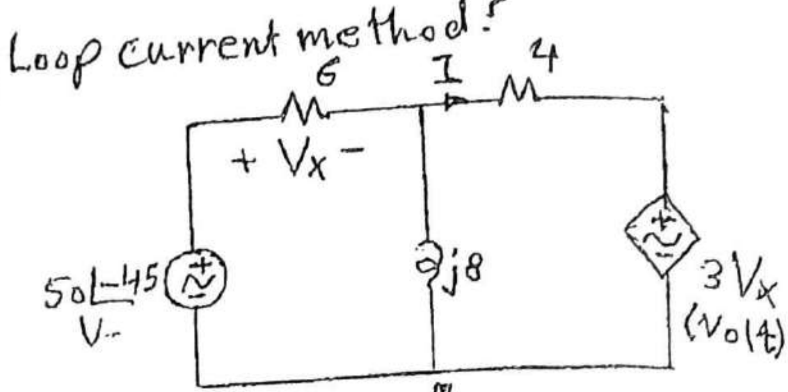
$\therefore 10\angle 65 = (4+j2)I_2 + 4\times 4$.

$\therefore I_2 = \dfrac{-16 + 10\angle 65}{4+j2}$

But $I_2 = -I$

$\therefore I = \dfrac{16 - 10\angle 65}{(4+j2)} = 3.32\angle -64.2$ Amp. $\rightarrow$ Same result.

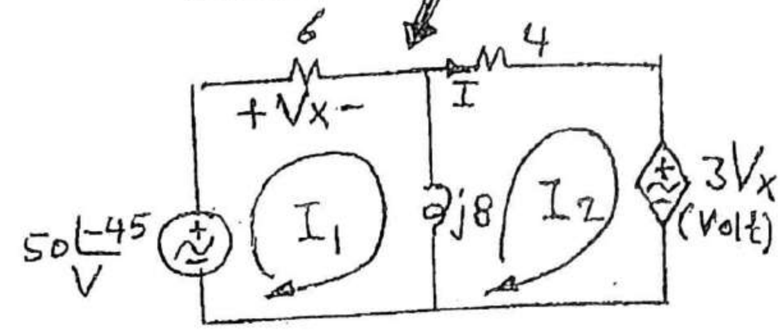
Ex: Find (I) by using Loop current method?



Solution:

From the circuit:

$I = I_2$.



Loop(1):

$$50\angle{-45} = (6+j8)I_1 - j8I_2 \quad \text{------------ (1)}$$

Loop(2):

$$-3V_x = (4+j8)I_2 - j8I_1 \quad \text{------------ (2)}$$

But $V_x = 6I_1$

$$\therefore -18I_1 = (4+j8)I_2 - j8I_1$$

Solving eq(1) & eq(2).

$$\therefore I = I_2 = \underline{4.37\angle{27.2}\ \text{Amp}}.$$

Ex: Draw the circuit having the Loop equations as-

Loop(1):

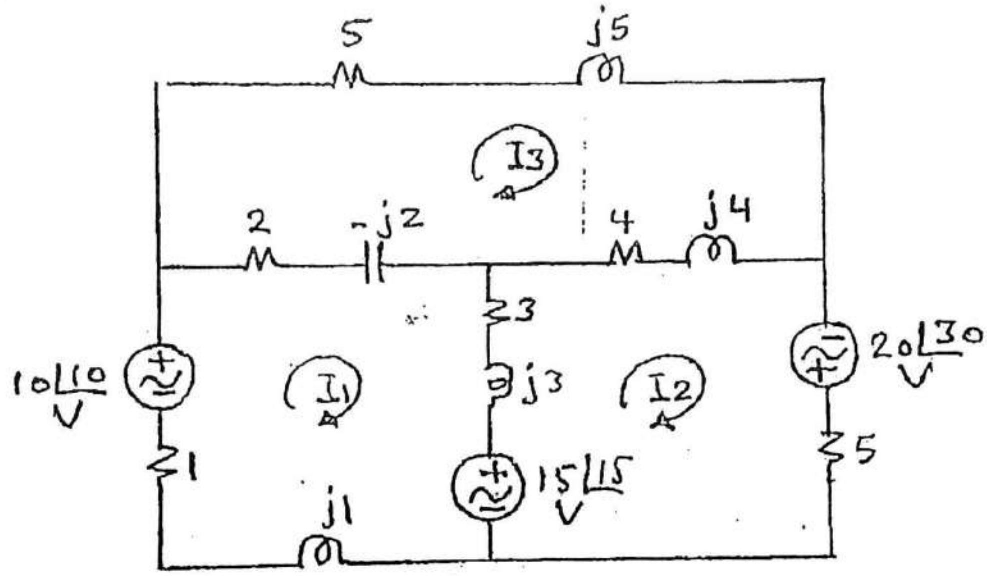
$$10\angle 10 - 15\angle 15 = (6+j2)I_1 - (3+j3)I_2 - (2-j2)I_3 \quad \text{--- (1)}$$

Loop(2):

$$15\angle 15 + 20\angle 30 = (12+j7)I_2 - (3+j3)I_1 - (4+j4)I_3 \quad \text{--- (2)}$$

Loop(3):

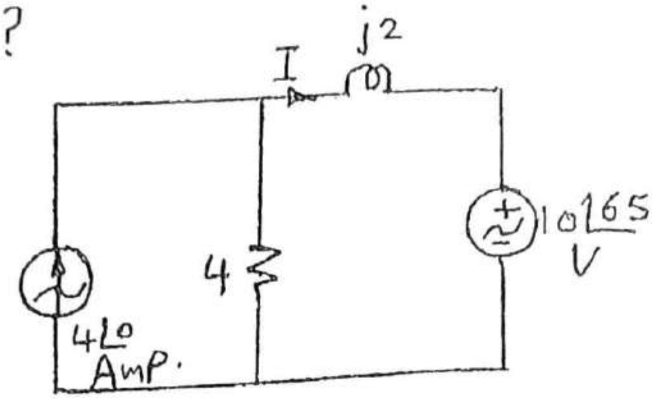
$$0 = (11+j7)I_3 - (2-j2)I_1 - (4+j4)I_2$$



Scanned with CamScanner

Super position method

Ex:- Find the current (I)?

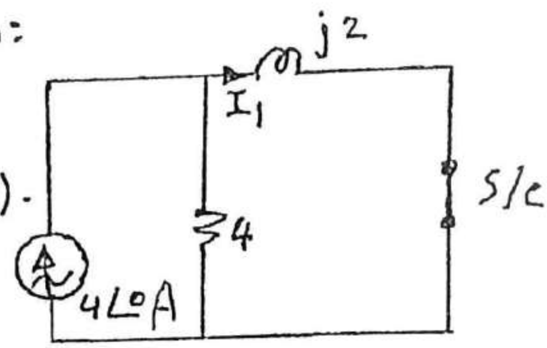


Solution:-

** By effect of $4\angle 0$ Amp.

make $10\angle 65$ V as (short-circuit) (S/c) why?

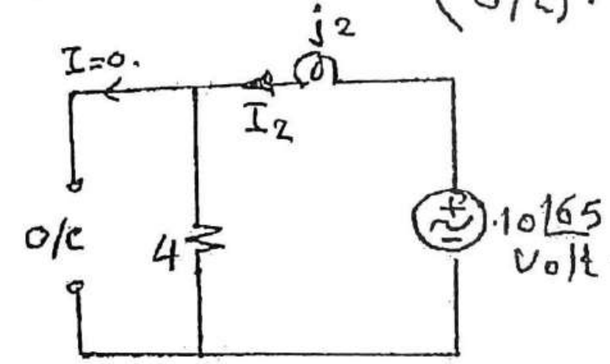
then the circuit is shown:



$\therefore I_1 = 4\angle 0 \; \frac{4}{4+j2}$ .... (C.D.R).

$= \frac{16\angle 0}{4+j2}$ Amp.

* By effect of $10\angle 65$ Volt → $4\angle 0$ becomes (open-circuit) (o/c).



$I_2 = \frac{10\angle 65}{4+j2}$ Amp.

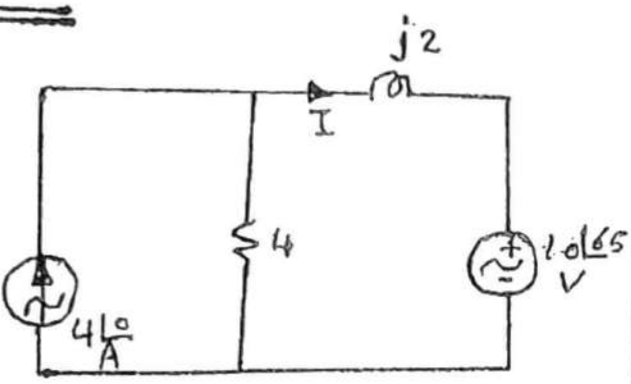
$\therefore I = I_1 - I_2$

$= \frac{16\angle 0}{4+j2} - \frac{10\angle 65}{4+j2}$

$= \frac{16\angle 0 - 10\angle 65}{4+j2} = 3.32\angle -64.2$ Amp.

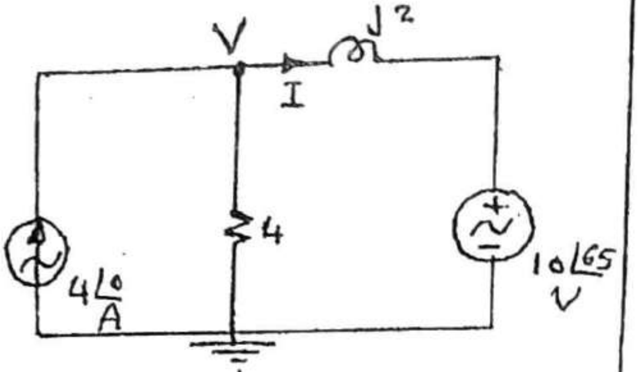
Nodal Voltage method

Ex: Find the current (I) ?



Solution:

$I = \frac{V - 10\angle 65}{j2}$. (ohm's Law).



Node (V):

$$\left(\frac{1}{4} + \frac{1}{j2}\right)V - \frac{10\angle 65}{j2} = 4\angle 0$$

$$(0.25 - j0.5)V - 5\angle -25 = 4\angle 0 .$$

$$(0.25 - j0.5)V = 4 + 5\angle -25 = 8.53 - j2.11 .$$

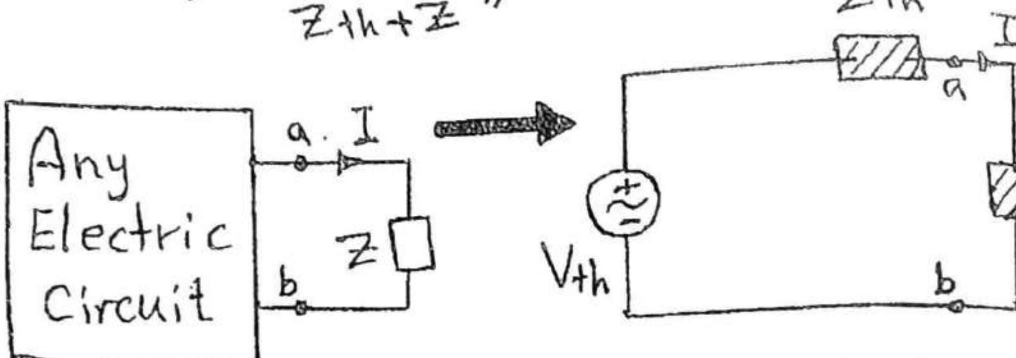
$$\therefore V = \frac{8.53 - j2.11}{0.25 - j0.5} = 15.72\angle 49.54 \text{ Volt.}$$

$$\therefore I = \frac{15.72\angle 49.54 - 10\angle 65}{j2} = \frac{5.98 + j2.9}{j2}$$

$$= 3.32\angle -64.2 \text{ Amp.}$$

Thevenin's Theorem

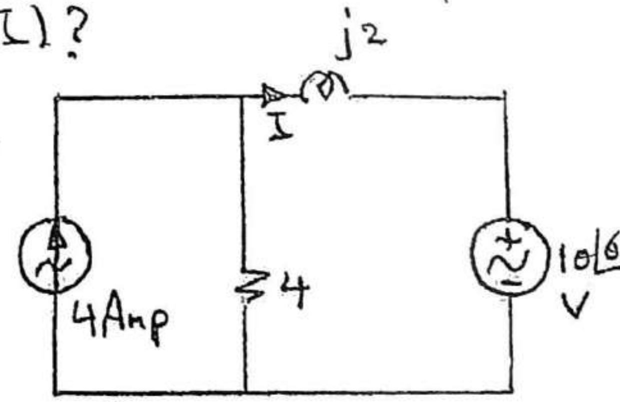
This theory states as « The current (I) passing through an impedance (Z) of any electric circuit is: $I = \frac{V_{th}}{Z_{th}+Z}$ ».



Where: V_{th} :- The voltage between a & b with (Z) disconnected.

Z_{th} :- The circuit impedance between a & b with (Z) also disconnected.

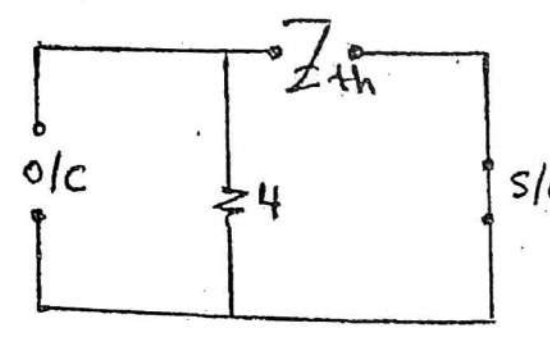
Ex: For the circuit find (I)?



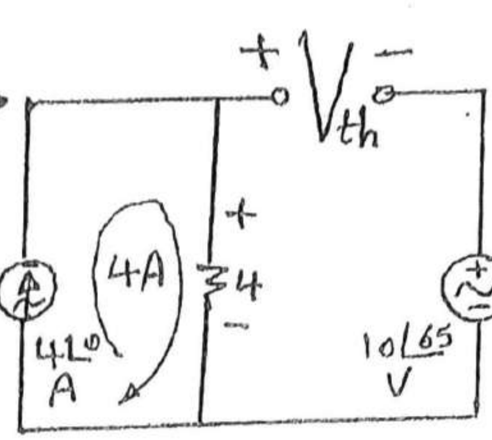
Solution:

$$I = \frac{V_{th}}{Z_{th}+j2}$$

$Z_{th} = 4\Omega$



$V_{th} = 16 - 10\angle 65$ Volt

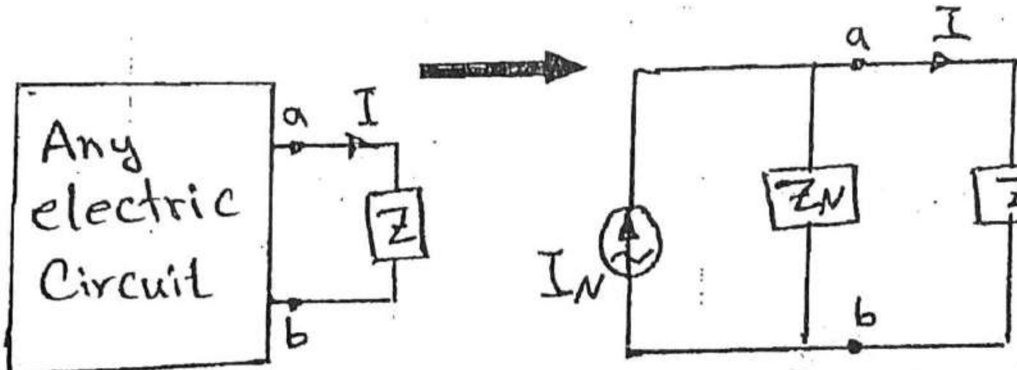


$\therefore I = \dfrac{16 - 10\angle 65}{4 + j2}$ Amp.

$= 3.32\angle -64.2$ Amp.

Norton Theory

. This theory states as ((The current (I) passing through an inpedanse (Z) of any electric circuit is: $I = I_N \dfrac{Z_N}{Z_N + Z}$)).



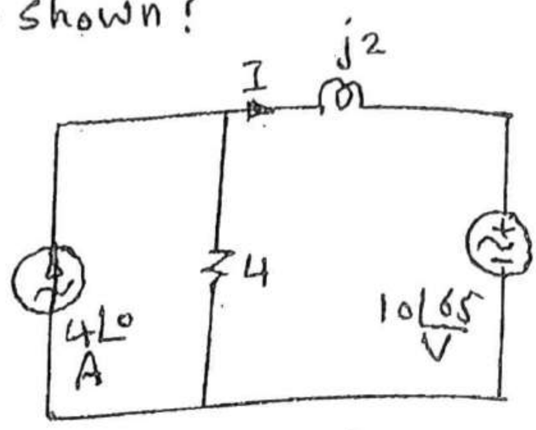
$I = I_N \dfrac{Z_N}{Z_N + Z}$ ---- (C.D.R)

Where:

I_N :- The current passing through a & b after replacing the impedance (Z) by a short-Circuit

Z_N :- The circuit impedance between a & b with (Z) disconnected.

Ex: Find (I) for the circuit shown?

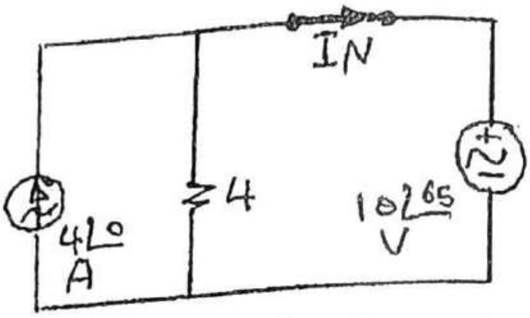


solution:

$$I = I_N \frac{Z_N}{Z_N + Z}$$

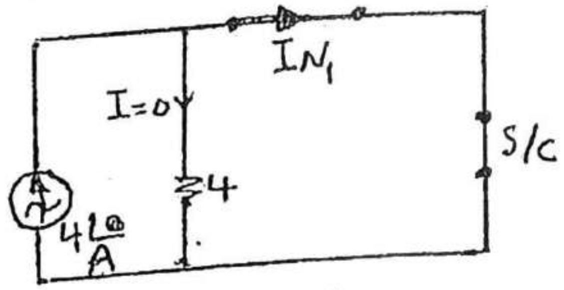
$Z_N = Z_{th} = 4\,\Omega$.

$I_N \longrightarrow$



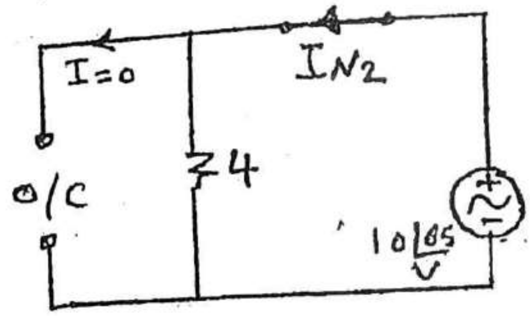
$I_{N_1} \longrightarrow$ by effect of 4(A)
$10\angle 65$ V (S/C)

$\therefore I_{N_1} = 4\angle 0$ Amp



$I_{N_2} \longrightarrow$ by effect of $10\angle 65$ V
$4\angle 0$ Amp (O/C).

$$I_{N_2} = \frac{10\angle 65}{4}.$$



$$\therefore I_N = I_{N_1} - I_{N_2}.$$

$$= 4\angle 0 - \frac{10\angle 65}{4}$$

$$= \frac{16 - 10\angle 65}{4} \text{ Amp.}$$

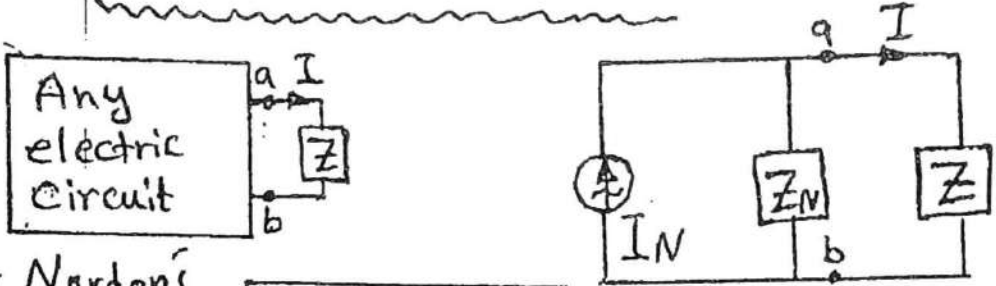
$$\therefore I = \frac{16 - 10\angle 65}{4}\left(\frac{4}{4+j2}\right).$$

$$= \frac{16 - 10\angle 65}{4+j2} = 3.32\angle -64.2 \text{ Amp.}$$

Voltage Source ⇌ Current Source Transformation
V.S ⇌ C.S Transformations

For thevenin's Theorem

$$\boxed{I = \frac{V_{th}}{Z_{th} + Z}} \quad \text{-------------} (1)$$



For Norton's Theorem

$$\boxed{I = I_N \frac{Z_N}{Z_N + Z}} \quad \text{-------------} (2)$$

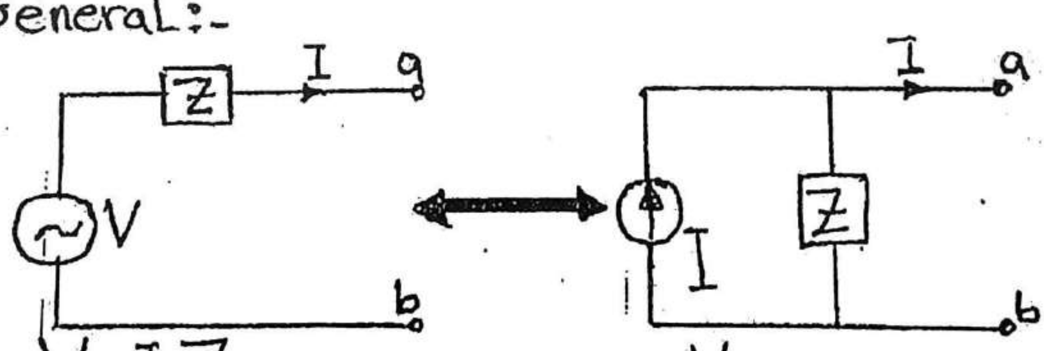
Since $Z_{th} = Z_N$

$\therefore$ from eq(1) and eq(2):

$$V_{th} = I_N \cdot Z_N .$$

OR $\boxed{Z_{th} = Z_N = \frac{V_{th}}{I_N}}$

In General:-



$V = I \cdot Z$ $\qquad$ $I = \frac{V}{Z}$.

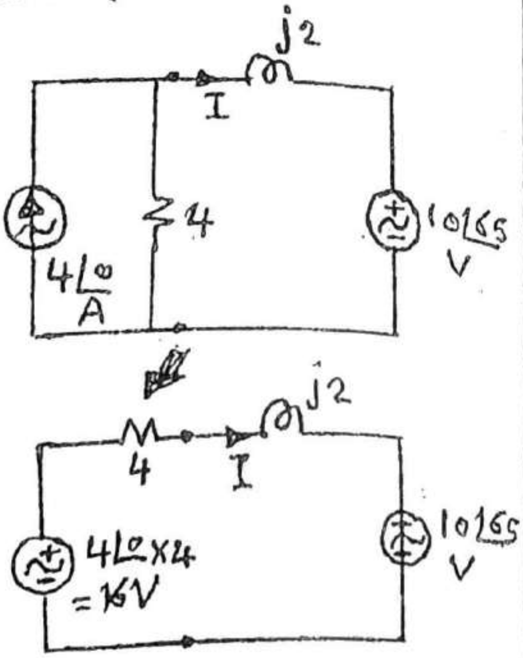
Ex: Find (I) for the circuit shown?

Solution:

Change the current source (4A) to voltage source

$V = I \cdot Z$

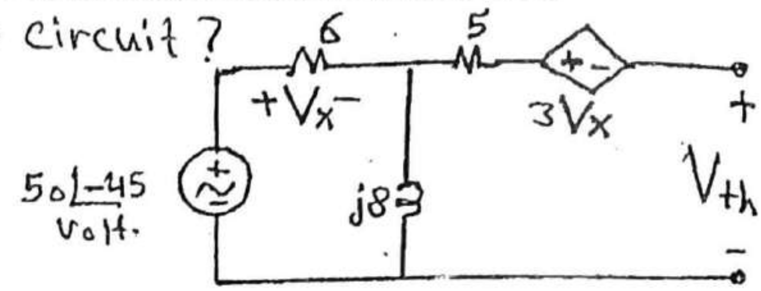
$= 4\angle 0 \times 4 = 16$ Volt.



$\therefore I = \dfrac{16 - 10\angle 65}{4 + j2} = 3.32\angle -64.2$ A

Ex: Find (V_{th}) for the circuit?

Solution:



$V_{th} = V_{j8} - 3V_x$

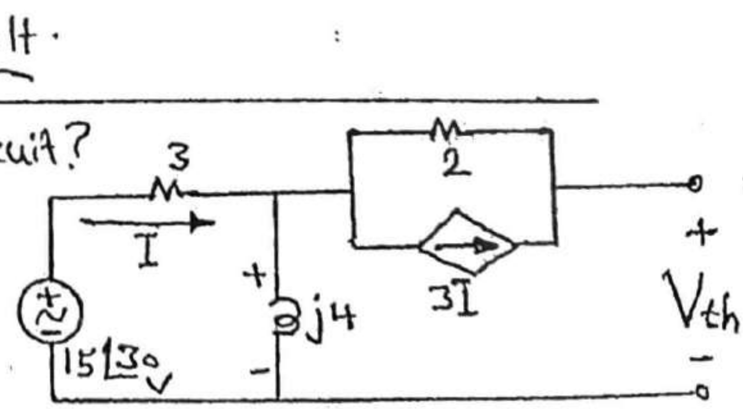
But $V_x = \dfrac{50\angle -45}{6 + j8} \times 6 = 30\angle -98.1$ Volt.

$\therefore V = \dfrac{50\angle -45}{6 + j8} \times (j8) - 3(30\angle -98.1)$

$= 98.49\angle 57.93$ Volt.

Ex: Find (V_{th}) for the circuit?

Solution:



$V_{th} = V_{j4\Omega} + V_{2\Omega}$

$I = \dfrac{15\angle 30}{3 + j4} = 3\angle -23.1$ Amp.

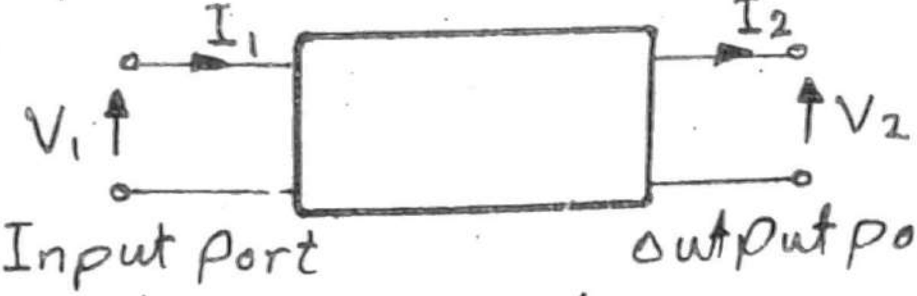
$\therefore V = I \times j4 + 2 \times 3I = I \cdot (6 + j4) = 3\angle -23.1 \times (6 + j4)$

$= 21.6\angle 10.6$ Volt.

FOUR_TERMINAL NETWORKS (1)

Two_Port Networks

Two-ports are electric circuits with two input and two output terminals as shown



In two-ports, the input voltage and current (V_1 & I_1) are related with the output voltage and current (V_2 & I_2) by the fallowing equations:-

$$\left.\begin{aligned} V_1 &= Z_{11} I_1 - Z_{12} I_2 \\ V_2 &= Z_{21} I_1 - Z_{22} I_2 \end{aligned}\right\} \text{Z - parameters.}$$

$$\left.\begin{aligned} I_1 &= y_{11} V_1 - y_{12} V_2 \\ I_2 &= y_{21} V_1 - y_{22} V_2 \end{aligned}\right\} \text{y - parameters.}$$

$$\left.\begin{aligned} V_1 &= h_{11} I_1 - h_{12} V_2 \\ I_2 &= h_{21} I_1 - h_{22} V_2 \end{aligned}\right\} \text{h - parameters.}$$

$$\left\{\begin{aligned} V_1 &= A\, V_2 + B\, I_2 \\ I_1 &= C\, V_2 + D\, I_2 \end{aligned}\right\} \text{ABCD parameters.}$$

The coefficients of the current and/or voltage on the right-hand side of the above equations are called the parameters of the two-port circuit.

* If ports can be interchanged without disturbing the values of the terminal currents and voltage then the two-port circuit is called Symmetrical.

For Symmetrical two-ports:-

$$A = D.$$

Also the ABCD parameters are related by the fallowing equation:

$$AD - BC = 1.$$

A and D - no units , B in (Ω) , C in ($\mho$).

* We can find the ABCD parameters interms of Z-parameters for example as follows:-

$$V_1 = Z_{11} I_1 - Z_{12} I_2 \quad \text{------ (1)}$$
$$V_2 = Z_{21} I_1 - Z_{22} I_2 \quad \text{------ (2)}.$$

from eq (2):

$$I_1 = \frac{V_2}{Z_{21}} + \frac{Z_{22}}{Z_{21}} I_2. \quad \text{------ (3)}.$$

Sub. for (I_1) in eq (1) we get.

$$V_1 = Z_{11}\left(\frac{V_2}{Z_{21}} + \frac{Z_{22}}{Z_{21}} I_2\right) - Z_{12} I_2.$$

$$\therefore V_1 = \left(\frac{Z_{11}}{Z_{21}}\right) V_2 + \left(\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}\right) I_2$$

Since $V_1 = A V_2 + B I_2$ (3)

$I_1 = C V_2 + D I_2$

$A = \dfrac{z_{11}}{z_{21}}$ & $B = \dfrac{z_{11} z_{22} - z_{12} z_{21}}{z_{21}}$

$C = \dfrac{1}{z_{21}}$ & $D = \dfrac{z_{22}}{z_{21}}$

* When interchanging the places between the source and the load in a two-port the following equations should be used :-

$V_2 = D V_1 + B I_1$

$I_2 = C V_1 + A I_1$

ABCD - parameters Calculations

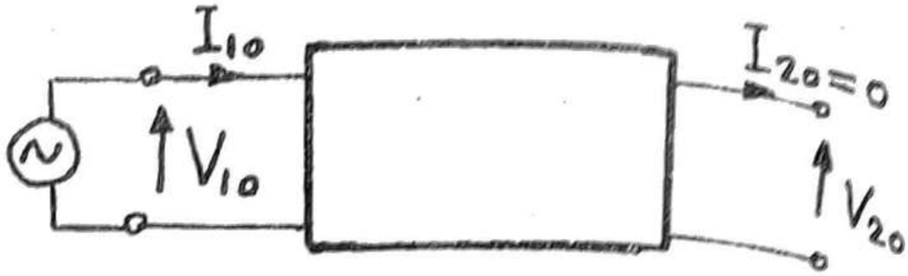
The ABCD parameters could be found by experiment as well as by calculation. When finding the ABCD parameters by calculation, circuit components and their interconnection must be known.

For the experimental determination of the parameters we must conduct -3- experiments (because we have only -3- independant parameters). The simplest method used is by conducting <u>Short</u> and <u>open</u> circuit experiments

Experiment № 1.

(4)

In this experiment the source is connected to the input terminals, the output terminals are open-circuited.



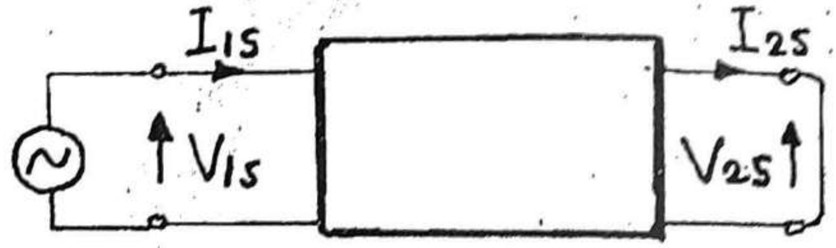
$V_{10} = A\,V_{20}$

$I_{10} = C\,V_{20}$

$$\therefore \boxed{\frac{V_{10}}{I_{10}} = Z_{10} = \frac{A}{C}} \quad \text{.............} \; (1)$$

Experiment № 2.

In this experiment the source is connected to the input terminals, the output terminals are short-circuited.



$V_{2S} = 0$

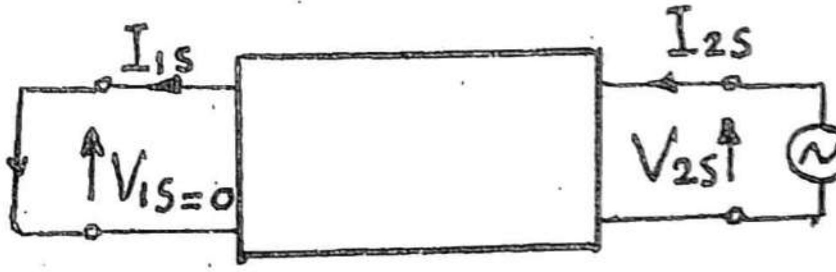
$V_{1S} = B\,I_{2S}.$

$I_{1S} = D\,I_{2S}.$

$$\therefore \boxed{\frac{V_{1S}}{I_{1S}} = Z_{1S} = \frac{B}{D}} \quad \text{.............} \; (2)$$

Experiment No 3.

In this experiment the source is connected to the output terminals, the input terminals are short_circuited.



$V_{1S}=0$

$V_2 = DV_1 + BI_1$

$I_2 = CV_1 + AI_1$

$\therefore \; V_{2S} = B I_{1S}$.

$I_{2S} = A I_{1S}$.

$$\therefore \; \boxed{\frac{V_{2S}}{I_{2S}} = Z_{2S} = \frac{B}{A}} \text{ - - - - - - - - - (3)}$$

Since we have : $AD - BC = 1$

$$\therefore \; 1 - \frac{BC}{AD} = \frac{1}{AD} \qquad \therefore \; 1 - \frac{Z_{1S}}{Z_{1O}} = \frac{1}{AD}.$$

$$\therefore \; \frac{Z_{1O} - Z_{1S}}{Z_{1O}} = \frac{1}{AD} \text{ - - - - - - - - (4)}.$$

$$\frac{Z_{2S}}{Z_{1S}} = \frac{B}{A} \times \frac{D}{B} = \frac{D}{A} \text{ - - - - - - - - (5)}$$

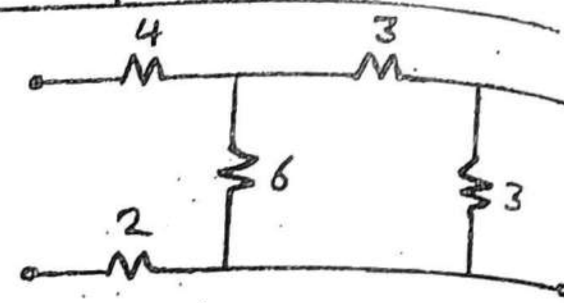
Multiplying eq (4) By eq (5) we get

$$\frac{Z_{2S}}{Z_{1S}} \times \frac{Z_{1O} - Z_{1S}}{Z_{1O}} = \frac{1}{A^2}.$$

$$\therefore \boxed{A = \sqrt{\frac{Z_{10} Z_{1S}}{Z_{2S}(Z_{10} - Z_{1S})}}} \quad \text{------} \quad (6)$$

After finding A from eq(6), we can find (C) from eq(1), from eq(3) we can find (B) then (D) can be find from eq(2).

Ex: Find the ABCD Parameters for the Shown Circuit



$$Z_{10} = 4 + 2 + (6 // 3 + 3) = 9\,\Omega.$$

$$Z_{1S} = 4 + 2 + (6 // 3) = 8\,\Omega.$$

$$Z_{2S} = \{(4 + 2 // 6) + 3\} // 3 = 2\,\Omega.$$

$$A = \sqrt{\frac{Z_{10} Z_{1S}}{Z_{2S}(Z_{10} - Z_{1S})}} = \sqrt{\frac{9 \times 8}{2(9-8)}} = 6.$$

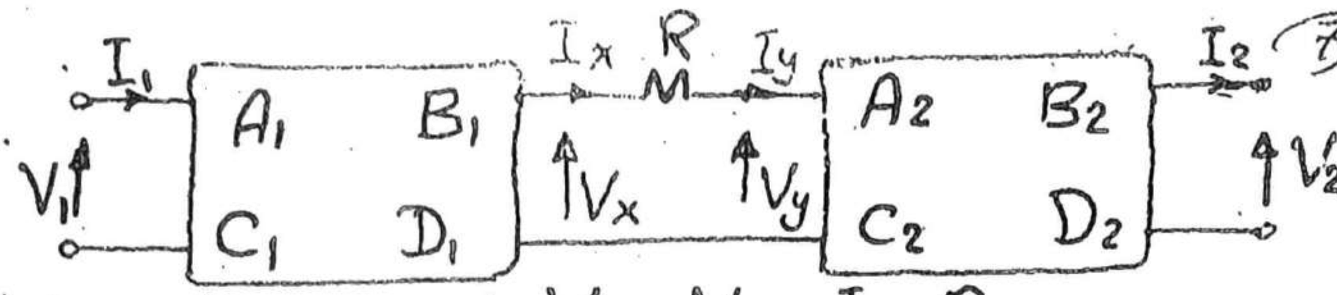
$$Z_{10} = \frac{A}{C} \qquad \therefore C = \frac{A}{Z_{10}} = \frac{6}{9} = \frac{2}{3} \text{ mho}.$$

$$Z_{2S} = \frac{B}{A} \qquad \therefore B = Z_{2S} \times A = 2 \times 6 = 12\,\Omega.$$

$$Z_{1S} = \frac{B}{D} \qquad \therefore D = \frac{B}{Z_{1S}} = \frac{12}{8} = \frac{3}{2}$$

check $AD - BC = 1.$

7



$I_x = I_y$ and $V_x = V_y + I_x.R$

$V_1 = A_1 V_x + B_1 I_x \qquad V_y = A_2 V_2 + B_2 I_2$

$I_1 = C_1 V_x + D_1 I_x \qquad I_x = I_y = C_2 V_2 + D_2 I_2.$

$\therefore V_1 = A_1 (A_2 V_2 + B_2 I_2 + I_x.R) + B_1 I_x$

$= A_1 A_2 V_2 + A_1 B_2 I_2 + (A_1.R + B_1) I_x$

$= A_1 A_2 V_2 + A_1 B_2 I_2 + (A_1.R + B_1)(C_2 V_2 + D_2 I_2).$

$\therefore V_1 = (A_1 A_2 + B_1 C_2 + A_1 C_2.R) V_2 + (A_1 B_2 + B_1 D_2 + A_1 D_2.R) I$

but $V_1 = A V_2 + B I_2.$

$\therefore A = A_1 A_2 + B_1 C_2 + A_1 C_2.R \qquad B = A_1 B_2 + B_1 D_2 + A_1 D_2.R$

$I_1 = C_1 (A_2 V_2 + B_2 I_2 + I_x.R) + D_1 I_x$

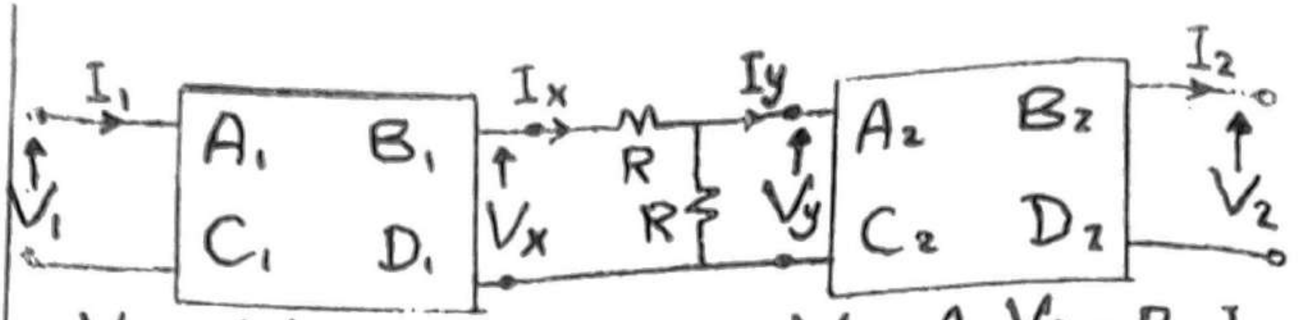
$= C_1 A_2 V_2 + C_1 B_2 I_2 + (C_1.R + D_1) I_x.$

$= C_1 A_2 V_2 + C_1 B_2 I_2 + (C_1.R + D_1)(C_2 V_2 + D_2 I_2).$

$I_1 = (C_1 A_2 + D_1 C_2 + C_1 C_2.R) V_2 + (C_1 B_2 + D_1 D_2 + C_1 D_2.R) I_2.$

but $I_1 = C V_2 + D I_2.$

$C = C_1 A_2 + D_1 C_2 + C_1 C_2.R \qquad D = C_1 B_2 + D_1 D_2 + C_1 D_2.R.$



$$\left.\begin{aligned} V_1 &= A_1 V_x + B_1 I_x \\ I_1 &= C_1 V_x + D_1 I_x \end{aligned}\right\} \qquad \left.\begin{aligned} V_y &= A_2 V_2 + B_2 I_2 \\ I_y &= C_2 V_2 + D_2 I_2 \end{aligned}\right\}$$

$$I_x = I_y + \frac{V_y}{R} \qquad V_x = V_y + I_x . R$$

$$\therefore V_x = V_y + \left(I_y + \frac{V_y}{R}\right).R \qquad \therefore V_x = 2V_y + I_y . R$$

$$\therefore V_1 = A_1 (2V_y + I_y . R) + B_1 \left(I_y + \frac{V_y}{R}\right)$$

$$I_1 = C_1 (2V_y + I_y . R) + D_1 \left(I_y + \frac{V_y}{R}\right)$$

$$\therefore V_1 = \left(2A_1 + \frac{B_1}{R}\right) V_y + (A_1 . R + B_1) I_y$$

$$I_1 = \left(2C_1 + \frac{D_1}{R}\right) V_y + (C_1 . R + D_1) I_y$$

$$V_1 = \left(2A_1 + \frac{B_1}{R}\right)(A_2 V_2 + B_2 I_2) + (A_1 . R + B_1)(C_2 V_2 + D_2 I_2)$$

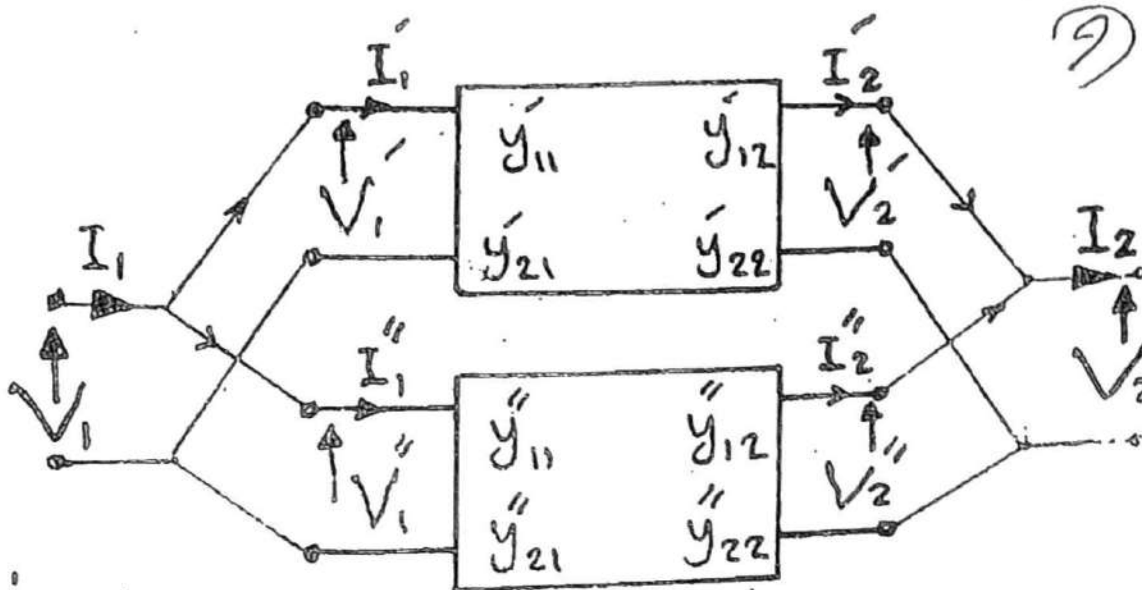
$$I_1 = \left(2C_1 + \frac{D_1}{R}\right)(A_2 V_2 + B_2 I_2) + (C_1 . R + D_1)(C_2 V_2 + D_2 I_2)$$

$$\therefore A = 2A_1 A_2 + B_1 C_2 + A_1 C_2 . R + \frac{B_1 A_2}{R}$$

$$B = 2A_1 B_2 + B_1 D_2 + A_1 D_2 . R + \frac{B_1 B_2}{R}$$

$$C = 2C_1 A_2 + D_1 C_2 + C_1 C_2 . R + \frac{D_1 A_2}{R}$$

$$D = 2C_1 B_2 + D_1 D_2 + C_1 D_2 . R + \frac{D_1 B_2}{R}$$



We have:

$I_1 = y_{11}V_1 - y_{12}V_2$

$I_2 = y_{21}V_1 - y_{22}V_2$

and

$V_1 = V_1' = V_1''$.

$V_2 = V_2' = V_2''$.

$I_1 = I_1' + I_1''$.

$I_2 = I_2' + I_2''$.

$I_1' = y_{11}'V_1 - y_{12}'V_2$ & $I_1'' = y_{11}''V_1 - y_{12}''V_2$

$\therefore I_1 = (y_{11}' + y_{11}'')V_1 - (y_{12}' + y_{12}'')V_2$

$\therefore y_{11} = y_{11}' + y_{11}''$ & $y_{12} = y_{12}' + y_{12}''$

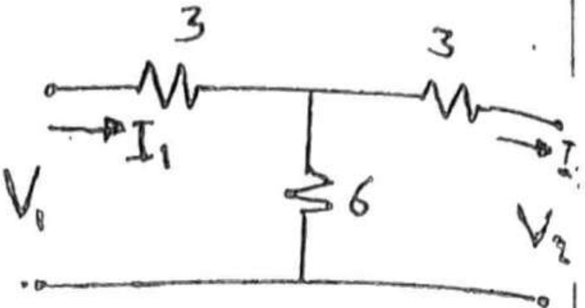
$I_2' = y_{21}'V_1 - y_{22}'V_2$ & $I_2'' = y_{21}''V_1 - y_{22}''V_2$

$\therefore I_2 = (y_{21}' + y_{21}'')V_1 - (y_{22}' + y_{22}'')V_2$

$\therefore y_{21} = y_{21}' + y_{21}''$ & $y_{22} = y_{22}' + y_{22}''$

Example:

Find the ABCD parameters for the shown two-port circuit.



$$Z_{10} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 3+6 = 9\,\Omega.$$

$$Z_{1S} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 3 + \frac{3\times 6}{9} = 5\,\Omega.$$

$$Z_{2S} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = 3+2 = 5\,\Omega.$$

$$\therefore \quad A = \sqrt{\frac{Z_{10}\,Z_{1S}}{Z_{2S}(Z_{10}-Z_{1S})}} = \sqrt{\frac{9\times 5}{5(9-5)}} = 1.5$$

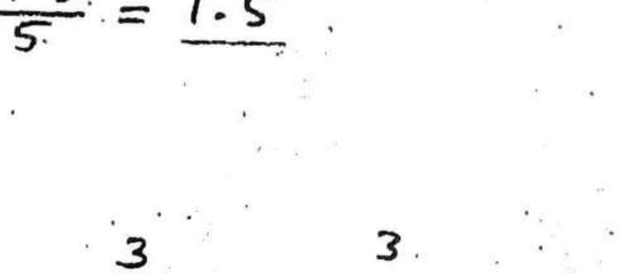
$$Z_{10} = \frac{A}{C} \qquad \therefore C = \frac{A}{Z_{10}} = 0.1666\ \mho$$

$$Z_{2S} = \frac{B}{A} \qquad \therefore B = Z_{2S}\times A = 7.5\,\Omega.$$

$$Z_{1S} = \frac{B}{D} \qquad \therefore D = \frac{B}{Z_{1S}} = \frac{7.5}{5} = 1.5$$

$$V_1 = 1.5\,V_2 + 7.5\,I_2.$$
$$I_1 = 0.166\,V_2 + 1.5\,I_2$$

** $I_1 = I_2 + I_3$



$$I_1 = I_2 + \frac{V_2 + 3I_2}{6}.$$

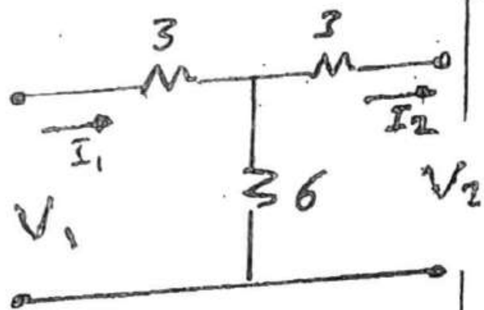
$$\therefore I_1 = 0.166\,V_2 + 1.5\,I_2.$$

and

$$V_1 = 3I_1 + 3I_2 + V_2.$$

$$= 3(0.166V_2 + 1.5I_2) + 3I_2 + V_2.$$

$$\therefore V_1 = 1.5V_2 + 7.5I_2.$$



$$V_1 = Z_{11}I_1 - Z_{12}I_2.$$
$$V_2 = Z_{21}I_1 - Z_{22}I_2.$$

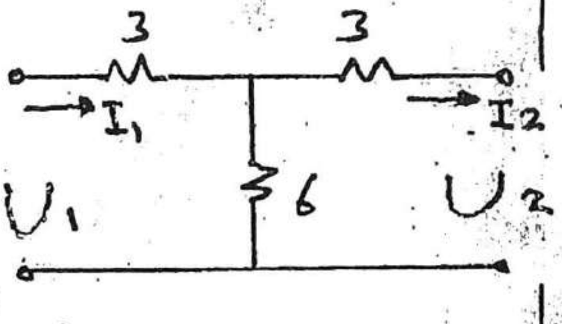
$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = 3+6 = \underline{9\,\Omega}.$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} = \frac{V_1(6/9)}{I_1} = \frac{6\times 9}{9} = \underline{6\,\Omega}.$$

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0} = 3+6 = \underline{9\,\Omega}.$$

$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} = \frac{V_2(6/9)}{I_2} = \underline{6\,\Omega}$$

$$\boxed{\begin{aligned} V_1 &= 9I_1 - 6I_2. \\ V_2 &= 6I_1 - 9I_2 \end{aligned}}$$



$$I_1 = y_{11}V_1 - y_{12}V_2$$
$$I_2 = y_{21}V_1 - y_{22}V_2.$$

$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = \frac{1}{5} = 0.2\,\mho$$

$$y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} = \frac{I_1(6/9)}{V_1} = \frac{1}{5}\times\frac{6}{9} = 0.1333\,\mho$$

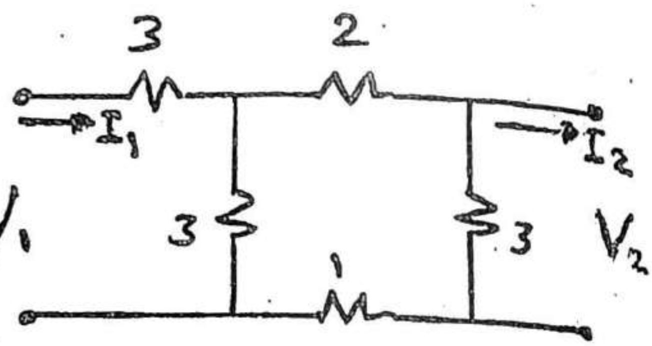
$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{5} = 0.2 \ \mho.$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{I_2(6/9)}{V_2} = \frac{1}{5}\times\frac{6}{9} = 0.1333 \ \mho.$$

$$\therefore \boxed{\begin{aligned} I_1 &= 0.2V_1 - 0.133V_2 \\ I_2 &= 0.133V_1 - 0.2V_2 \end{aligned}}$$

Example:

Find the ABCD parameters.



$$Z_{10} = 3 + \frac{3\times 6}{3+6} = 5\ \Omega.$$

$$Z_{15} = 3 + \frac{3\times 3}{3+3} = 4.5\ \Omega.$$

$$Z_{25} = \left[(3//3)+3\right]//3 = 1.8\ \Omega.$$

$$A = \sqrt{\frac{5\times 4.5}{1.8(5-4.5)}} = \underline{5}.$$

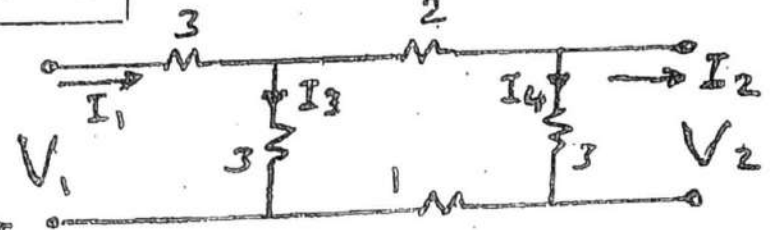
$$Z_{10} = \frac{A}{C} \qquad \therefore C = \underline{1\ \mho}.$$

$$Z_{25} = \frac{B}{A} \qquad \therefore B = \underline{9\ \Omega}.$$

$$Z_{15} = \frac{B}{D} \qquad \therefore D = \underline{2}.$$

$$\therefore \boxed{\begin{aligned} V_1 &= 5V_2 + 9I_2. \\ I_1 &= V_2 + 2I_2. \end{aligned}}$$

**



$$I_1 = I_2 + I_3 + I_4.$$

$$= I_2 + \frac{V_2}{3} + \frac{V_2 + 3(I_2 + V_2/3)}{3}.$$

$$\therefore I_1 = I_2 + \frac{V_2}{3} + \frac{V_2}{3} + I_2 + \frac{V_2}{3}.$$

$$\therefore \boxed{I_1 = V_2 + 2I_2.}$$

and;

$$V_1 = 3I_1 + V_2 + 3\left(I_2 + \frac{V_2}{3}\right).$$

$$= 3I_1 + V_2 + 3I_2 + V_2.$$

$$= 3(V_2 + 2I_2) + 2V_2 + 3I_2.$$

$$\therefore \boxed{V_1 = 5V_2 + 9I_2.}$$